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Timer option pricing of stochastic volatility model with changing coefficients under time-varying interest rate

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Abstract

Considering economic variables changing from time to time, the time-varying models can fit the financial data better. In this paper, we construct stochastic volatility models with time-varying coefficients. Furthermore, the interest rate risk is one of important factors for timer options pricing. Therefore, we study the timer options pricing for stochastic volatility models with changing coefficients under time-varying interest rate. Firstly, the partial differential equation boundary value problem is given by using Δ-hedging approach and replicating a timer option. Secondly, we obtain the joint distribution of the variance process and the random maturity under the risk neutral probability measure. Thirdly, the explicit formula of timer option pricing is proposed which can be applied to the financial market directly. Finally, numerical analysis is conducted to show the performance of timer option pricing proposed.

Keywords: Timer option pricing, stochastic volatility model, risk neutral measure, Δ-hedging, time-varying interest rate.

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1. Introduction

Timer options are barrier style options, which depend on the realized volatility of the underlying asset. Timer call (put) options entitle investors to the right to buy (sell) the underlying asset at random maturity when occurs at the first time that a prescribed variance budget is exhausted. In 2007, timer options were first traded by Société Généralal Corporate and Investment Banking. In fact, timer options were discussed in academic literature before they appeared in the financial market. For example, Neuberger [13] proposed the "mileage option" and Bick [2] studied timer options for the continuous time models and so on.

Stochastic volatility models are widely used in option pricing (see [3, 4, 8–10] and so on). In recent years, timer options pricing for stochastic volatility have been proposed by lots of researchers. Carr and Lee [5] discussed the timer options under the risk-free rate being zero. Saunders [14] studied an

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asymptotic expansion of time options for fast mean reverting stochastic volatility models. Li and Mercurio [12] proposed the time options pricing for general stochastic volatility models around small volatility of variance. Li [11] proposed a Black-Scholes-Merton-type formula of timer options pricing by using the joint distribute of the first-passage time of the realized variance and the corresponding variance.

To the best of our knowledge, all the literatures about time option pricing assume that the interest rates and the coefficients in models are both constants. However, due to economic variables are varying time to time, their models do not fully fit all financial data well. Furthermore, the interest rate is an important risk factor for time options (see [1]). Those motivate us to consider a large class of stochastic volatility models, such as considering time-varying coefficient stochastic volatility models. In this paper, we study the timer option pricing for stochastic volatility models with changing coefficients under time varying interest rate. We first obtain the partial differential equation boundary value problem by using replicating a timer option and Δ-hedging approach. Then we propose the explicit formula of timer option pricing through the joint distribution of the variance process and the first-passage time of the realized variance.

The organization of the rest of this paper is as follows. In section 2, we construct the stochastic volatility models with time-varying coefficients and propose the price of timer option for our models. The explicit formula of timer option pricing is obtained in Section 3. In Section 4, we construct numerical analysis for timer option pricing. Conclusions are given in Section 5.

2. Model and timer option pricing

On a filtered probability space $(\Omega, F, (F_t), P)$, assume that the asset S_t and its volatility V_t satisfy the following stochastic volatility model with changing coefficients:

$$dS_t = \mu(t)S_t dt + \sqrt{V_t}S_t(\rho dW_t^{(1)} + \sqrt{1-\rho^2}dW_t^{(2)}), \tag{2.1}$$

$$dV_t = \{\theta(t) - \zeta V_t\}dt + \sigma_v\sqrt{V_t}dW_t^{(1)}, \tag{2.2}$$

where $\mu(t)$ means the return of the asset at time t. $W_t^{(1)}$ and $W_t^{(2)}$ are Brownian motion, ζ is the speed of mean reversion of V_t, $\theta(t)$ is a drift function of V_t, σ_v is a function reflecting the volatility of V_t, and ρ is the correlation coefficient of $W_t^{(1)}$ and $W_t^{(2)}$.

In this paper, we suppose that the market is completed. Let Q be a risk-neutral probability measure. Under the measure Q, equations (2.1) and (2.2) can be expressed as the following.

$$dS_t = r(t)S_t dt + \sqrt{V_t}S_t(\rho dB_t^{(1)} + \sqrt{1-\rho^2}dB_t^{(2)}), \tag{2.3}$$

$$dV_t = (\alpha(t) - \beta V_t)dt + \sigma_v\sqrt{V_t}dB_t^{(1)}, \tag{2.4}$$

where $r(t)$ is the risk-free interest rate at time t. $B_t^{(1)}$ and $B_t^{(2)}$ are standard Brownian motion. We assume that B is a pre-specified variance budget. Denote τ as the random time that the first time when the realized variance exceeds the level B, i.e.,

$$\tau = \inf\{t > 0 : \int_0^t V_u du = B\}. \tag{2.5}$$

The price of variance swap $G_t = G(t, V_t, I_t)$ satisfies the following PDE (see [4]),

$$\frac{\partial G}{\partial t} + (\alpha(t) - \beta v)\frac{\partial G}{\partial v} + v\frac{\partial G}{\partial o} + \frac{1}{2}\sigma_v^2 v\frac{\partial^2 G}{\partial v^2} = r(t)G, \quad \text{and} \quad d(e^{-\int_0^t r(s)ds}G_t) = e^{-\int_0^t r(s)ds}\frac{\partial G}{\partial v}\sigma_v\sqrt{V_t}dB_t^{(1)},$$

where I_t is the accumulated variance, that is $I_t = \int_0^t V_s ds$. Suppose that an investor holds Δ_t^S shares of the asset with price G_t at time t. Let $\Pi_t - \Delta_t^S S_t - \Delta_t^G G_t$ is the remainder of the portfolio value, which is fully invested in the risk-free market. We can replicate a timer option. According to the portfolio being self-financing, we obtain

$$d\Pi_t = \Delta_t^S dS_t + \Delta_t^G dG_t + r(t)(\Pi_t - \Delta_t^S S_t - \Delta_t^G G_t)dt. \tag{2.6}$$

Equation (2.6) is equivalent to

$$\begin{aligned} d(e^{-\int_0^t r(s)ds}\Pi_t) &= e^{-\int_0^t r(s)ds}\Big[\Delta_t^S(dS_t - r(t)S_t dt) + \Delta_t^G(dG_t - r(t)G_t dt)\Big] \\ &= e^{-\int_0^t r(s)ds}\Big[\Delta_t^G \frac{\partial G}{\partial v}\sigma_v\sqrt{V_t}dB_t^{(1)} + \Delta_t^S\sqrt{V_t}S_t(\rho dB_t^{(1)} + \sqrt{1-\rho^2}dB_t^{(2)})\Big]. \end{aligned}$$

Let K be struck price of the timer option. The payoff function $H(s)$ of the timer option can be expressed as $H(s) = \max\{K - s, 0\}$ for a timer put option and $H(s) = \max\{s - K, 0\}$ for a timer call option. Denote $P_t = P(t, s, v, x)$ by the price of the timer option. We obtain

$$\begin{aligned} d(e^{-\int_0^t r(s)ds}P_t) = e^{-\int_0^t r(s)ds}\Big[&\frac{\partial P}{\partial t} + (\alpha(t) - \beta V_t)\frac{\partial P}{\partial v} + r(t)S_t\frac{\partial P}{\partial S} + V_t\frac{\partial P}{\partial x} \\ &+ \frac{1}{2}\sigma_v^2 V_t\frac{\partial^2 P}{\partial v^2} + \frac{1}{2}S_t^2 V_t\frac{\partial^2 P}{\partial s^2} + \rho\Delta_v S_t V_t\frac{\partial^2 U}{\partial s\partial v} - r(t)\rho\Big]dt \\ &+ e^{-\int_0^t r(s)ds}\Big[\frac{\partial P}{\partial v}\sigma_v\sqrt{V_t}dB_t^{(1)} + \frac{\partial P}{\partial s}\sqrt{V_t}S_t(\rho dB_t^{(1)} + \sqrt{1-\rho^2}dB_t^{(2)})\Big]. \end{aligned}$$

According to the following replicating strategy $\Delta_t^S = \partial P/\partial s$ and $\Delta_t^G = \frac{\partial P}{\partial v}/\frac{\partial F}{\partial v}$ replication yields that

$$d(e^{-\int_0^t r(s)ds}P_t) = d(e^{-\int_0^t r(s)ds}\Pi_t),$$

which results the following PDE

$$\frac{\partial P}{\partial t} + (\alpha(t) - \beta v)\frac{\partial P}{\partial v} + r(t)s\frac{\partial P}{\partial s} + v\frac{\partial U}{\partial x} + \frac{1}{2}\sigma_v^2 v\frac{\partial P^2}{\partial v^2} + \frac{1}{2}s^2 v\frac{\partial P^2}{\partial s^2} + \rho\sigma_v sv\frac{\partial P^2}{\partial s\partial v} - ru = 0, \quad P(t, s, v, B) = H(s).$$

Theorem 2.1. *Under models (2.3) and (2.4), the initial arbitrage-free price of the timer put option and the timer call option satisfy*

$$\begin{aligned} P_0 &= E^Q\Big[e^{-\int_0^\tau r(s)ds}\max(K - S_\tau, 0)\Big] \\ &= E^Q\Big[Ke^{-\int_0^\tau r(s)ds}\Phi\big(-d_2(V_\tau, \tau)\big) - S_0\Big(1 - e^{d_0(V_\tau,\tau)}\Phi\big(d_1(V_\tau, \tau)\big)\Big)\Big], \end{aligned} \tag{2.7}$$

and

$$C_0 = E^Q\Big[e^{-\int_0^\tau r(s)ds}\max(S_\tau - K, 0)\Big] = E^Q\Big[S_0 e^{d_0(V_\tau,\tau)}\Phi\big(d_1(V_\tau, \tau)\big) - Ke^{-\int_0^\tau r(s)ds}\Phi\big(d_2(V_\tau, \tau)\big)\Big], \tag{2.8}$$

respectively, where τ is defined by equation (2.5). Here

$$\begin{aligned} d_0(V_\tau, \tau) &= \frac{\rho}{\sigma_v}(V_\tau - V_0 - \int_0^\tau \alpha(s)ds + \beta B) - \frac{1}{2}\rho^2 B, \\ d_1(V_\tau, \tau) &= \frac{1}{\sqrt{(1-\rho^2)B}}\big[\log(\frac{S_0}{K}) + \int_0^\tau r(s)ds + \frac{1}{2}B(1-\rho^2) + d_0(V_\tau, \tau)\big], \end{aligned}$$

and

$$d_2(V_\tau, \tau) = \frac{1}{\sqrt{(1-\rho^2)B}}\big[\log(\frac{S_0}{K}) + \int_0^\tau r(s)ds - \frac{1}{2}B(1-\rho^2) + d_0(V_\tau, \tau)\big].$$

Proof. We will prove equation (2.7) as following. Equation (2.8) can be proved similarly. The solutions of stochastic differential equations (2.3) and (2.4) can be expressed as

$$\begin{aligned} S_t &= S_0 \exp\left\{\int_0^t r(s)ds - \frac{1}{2}\int_0^t V_s ds + \rho\int_0^t \sqrt{V_s}dB_t^{(1)} + \sqrt{1-\rho^2}\int_0^t \sqrt{V_s}dB_s^{(2)}\right\},\\ V_t &= V_0 + \int_0^t \alpha(s)ds - \beta\int_0^t V_s ds + \sigma_v\int_0^t \sqrt{V_s}dB_s^{(1)}. \end{aligned}$$

According to the above two equations, we have

$$\begin{aligned} S_\tau &= S_0 \exp\left\{\int_0^\tau r(s)ds - \frac{1}{2}\int_0^\tau V_s ds + \frac{\rho}{\sigma_v}(V_\tau - V_0 - \int_0^\tau \alpha(s)ds + \beta\int_0^\tau V_s ds) + \sqrt{1-\rho^2}\int_0^t \sqrt{V_s}dB_s^{(2)}\right\}\\ &= S_0 \exp\left\{\int_0^\tau r(s)ds - \frac{1}{2}B + \frac{\rho}{\sigma_v}(V_\tau - V_0 - \int_0^\tau \alpha(s)ds + \beta B) + \sqrt{1-\rho^2}\int_0^t \sqrt{V_s}dB_s^{(2)}\right\}. \end{aligned}$$

Since $\{V_t\}$ is independent of standard Brownian motion, $B_t^{(2)}$ and Example 4.7.3 of [15], we obtain

$$\int_0^\tau \sqrt{V_s}dB_s^{(2)}\Big| F_\tau^V \overset{D}{=} N(0, \int_0^\tau V_s ds)\Big| F_\tau^V \equiv N(0,B)| F_\tau^V,$$

where $\{F_\tau^V\}$ means the filtration generated by process $\{V_t\}$. Since V_τ and τ are F_τ^V-measurable, we have

$$P_0 = E^Q\left[E^Q[e^{-\int_0^\tau r(s)ds}\max\{K - S_\tau, 0\}\Big|F_\tau^V]\right] = E^Q\left[E^Q[e^{-\int_0^\tau r(s)ds}\max\{K - S_0\exp\{m+n\xi\}, 0\}\Big|F_\tau^V]\right],$$

where ξ is the standard normal variable, which is independent of F_τ^V,

$$m = \int_0^\tau r(s)ds - \frac{1}{2}B + \frac{\rho}{\sigma_v}(V_\tau - V_0 - \int_0^\tau \alpha(s)ds + \beta B)$$

and $n = \sqrt{(1-\rho^2)B}$. Thus, we obtain

$$\begin{aligned} &E^Q\left[e^{-\int_0^\tau r(s)ds}\max\{K - S_0\exp\{m+n\xi\}, 0\}\Big|F_\tau^V\right]\\ &= e^{-\int_0^\tau r(s)ds}E^Q\left[(K - S_0\exp\{m+n\xi\})I_{\{S_0\exp\{m+n\xi\}\leqslant K\}}\Big|V_\tau, \tau\right]\\ &= e^{-\int_0^\tau r(s)ds}KQ\left(\xi \leqslant \frac{1}{n}(\log\frac{K}{S_0} - m)\Big|V_\tau, \tau\right) - e^{-\int_0^\tau r(s)ds}S_0E^Q\left[\exp\{m+n\xi\}I_{\{\xi\leqslant\frac{1}{n}(\log\frac{K}{S_0}-m)\}}\Big|V_\tau, \tau\right]. \end{aligned}$$

So, the time put options pricing (2.7) can be obtained by the above two terms on the standard normal distribution. Equations (2.7) and (2.8) in Theorem 2.1 both include the conditional expectation, which are difficult to be calculated. In the next section, we will give the explicit formula of timer option pricing. □

3. Explicit formula of timer option pricing

Equations (2.7) and (2.8) in Theorem 2.1 both include the conditional expectations. In this section, we will discuss the explicit formula of timer option pricing. The following Lemma 3.1 gives the distributional identity of the bivariate random variable(V_τ, τ).

Lemma 3.1. *under the risk neutral probability measure* Q, *we have*

$$(V_\tau, \tau) \overset{D}{=} (\sigma_v X_B, \int_0^B \frac{1}{\sigma_v X_v}ds), \tag{3.1}$$

where τ is defined by equation (2.5), and B is a variance budget. Here $\{X_t\}$ is a Bessel process which satisfies the following stochastic differential equation

$$dX_t = \Big(\frac{\alpha(t)}{\sigma_v^2 X_t} - \frac{\beta}{\sigma_v}\Big)dt + dB_t, \qquad X_0 = \frac{V_0}{\sigma_v},$$

where $\{B_t\}$ is standard Brownian motion.

The proof of Lemma 3.1 is similar to that of Proposition 4.1 in [11]. Now we obtain the following Theorem 3.2.

Theorem 3.2. *Under the time-varying stochastic volatility model (2.3) and (2.4), for varying budget B and strike price K, the initial price of timer call option and timer put option are given by*

$$C_0 = S_0\Omega_1^c - K\Omega_2^c \tag{3.2}$$

and

$$P_0 = K\Omega_1^P - S_0\Omega_2^P, \tag{3.3}$$

respectively, where

$$\Omega_1^C = \int_0^{+\infty}\int_0^{+\infty} \Phi\Big(d_1\big(\sigma_v x, \frac{t}{\sigma_v}\big)\Big)\exp\Big\{d_0\big(\sigma_v x, \frac{t}{\sigma_v}\big) + C\big(\sigma_v x, \frac{t}{\sigma_v}\big)\Big\} f(x,t;B)\,dx\,dt,$$
$$\Omega_2^C = \int_0^{+\infty}\int_0^{+\infty} \Phi\Big(d_2\big(\sigma_v x, \frac{t}{\sigma_v}\big)\Big)\exp\Big\{-\frac{1}{\sigma_v}\int_0^t r(s)ds + C\big(\sigma_v x, \frac{t}{\sigma_v}\big)\Big\} f(x,t;B)\,dx\,dt,$$

and

$$\Omega_1^P = \int_0^{+\infty}\int_0^{+\infty} \Big[1-\Phi\Big(d_1\big(\sigma_v x, \frac{t}{\sigma_v}\big)\Big)\exp\Big\{d_0\big(\sigma_v x, \frac{t}{\sigma_v}\big) + C\big(\sigma_v x, \frac{t}{\sigma_v}\big)\Big\}\Big]\exp\Big\{C\big(\sigma_v x, \frac{t}{\sigma_v}\big)\Big\} f(x,t;B)\,dx\,dt,$$
$$\Omega_2^P = \int_0^{+\infty}\int_0^{+\infty} \Phi\Big(-d_2\big(\sigma_v x, \frac{t}{\sigma_v}\big)\Big)\exp\Big\{-\frac{1}{\sigma_v}\int_0^t r(s)ds + C\big(\sigma_v x, \frac{t}{\sigma_v}\big)\Big\} f(x,t;B)\,dx\,dt.$$

Here

$$C(x,t) = \frac{\beta}{\sigma_v^2}(V_0 - x) + \frac{\beta}{\sigma_v^2}\int_0^t \alpha(s)ds - \frac{\beta^2}{2\sigma_v^2}B,$$

and $f(x,t;B)$ is the transition density of a Standard Bessel Process (see Proposition 4.2 in [11]). $d_0(x,t)$, $d_1(x,t)$, and $d_2(x,t)$ are defined by Theorem 2.1.

Proof. Based on Theorem 2.1 and Lemma 3.1, we have

$$C_0 = E^Q\Big[S_0 e^{d_0(\sigma_v X_B, \int_0^B \frac{ds}{\sigma_v X_s})}\Phi\Big(d_1\big(\sigma_v X_B, \int_0^B \frac{1}{\sigma_v X_s}ds\big)\Big) - Ke^{-\int_0^B \frac{r(s)}{\sigma_v X_s}ds}\Phi\Big(d_2\big(\sigma_v X_B, \int_0^B \frac{1}{\sigma_v X_s}ds\big)\Big)\Big] \tag{3.4}$$

and

$$P_0 = E^Q\Big[Ke^{-\int_0^\tau r(s)ds}\Phi\Big(-d_2\big(\sigma_v X_B, \int_0^B \frac{1}{\sigma_v X_s}ds\big)\Big) - S_0\Big(1 - e^{d_0(\sigma_v X_B, \int_0^B \frac{ds}{\sigma_v X_s})}\Phi\Big(d_1\big(\sigma_v X_B, \int_0^B \frac{1}{\sigma_v X_s}ds\big)\Big)\Big)\Big].$$

Now, we change the probability measure Bessel process. Let $\hat{B}_t = B_t - t\beta/\sigma_v$. By using the Girsanov theorem, $\hat{B}_t$ is a standard Brownian motion under a new probability measure $\hat{Q}$ whose Radon-Nikodym derivative is

$$\frac{d\hat{Q}}{dQ}\Big|_{F_t} = \exp\Big\{\frac{\beta}{\sigma_v}B_t - \frac{1}{2}\Big(\frac{\beta}{\sigma_v}\Big)^2 t\Big\}.$$

So, under probability measure $\hat{Q}$, X_t is a standard Bessel process which satisfies the following equation

$$dX_t = \frac{\alpha(t)}{\sigma_v^2 X_t}dt + d\hat{B}_t, \quad X_0 = V_0/\sigma_v.$$

Thus, we obtain

$$\frac{d\hat{Q}}{dQ}\Big|_{F_t} = \exp\Big\{\frac{\beta}{\sigma_v}(X_t - \frac{V_0}{\sigma_v}) - \frac{\beta}{\sigma_v}\int_0^t \frac{\alpha(s)}{\sigma_v^2 X_s}ds + \frac{1}{2}(\frac{\beta}{\sigma_v})^2 t\Big\}.$$

It follows from (3.4) that

$$C_0 = E^{\hat{Q}}\Big\{\Big[S_0 e^{d_0(\sigma_v X_B, \int_0^B \frac{ds}{\sigma_v X_s})}\Phi\big(d_1(\sigma_v X_B, \int_0^B \frac{ds}{\sigma_v X_s})\big) - K e^{-\int_0^B \frac{r(s)}{\sigma_v X_s}ds}\Phi\big(\sigma_v X_B, \int_0^B \frac{ds}{\sigma_v X_s}\big)\Big]\frac{dQ}{d\hat{Q}}\Big|_{F_t}\Big\}, \tag{3.5}$$

so, the timer call option price (3.1) can be obtained by using equation (3.4), (3.5), and the joint density function $f(x, t; B)$ given by Proposition 4.2 in [11]. The formula of timer option pricing (3.3) can be similarly proved. □

Remark 3.3. When $r(t) = 0\%$, a much simpler expression of the formulas for timer option pricing are obtained by Theorem 3.2. Indeed, we assume that variance budget $B = \sigma_0^2 T$, where T is an expected investment horizon and σ_0 is the investment period. K is a strike price. We easily obtain the price of timer call option as the following

$$C_0 = E^Q\Big[\max\{S_\tau - K, 0\}\Big] = C(S_0, K, T, \sigma_0, 0), \tag{3.6}$$

where

$$C(S_0, K, T, \sigma_0, r) = S_0\Phi(d_1) - K e^{\int_0^T r(s)ds}\Phi(d_2). \tag{3.7}$$

Here

$$d_1 = \frac{1}{\sigma_0 T}\Big[\log(\frac{S_0}{K}) + \int_0^T r(s)ds + \frac{1}{2}\sigma_0^2 T\Big] \quad \text{and} \quad d_2 = \frac{1}{\sigma_0 T}\Big[\log(\frac{S_0}{K}) + \int_0^T r(s)ds - \frac{1}{2}\sigma_0^2 T\Big].$$

Equation (3.7) is the price of European call option. The price of timer call option (3.6) yields to equation (3.7) with $r(t) = 0\%$.

4. Numerical analysis

In this section, we present numerical examples to illustrate our results. We chose the time point $t_i = i\Delta t$ for $i = 1, 2, \ldots$, where $\Delta t = T/n$. First, the transition of the following diffusion process

$$dV_t = (\alpha(t) - \beta V_t)dt + \sigma_v\sqrt{V_t}dB_t^{(1)}$$

follows a noncentral chi-squared distribution (see [6]). More precisely, given V_u for $0 < u < t$,

$$V_t = \frac{\sigma_v^2(1 - e^{\beta(t-u)})}{4\beta}\chi_d^2\left(\frac{4\beta e^{\beta(t-u)}}{\sigma_v^2}(1 - e^{\beta(t-u)})V_u\right),$$

where the degree of freedom is $d = 4\alpha(t)/\sigma_v^2$. The total variance can be approximated by using a trapezoidal rule, i.e.,

$$\int_0^{i\Delta t} V_s ds \approx \Delta t\left[\frac{V_0 + V(i\Delta t)}{2} + \sum_{j=1}^{i-1} V(j\Delta t)\right].$$

We chose the first time when the variance budget is exhausted by the following

$$\tau = \inf\left\{t : \Delta t\left[\frac{V_0 + V(i\Delta t)}{2} + \sum_{j=1}^{i-1} V(j\Delta t)\right] \geqslant B\right\}.$$

Now, we set the following parameters. Let $S_0 = 100$, $\rho = -0.3$, $V_0 = 0.09$, $\alpha(t) = 0.17 + 0.002t$, $\beta = 2$,

$\sigma_\nu = 1$, $\sigma_0 = 0.22$(or 0.25), $T = 0.96$(or 2.9), $B = \sigma_0^2 = 0.046$ (or 0.181). Figures 1 and 2 point the analytical values and simulation values of the price for time call option when strike price is $K = 90, 100, 110$. The analytical values are computed by equation (3.2), and simulation values are obtained by the asymptotically optimal rule (see [7]). The symbol "$*$" and "$+$" denote the analytical values and simulation values, respectively. From Figures 1 and 2, we can see that the formula of time option pricing performs well.

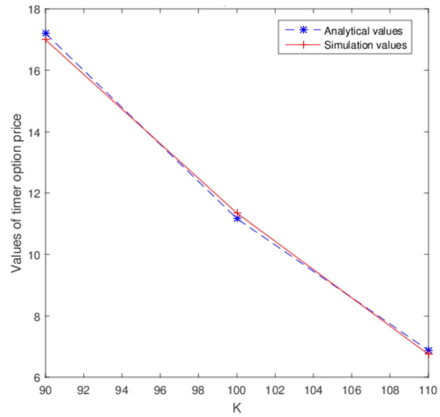

Figure 1: Analytical values and simulation values of price for timer call option with B=0.046.

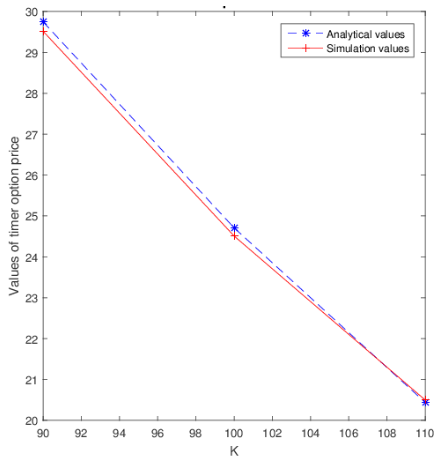

Figure 2: Analytical values and simulation values of price for timer call option with B=0.181.

5. Conclusion

In this paper, we study the timer options pricing for stochastic volatility models with changing coefficients under time-varying interest rate. The partial differential equation boundary value problem is given by using Δ-hedging approach and replicating a timer option. According to the joint distribution of the variance process and the random maturity under the risk neutral probability measure, the explicit formula of timer option pricing is proposed which can be applied to the financial market directly. Numerical analysis is conducted to show the performance of timer option pricing proposed.

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