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Lag synchronization of uncertain complex dynamical networks with derivative coupling

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Abstract

In this study, uncertain complex dynamical network model with time varying coupling delay and derivative coupling delay is considered. The lag synchronization between two such uncertain networks with different nodes is investigated. An adaptive control method is designed by using Lyapunov stability theory for achieving the lag synchronization and some corollaries are also given. In addition, on the basis of the adaptive update law, unknown parameters of the networks are estimated. The analytical results show that the states of the dynamical network with derivative delay coupling can be asymptotically synchronized under the designed control. The numerical simulation results also demonstrate the validity of the designed method.

Keywords: Lag synchronization, derivative coupling, complex dynamical networks, adaptive control.

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1. Introduction

In recent decades, many researchers have concentrated on synchronization behavior of dynamical network, due to its ability to explain many natural phenomenons and its application in different disciplines [3, 18, 20]. When the synchronization of complex networks cannot be attained, many control schemes have been designed effectively for achieving network synchronization. In fact, after the pioneering work of Pecora and Carroll was published, many researches have been put great efforts to investigate network synchronization phenomenon [4, 6, 11, 12, 16, 17, 19, 21, 22, 29]. In these studies, lag synchronization behavior is one of the most interesting type. This behavior has appeared in lasers, neural models, electronic applications, and secure communication [7, 13], which can cause instability and poor performance. Therefore, lag synchronization has become a hot topic in the research of complex networks and many works have been presented [1, 14, 23, 24, 31].

It should be pointed out that all the above mentioned results are concerned with the synchronization in dynamical networks when the system parameters are well known beforehand. In practical implementation, knowing the exact values of the systems parameters is difficult which may cause undesirable

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dynamic behaviors and destroy the systems stability. Thus, the main motivation of this work is to study how to synchronize the networks with unknown parameters and estimate theses parameters effectively.

On the other hand, adaptive control method is an effective way to estimate the unknown parameters due to its advantages leading to stability. Therefore, it has been designed and applied effectively to explore the synchronization behavior in dynamical network with unknown parameters. In [25], exponential outer synchronization problem between two uncertain nonlinearly coupled networks with constant time delays was discussed. Ji et al. examined lag synchronization in drive-response uncertain complex dynamical network having delayed coupling and unknown parameters by adaptive control [8]. Based on a hybrid feedback control, lag synchronization in drive-response dynamical networks with non-delay coupling and unknown parameters was discussed in [15]. In [2], adaptive control method was designed to achieve the projective lag synchronization in DRDN with constant and time-varying coupling delay, also the uncertain parameters of both the network node and drive system were identified.

For a better way of describing the real world, network model should also include information of the past change rate of the nodes, such as population ecology, biologic system and ecosystem [10], where each network is shown by the present and historical fluctuating rate information. In [26], Xu et al. were the first to explore the synchronization of complex networks with derivative and non-derivative coupling. Following that pioneering work, many studies have been examined. Topological structure identification, pinning synchronization, finite-time, and pinning impulsive synchronization were studied in [5, 27, 28, 30]. In [9], Jian et al. studied the synchronization of dynamical network with time varying coupling delay and derivative coupling. To the best of our knowledge, we have not come across any theoretical results considering the problem of lag synchronization between two uncertain dynamical networks with time varying delay non-derivative and derivative coupling.

In the light of the above discussion, delayed uncertain complex dynamical network model with derivative and non-derivative coupling is proposed in this paper. The adaptive control method is developed for investigating lag synchronization and the unknown parameters are estimated. Numerical simulations results are given to prove the efficiency of the designed control.

The paper is organized as follows. Section 2 introduced the network model and some necessary preliminaries. The main results are given and novel criteria are derived in Section 3. Section 4 presented examples and their simulations. Finally, the conclusions are drawn in Section 5.

2. Model description

A general complex dynamical network model consisting of N linearly coupled nodes with uncertain parameters and delay derivative coupling can be described as

$$\dot{x_i}(t) = f_i(x_i(t)) + F_i(x_i(t))\alpha_i + \sum_{j=1}^{N} a_{ij}\Gamma x_j(t-\eta(t)) + \sum_{j=1}^{N} b_{ij}\Gamma \dot{x_j}(t-\eta(t)), \tag{2.1}$$

Here, $i \in \mathbf{N} \triangleq \{1, 2, \dots, N\}$, N corresponds to the number of units in the delayed network dynamic system, $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbf{R}^n$ denotes the state vector of the i^{th} node, and $f_i : \mathbf{R}^n \longrightarrow \mathbf{R}^n$ and $F_i : \mathbf{R}^n \longrightarrow \mathbf{R}^{n\times m_i}$ are the continuous nonlinear function matrices. The α_i's are the unknown constant parameter vector, $\eta(t) \geqslant 0$ is the time varying coupling delay and Γ is the inner coupling matrix. Also $A = (a_{ij}) \in \mathbf{R}^{N\times N}$ and $B = (b_{ij}) \in \mathbf{R}^{N\times N}$ are the coupling configuration matrices representing the coupling weights and topological structure for non delayed configuration and delayed one, where the diagonal elements of of them are defined as

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}, \quad b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij} \quad i = 1, 2, \dots, N.$$

We refer to model (2.1) as the drive network, and a response network is described as

$$y_i(t) = g_i(y_i(t)) + G_i(y_i(t))\beta_i + \sum_{j=1}^{N} a_{ij}\Gamma y_j(t-\eta(t)) + \sum_{j=1}^{N} b_{ij}\Gamma y_j(t-\eta(t)) + u_i(t), \tag{2.2}$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \dots, y_{in}(t))^T \in \mathbf{R}^n$ is response state of the i^{th} node, and $g_i : \mathbf{R}^n \longrightarrow \mathbf{R}^n$ and $G_i : \mathbf{R}^n \longrightarrow \mathbf{R}^{n\times m_i}$ are the continuous nonlinear function matrices. The β_i's are the unknown constant parameter vector and $u_i \in \mathbf{R}^n$ is the control input.

Remark 2.1. Our network model has unknown parameters. Therefore, it is different from the model that was studied in [9].

Remark 2.2. Assume the model do not contain unknown parameters and $\eta(t) = 0$. Then, we can get the complex network model which was when the controller u_i can be pinning control [5], finite time control [27], and pinning impulsive control [30].

Define the lag synchronization error as

$$e_i(t) = y_i(t) - x_i(t-\tau), \quad i = 1, \dots, N,$$

where $\tau > 0$ is a constant representing time delay or lag. Our objective in this paper is to design the controller $u_i(t)$ that makes the drive network and response network asymptotically synchronized, i.e,

$$\lim_{t\longrightarrow\infty} \|y_i(t) - x_i(t-\tau)\| = 0.$$

Assumption 2.3 ([6])**.** *Time delay* $\eta(t)$ *is a differentiable function with* $0 \leqslant \dot{\eta}(t) \leqslant \varepsilon < 1$. *Clearly, this assumption is certainly ensured if the coupling delay* $\eta(t)$ *is a constant.*

Lemma 2.4 ([16])**.** *For any vector* $x, y \in \mathbf{R}^n$ *and positive definite matrix* $Q \in \mathbf{R}^{n\times n}$, *the following matrix inequality holds,*

$$2x^T y \leqslant x^T Q x + y^T Q^{-1} y.$$

3. Main results

In this section, we design an adaptive control method for achieving lag synchronization between two uncertain complex dynamical networks with time varying delayed coupling and derivative delayed coupling.

According to the networks (2.1) and (2.2), the error dynamical network for lag synchronization can be obtained as following:

$$\begin{aligned} \dot{e}_i(t) = {} & g_i(y_i(t)) + G_i(y_i(t))\beta_i + \sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\eta(t)) + \sum_{j=1}^{N} b_{ij}\Gamma \dot{e}_j(t-\eta(t)) \\ & - \Big(f_i(x_i(t-\tau)) + F_i(x_i(t-\tau))\alpha_i\Big) + u_i(t). \end{aligned} \tag{3.1}$$

Theorem 3.1. *Suppose that Assumption (2.3) holds. If there exist positive constant* τ, *the drive network and response network can be achieve lag synchronization by using the following controllers*

$$\begin{aligned} u_i(t) = {} & f_i(x_i(t-\tau)) + F_i(x_i(t-\tau))\hat{\alpha}_i(t) - g_i(y_i(t)) - G_i(y_i(t))\hat{\beta}_i(t) - \omega_i(t)e_i(t) \\ & - \sum_{j=1}^{N} r_i(t)\Gamma \dot{e}_j(t-\eta(t)), \end{aligned} \tag{3.2}$$

$$\dot{\hat{\alpha}}_i(t) = -\kappa_1 F_i^T(x_i(t-\tau))e_i(t), \tag{3.3}$$

$$\dot{\hat{\beta}}_i(t) = \kappa_2 G_i^T(y_i(t))e_i(t), \tag{3.4}$$
$$\dot{\omega}_i(t) = \kappa_3 e_i^T(t)e_i(t), \tag{3.5}$$
$$\dot{r}_i(t) = \kappa_4 e_i^T(t)\Gamma\dot{e}_i(t-\eta(t)), \tag{3.6}$$

where $\kappa_1, \kappa_2, \kappa_3$, and κ_4 are positive constants. Here $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$ are the estimated parameters for the drive network (2.1) and response network (2.2), respectively.

Proof. Select a Lyapunov function candidate as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2\kappa_1}\sum_{i=1}^{N}\tilde{\alpha}_i^T(t)\tilde{\alpha}_i(t) + \frac{1}{2\kappa_2}\sum_{i=1}^{N}\tilde{\beta}_i^T(t)\tilde{\beta}_i(t) + \frac{1}{2\kappa_3}\sum_{i=1}^{N}(\omega_i(t)-\omega_i^*)^2$$
$$+ \frac{1}{2\kappa_4}\sum_{i=1}^{N}(r_i(t) - \sum_{i=1}^{N} b_{ij})^2 + \frac{1}{2(1-\varepsilon)}\int_{t-\eta(t)}^{t}\sum_{i=1}^{N} e_i^T(s)e_i(s)ds,$$

where $\tilde{\alpha}_i(t) = \hat{\alpha}_i(t) - \alpha$, $\tilde{\beta}_i(t) = \hat{\beta}_i(t) - \beta_i$, and ω_i^* is positive constant.

The time derivative of V is obtained as

$$\dot{V} = \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \frac{1}{\kappa_1}\sum_{i=1}^{N}\dot{\hat{\alpha}}_i^T(t)\tilde{\alpha}_i(t) + \frac{1}{\kappa_2}\sum_{i=1}^{N}\dot{\hat{\beta}}_i^T(t)\tilde{\beta}_i(t) + \frac{1}{\kappa_3}\sum_{i=1}^{N}(\omega_i(t)-\omega_i^*)\dot{\omega}_i(t)$$
$$+ \frac{1}{\kappa_4}\sum_{i=1}^{N}(r_i(t) - \sum_{i=1}^{N} b_{ij})\dot{r}_i(t) + \frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N} e_i^T(t)e_i(t)$$
$$- \frac{1-\dot{\eta}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N} e_i^T(t-\eta(t))e_i(t-\eta(t)).$$

Apply of the control function (3.2) to error dynamics (3.1), we obtain

$$\dot{V} = \sum_{i=1}^{N} e_i^T(t)\Big(F_i(x_i(t-\tau))\tilde{\alpha}_i(t) - G_i(y_i(t))\tilde{\beta}_i(t) + \sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\eta(t))$$
$$+ \sum_{j=1}^{N} b_{ij}\Gamma\dot{e}_j(t-\eta(t)) - \omega_i(t)e_i(t) - \sum_{j=1}^{N} r_i(t)\Gamma\dot{e}_j(t-\eta(t))\Big)$$
$$+ \frac{1}{\kappa_1}\sum_{i=1}^{N}\dot{\hat{\alpha}}_i^T(t)\tilde{\alpha}_i(t) + \frac{1}{\kappa_2}\sum_{i=1}^{N}\dot{\hat{\beta}}_i^T(t)\tilde{\beta}_i(t) + \frac{1}{\kappa_3}\sum_{i=1}^{N}(\omega_i(t)-\omega_i^*)\dot{\omega}_i(t)$$
$$+ \frac{1}{\kappa_4}\sum_{i=1}^{N}(r_i(t) - \sum_{i=1}^{N} b_{ij})\dot{r}_i(t) + \frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N} e_i^T(t)e_i(t)$$
$$- \frac{1-\dot{\eta}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N} e_i^T(t-\eta(t))e_i(t-\eta(t)).$$

By the adaptation updating laws (3.3)-(3.6), we have

$$\dot{V} = \sum_{i=1}^{N} e_i^T(t) + \sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\eta(t)) - \omega_i^*\sum_{i=1}^{N} e_i^T(t)e_i(t)$$
$$+ \frac{1}{2(1-\varepsilon)}\sum_{i=1}^{N} e_i^T(t)e_i(t) - \frac{1-\dot{\eta}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N} e_i^T(t-\eta(t))e_i(t-\eta(t)).$$

Let us define $\Omega^* = \mathrm{diag}(\omega_1^*, \omega_2^*, \dots, \omega_N^*)$, $P = (A \otimes \Gamma)$, $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$. Then we have

$$\dot{V} = -e^T(t)\Omega^* e(t) + e(t)^T P e(t-\eta(t)) + \frac{1}{2(1-\varepsilon)} e^T(t)e(t) - \frac{1-\dot{\eta}(t)}{2(1-\varepsilon)} e^T(t-\eta(t))e(t-\eta(t)).$$

Using Lemma (2.4), we have

$$\begin{aligned}\dot{V} \leqslant{} & -e^T(t)\Omega^* e(t) + \frac{1}{2}e(t)^T P P^T e(t) + \frac{1}{2}e^T(t-\eta(t))e(t-\eta(t)) + \frac{1}{2(1-\varepsilon)}e^T(t)e(t) \\ & - \frac{1-\dot{\eta}(t)}{2(1-\varepsilon)} e^T(t-\eta(t))e(t-\eta(t)).\end{aligned}$$

From Assumption (2.3), we get

$$\frac{1-\dot{\eta}(t)}{2(1-\varepsilon)} \geqslant \frac{1}{2}.$$

Thus, we obtain

$$\dot{V} \leqslant e^T(t)\Big(\frac{1}{2}PP^T + \frac{1}{2(1-\varepsilon)} - \Omega^*\Big)e(t).$$

Taking $\Omega_i^* = \frac{1}{2}PP^T + \frac{1}{2(1-\varepsilon)} + 1$, we obtain

$$\dot{V} \leqslant -e(t)^T e(t).$$

Based on Lyapunov stability theory, the error dynamics $e_i(t) \to 0$ as $t \to \infty$. That means the drive network (2.1) and response network (2.2) with delay derivative coupling achieve lag synchronization and the unknown parameters can be successfully estimated via adaptive control (3.2) and updating laws (3.3)-(3.6). □

Remark 3.2. When the model does not contain derivative coupling and $\eta(t)$ is constant, then lag synchronization between uncertain drive-response complex dynamical network with non-delay coupling was discussed in Ji et al. [8] by adaptive control.

Corollary 3.3. For any given positive propagation delay τ, if $\eta(t) = \eta$, then the two networks can achieve lag synchronization under the following controllers

$$\begin{aligned} u_i(t) ={} & f_i(x_i(t-\tau)) + F_i(x_i(t-\tau))\hat{\alpha}_i(t) - g_i(y_i(t)) - G_i(y_i(t))\hat{\beta}_i(t) - \omega_i(t)e_i(t) \\ & - \sum_{j=1}^{N} r_i(t)\Gamma\dot{e}_j(t-\eta), \end{aligned} \tag{3.7}$$

$$\dot{\hat{\alpha}}_i(t) = -\kappa_1 F_i^T(x_i(t-\tau))e_i(t), \tag{3.8}$$

$$\dot{\hat{\beta}}_i(t) = \kappa_2 G_i^T(y_i(t))e_i(t), \tag{3.9}$$

$$\dot{\omega}_i(t) = \kappa_3 e_i^T(t)e_i(t), \tag{3.10}$$

$$\dot{r}_i(t) = \kappa_4 e_i^T(t)\Gamma\dot{e}_i(t-\eta). \tag{3.11}$$

Corollary 3.4. For any given positive delay τ, if $A = 0$ then the two networks can achieve lag synchronization under the following controllers

$$\begin{aligned} u_i(t) &= f_i(x_i(t-\tau)) + F_i(x_i(t-\tau))\hat{\alpha}_i(t) - g_i(y_i(t)) - G_i(y_i(t))\hat{\beta}_i(t) - \sum_{j=1}^{N} r_i(t)\Gamma\dot{e}_j(t-\eta(t)), \\ \dot{\hat{\alpha}}_i(t) &= -\kappa_1 F_i^T(x_i(t-\tau))e_i(t), \\ \dot{\hat{\beta}}_i(t) &= \kappa_2 G_i^T(y_i(t))e_i(t), \\ \dot{r}_i(t) &= \kappa_4 e_i^T(t)\Gamma\dot{e}_i(t-\eta(t)). \end{aligned}$$

4. Numerical analysis

In this section, numerical examples are given to show the effective of the designed control methods obtained in the previous section. In the numerical simulations, the node of the drive dynamical equations are taken as the Lü chaotic system, which is given by

$$\dot{x}_1(t) = \alpha_1(x_2(t) - x_1(t)), \quad \dot{x}_2(t) = \alpha_3 x_2(t) - x_1(t)x_3(t), \quad \dot{x}_3(t) = x_1(t)x_2(t) - \alpha_2 x_3(t).$$

The node of the response dynamics described by the following Chen chaotic system

$$\dot{y}_1(t) = \beta_1(y_2(t) - y_1(t)), \quad \dot{y}_2(t) = (\beta_3 - \beta_1)y_1(t) + \beta_3 y_2(t) - y_1(t)y_3(t), \quad \dot{y}_3(t) = y_1(t)y_2(t) - \beta_2 x_3(t),$$

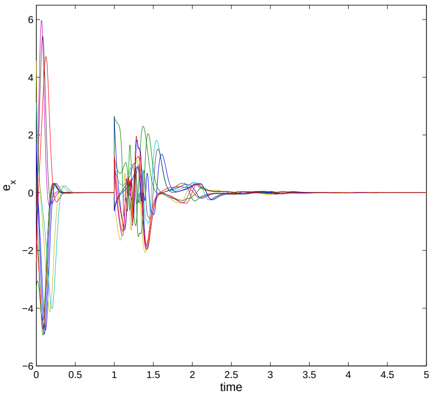
where the unknown parameter vectors are $\alpha_i = [\alpha_1 \;\; \alpha_2 \;\; \alpha_3]^T = [36 \;\; 3 \;\; 20]^T$, $\beta_i = [\beta_1 \;\; \beta_2 \;\; \beta_3]^T = [35 \;\; 3 \;\; 28]^T$. We take the propagation delay as $\tau = 1$, the inner coupling matrix Γ as the identity matrix and the outer coupling matrices as following:

$$A = \begin{pmatrix} -7 & 1 & 0 & 0 & 2 & 0 & 1 & 0 & 2 & 1 \\ 0 & -4 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -5 & 0 & 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -4 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & -4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & -4 & 0 & 1 \\ 2 & 0 & 1 & 1 & 0 & 1 & 0 & 2 & -8 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 \end{pmatrix},$$

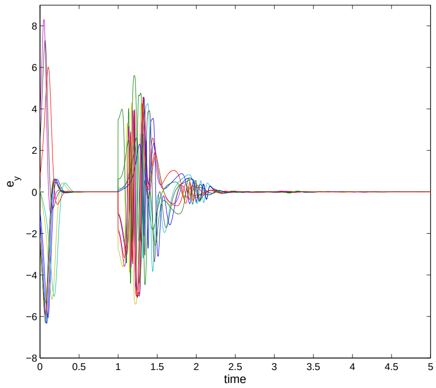
$$B == \begin{pmatrix} -4 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & -5 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & -6 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -4 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & -7 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & -5 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & -6 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & -5 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & -4 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -5 \end{pmatrix}.$$

According to Theorem 3.1, the adaptive laws gains are $\kappa_1 = \kappa_2 = \kappa_3 = 1$ and $\kappa_4 = 0.3$. The time-varying coupling delay is chosen as $\eta(t) = \frac{\exp(t)}{2(1+\exp(t))}$, then $\dot{\eta}(t) = \frac{\exp(t)}{2(1+\exp(t))^2} \in (0, \frac{1}{2}]$, where the Assumption 2.3 holds. The initial values are $\hat{\alpha}_i = \hat{\beta}_i = 0, \omega_i = 2, r_i = 3$. We take the initial states as $x_i(0)$ and $y_i(0)$ are randomly chosen. The numerical results are presented in Fig. 1 and Fig. 2. The lag synchronization error is depicted in Fig. 1, showing that the lag synchronization between the drive and response networks is achieved. Fig. 2 ((a) and (b)) shows the identification of the uncertain parameters $\tilde{\alpha}$ and $\tilde{\beta}$ converge to their real values, which means that the unknown parameters are successfully estimated. These results prove the effectiveness of our designed control (3.2) with adaptive law (3.3)-(3.6) for uncertain complex dynamical networks with time varying delay coupling and delay derivative coupling.

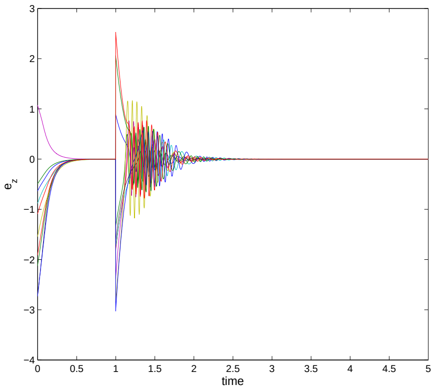
According to corollary 3.3, when the delay coupling is constant, we choose $\eta = 0.1$ and the adaptive laws gains are $\kappa_1 = \kappa_2 = 4, \kappa_3 = 1$ and $\kappa_4 = 0.3$. The initial values are $\hat{\alpha}_i = \hat{\beta}_i = 0, \omega_i = 4, r_i = 3$. We take the initial states as $x_i(0)$ and $y_i(0)$ are chosen randomly. In numerical simulation, the lag synchronization error is depicted in Fig. 3, which displays $e \longrightarrow 0$ with $t \longrightarrow \infty$. That means the required lag synchronization has been achieved with our designed control (3.7). The estimated parameters of the drive network nodes and response network nodes are shown in Fig. 4 ((a) and (b)) respectively, successfully estimated. These results prove the effectiveness of our designed control (3.7) with adaptive law (3.8)-(3.11) for uncertain complex dynamical networks with constant delay coupling and derivative coupling.



(a)

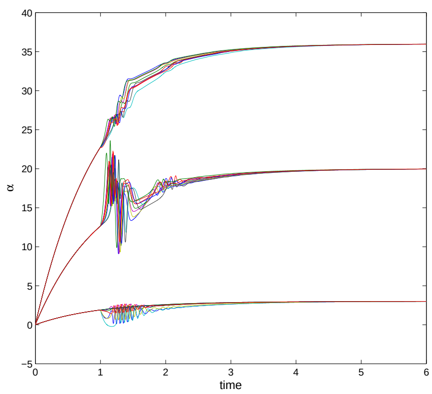


(b)

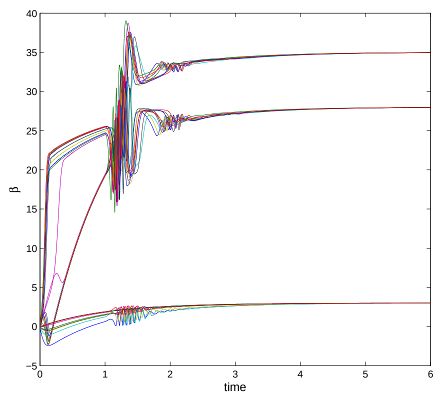


(c)

Figure 1: Time evolution of the lag synchronization error with time varying delay.

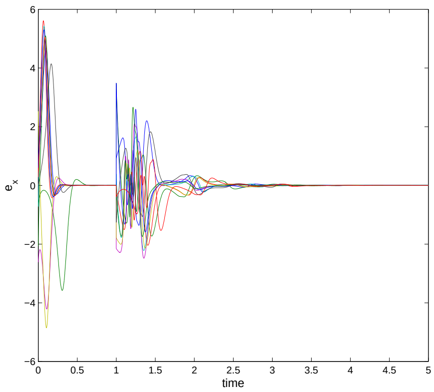


(a)

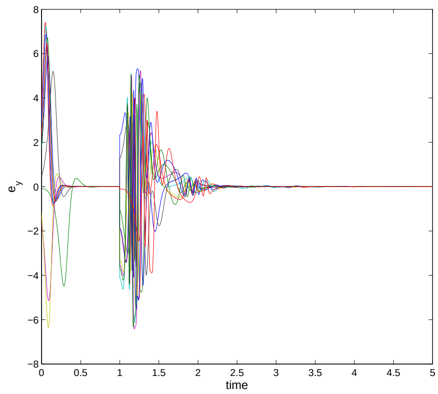


(b)

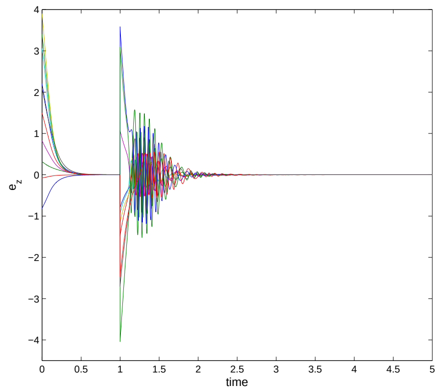
Figure 2: The estimated unknown parameter of (a) $\hat{\alpha}$, (b)$\hat{\beta}$.



(a)

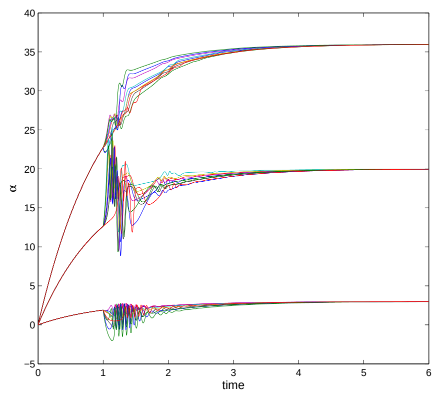


(b)

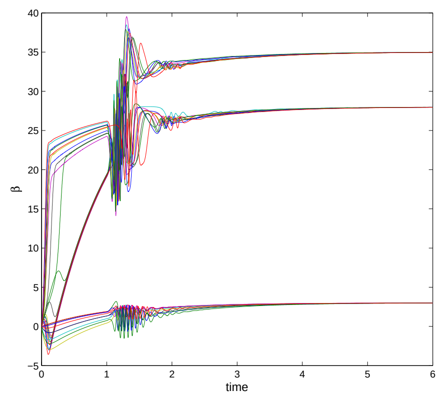


(c)

Figure 3: Time evolution of the lag synchronization error with constant delay coupling.



(a)



(b)

Figure 4: The estimated unknown parameter of (a): $\hat{\alpha}$; (b): $\hat{\beta}$.

5. Conclusion

In this paper, we explored a general uncertain complex dynamical networks model with time varying delayed coupling and derivative coupling delay. The adaptive lag synchronization method was studied between uncertain complex dynamical networks with different nodes. Based on the Lyapunov stability theory and adaptive control, lag synchronization criterion was obtained and the unknown parameters were identified. The numerical simulation results showed the efficiency of the proposed method.

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