# Stability analysis for a class of nonlinear impulsive switched systems 

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#### Abstract

In this note, we show a new sufficient condition for exponentially stability for a class of nonlinear impulsive switched systems. Based on the result obtained, an effective computational method is devised for the construction of switched linear stabilizing feedback controllers. Finally, a numerical example is given to illustrate the feasibility of the proposed methods. Compared with the results shown by Xu and Teo [H.-L. Xu, K. L. Teo, IEEE Trans. Automat. Control, 55 (2010), 2429-2433], the form of our result is simpler and its computational cost is lower. © 2017 All rights reserved.


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## 1. Introduction

An important class of hybrid dynamical systems is nonlinear impulsive switched systems. This class of systems can be used in many fields, such as the automotive industry, mechanical systems and switching power converters and so on [11].

The fundamentally research issues for linear or nonlinear impulsive switched systems are stability analysis and controller design [11]. As a result, theory and methods for stability of linear or nonlinear impulsive switched systems have been extensively studied by many researchers and many research papers have been published in various academic journals, for examples, see $[2,28,30,31]$ and the references therein.

In [24, 27], the authors discussed linear impulsive switched systems with linear impulsive increments. In [23], Xu and Teo considered a general class of nonlinear impulsive switched systems with nonlinear impulsive increments and presented some sufficient conditions for exponentially stability. The authors of [ 7,15 ] studied exponential stability of nonlinear impulsive switched systems with delays. In [16, 17], the authors discussed finite-time stability for nonlinear impulsive switched systems. In [22, 29], the authors investigated the stability properties for a class of nonlinear impulsive switched systems which include both stable and unstable subsystems.

[^0]Motivated by the above mentioned works, in this note, we shall also consider stability analysis for a class of nonlinear impulsive switched systems, as in [23]. The main technique of this note is based on an original argument developed in [23], with some technical changes. The main contributions of this note is that some new sufficient conditions for exponentially stability are presented. The conditions of our results are simpler than ones shown in [23] and the computational cost of solving these conditions is also lower than solving those conditions shown in [23]. Some related papers those may be helpful to readers can be found in $[3-6,9,10,12-14,18-21,25,26]$.

## 2. Nonlinear impulsive switched systems

In this note we mainly adopt the notation and terminology in [23]. The nonlinear impulsive switched systems considered in [23] can be described by

$$
\left\{\begin{array}{l}
\dot{x}(t)=A_{i} x(t)+B_{i} \omega(t)+C_{i} u(t)+\phi_{i}(t, x(t)), \quad t \neq \tau_{k}  \tag{2.1}\\
\Delta x(t)=D_{k} x(t)+\psi_{k}(t, x(t)), \quad t=\tau_{k} \\
z(t)=E_{i} x(t) \\
x\left(\tau_{0}^{+}\right)=x_{0}
\end{array}\right.
$$

where $x \in R^{n}$ is the state variable, $\omega(t) \in R^{p}$ is the disturbance input, $u(t) \in R^{r}$ is the control input, and $z(t) \in R^{q}$ is the controlled output. The functions $\phi_{i}(t, x(t)):\left[\tau_{0}, \infty\right) \times R^{n} \rightarrow R^{n}$ are globally Lipschitz continuous, and $\psi_{k}(t, x(t)):\left[\tau_{0}, \infty\right) \times R^{n} \rightarrow R^{n}$ are nonlinear functions, and $\phi_{i}(t, 0)=\psi_{i}(t, 0)=0$. $A_{i} \in R^{n \times n}, B_{i} \in R^{n \times p}, C_{i} \in R^{n \times r}, D_{k} \in R^{n \times n}, E_{i} \in R^{q \times n}$, are known constant matrices.

$$
\Delta x\left(\tau_{k}\right)=x\left(\tau_{k}^{+}\right)-x\left(\tau_{k}^{-}\right)=x\left(\tau_{k}^{+}\right)-x\left(\tau_{k}\right)
$$

with $x\left(\tau_{k}^{+}\right)=\lim _{t \rightarrow \tau_{k}^{+}} x(t)$ and $x\left(\tau_{k}\right)=x\left(\tau_{k}^{-}\right)=\lim _{t \rightarrow \tau_{k}^{-}} x(t)$ meaning that the solution of system (2.1) is left continuous. $\tau_{k}, k \in N$ denote the moments when impulsive control occurs which satisfy

$$
0=\tau_{0}<\tau_{1}<\tau_{2}<\cdots, \quad \lim _{k \rightarrow \infty} \tau_{k}=\infty
$$

Under the control of a switching signal, coupling with the impulsive effects, system (2.1) enters from the $i$ subsystem to the $i+1$ subsystem at the time point $t=\tau_{k}$. Without loss of generality, we assume that

$$
\|\omega(t)\|^{2} \leqslant \alpha\|x(t)\|^{2}, \quad\left\|\phi_{i}(t, x(t))\right\|^{2} \leqslant g_{i}\|x(t)\|^{2}, \quad\left\|\psi_{k}(t, x(t))\right\|^{2} \leqslant \rho_{k}\|x(t)\|^{2}
$$

for all $\left[\tau_{0}, \infty\right)$, where $\|\cdot\|$ denotes the Euclidean norm of vectors and $\alpha, g_{i}, \rho_{k}$ are positive constants.
For exponential stabilization, Xu and Teo [23] also discussed a class of switched linear feedback controllers $u(t)=F_{i} x(t)$, where $F_{i} \in R^{r \times n}$ are constant matrices. Then, the following impulsive switched closed-loop system was obtained

$$
\left\{\begin{array}{l}
\dot{x}(t)=\left(A_{i}+C_{i} F_{i}\right) x(t)+B_{i} \omega(t)+\phi_{i}(t, x(t)), \quad t \neq \tau_{k}  \tag{2.2}\\
\Delta x(t)=D_{k} x(t)+\psi_{k}(t, x(t)), \quad t=\tau_{k} \\
z(t)=E_{i} x(t) \\
x\left(\tau_{0}^{+}\right)=x_{0}
\end{array}\right.
$$

In this note, we show a new sufficient condition for exponentially stability of system (2.1). Based on the result obtained, linear feedback gain matrices $F_{i}$ are constructed such that system (2.2) is exponentially stable. Finally, a numerical example is given to illustrate the feasibility of the proposed results.

## 3. Main results

In this section, we will give the main results of this note. To do this, we need the following lemmas [8].

Lemma 3.1. For any $x, y \in R^{n}$, then

$$
\left|x^{\top} y\right| \leqslant\|x\|\|y\|
$$

Lemma 3.2. Let H be a real symmetrical matrix, and $\lambda_{\max }(\mathrm{H}), \lambda_{\min }(\mathrm{H})$ be the largest and the smallest eigenvalues of $H$, respectively. Then for any $x \in R^{n}$, we have

$$
\lambda_{\min }(H) x^{\top} x \leqslant x^{\top} H x \leqslant \lambda_{\max }(H) x^{\top} x
$$

Theorem 3.3. Let $u(t)=0$ and suppose that the assumptions about $\omega(t), \phi_{i}(t, x(t)), \psi_{k}(t, x(t))$ are satisfied. If there exist symmetric and positive definite matrices $P_{i}$ such that the following conditions hold
(i): $\left[\begin{array}{cc}A_{i}^{\top} P_{i}+P_{i} A_{i}+\sigma_{i} I & P_{i} \\ P_{i} & -I\end{array}\right]<0 ;$
(ii): $\ln \beta_{k}+\eta\left(\tau_{k+1}-\tau_{k}\right) \leqslant 0$,
where

$$
\begin{gathered}
\sigma_{i}=\left(\sqrt{\alpha \lambda_{\max }\left(B_{i}^{\top} B_{i}\right)}+\sqrt{g_{i}}\right)^{2}, \eta=\max \eta_{i_{k}}<0 \\
\eta_{i_{k}}=\lambda_{\max }\left(P_{i}^{-1}\left(P_{i}^{2}+A_{i}^{\top} P_{i}+P_{i} A_{i}+\sigma_{i} I\right)\right)<0
\end{gathered}
$$

and

$$
\beta_{k}=\left(\sqrt{\lambda_{\max }\left(P_{i-1}^{-1}\left(I+D_{k}\right)^{T} P_{i}\left(I+D_{k}\right)\right)}+\sqrt{\rho_{k} \frac{\lambda_{\max }\left(P_{i}\right)}{\lambda_{\max }\left(P_{i-1}\right)}}\right)^{2}
$$

then the nonlinear impulsive switched system (2.1) is exponentially stable.
Proof. Let us construct switched Lyapunov functions

$$
V(t)=x(t)^{\top} P_{i} x(t)
$$

When $t \in\left(\tau_{k}, \tau_{k+1}\right]$, we have

$$
\begin{align*}
D^{+}(V(t)) & =\dot{x}(t)^{\top} P_{i} x(t)+x(t)^{\top} P_{i} \dot{x}(t) \\
& =x(t)^{\top}\left(A_{i}^{\top} P_{i}+P_{i} A_{i}\right) x(t)+2 x(t)^{\top} P_{i}\left(B_{i} \omega(t)+\varphi_{i}(t, x(t))\right) \tag{3.1}
\end{align*}
$$

Then by Lemmas 3.1, 3.2 and the arithmetic-geometric mean for scalars, we have

$$
\begin{align*}
2 x(t)^{\top} & P_{i}\left(B_{i} \omega(t)+\phi_{i}(t, x(t))\right) \\
\leqslant & 2 \sqrt{x(t)^{\top} P_{i}^{2} x(t)^{\top}\left(B_{i} \omega(t)+\phi_{i}(t, x(t))\right)^{\top}\left(B_{i} \omega(t)+\phi_{i}(t, x(t))\right)} \\
\leqslant & x(t)^{\top} P_{i}^{2} x(t)^{\top}+\left(B_{i} \omega(t)+\phi_{i}(t, x(t))\right)^{\top}\left(B_{i} \omega(t)+\phi_{i}(t, x(t))\right) \\
= & x(t)^{\top} P_{i}^{2} x(t)^{\top}+\omega(t)^{\top} B_{i}^{\top} B_{i} \omega(t)+2 \omega(t)^{\top} B_{i}^{\top} \phi_{i}(t, x(t)) \\
& +\phi_{i}(t, x(t))^{\top} \phi_{i}(t, x(t)) \\
\leqslant & x(t)^{\top} P_{i}^{2} x(t)^{\top}+\lambda_{\max }\left(B_{i}^{\top} B_{i}\right) \omega(t)^{\top} \omega(t)+2 \omega(t)^{\top} B_{i}^{\top} \phi_{i}(t, x(t))  \tag{3.2}\\
& +\phi_{i}(t, x(t))^{\top} \phi_{i}(t, x(t)) \\
\leqslant & x(t)^{\top} P_{i}^{2} x(t)^{\top}+\lambda_{\max }\left(B_{i}^{\top} B_{i}\right) \omega(t)^{\top} \omega(t)+\phi_{i}(t, x(t))^{\top} \phi_{i}(t, x(t)) \\
& +2 \sqrt{\omega(t)^{\top} B_{i}^{\top} B_{i} \omega(t) \phi_{i}(t, x(t))^{\top} \phi_{i}(t, x(t))} \\
\leqslant & x(t)^{\top} P_{i}^{2} x(t)^{\top}+\lambda_{\max }\left(B_{i}^{\top} B_{i}\right) \omega(t)^{\top} \omega(t)+\phi_{i}(t, x(t))^{\top} \phi_{i}(t, x(t)) \\
& +2 \sqrt{\lambda_{\max }\left(B_{i}^{\top} B_{i}\right)} \sqrt{\omega(t)^{\top} \omega(t) \phi_{i}(t, x(t))^{\top} \phi_{i}(t, x(t))} .
\end{align*}
$$

Thus, by the assumptions about $\omega(\mathrm{t}), \phi_{\mathrm{i}}(\mathrm{t}, \mathrm{x}(\mathrm{t}))$, (3.1), (3.2), we obtain

$$
\begin{aligned}
D^{+}(V(t)) \leqslant & x(t)^{\top}\left(A_{i}^{\top} P_{i}+P_{i} A_{i}\right) x(t)+x(t)^{\top} P_{i}^{2} x(t)^{\top} \\
& +\alpha \lambda_{\max }\left(B_{i}^{\top} B_{i}\right) x(t)^{\top} x(t)+g_{i} x(t)^{\top} x(t)+2 \sqrt{\alpha g_{i} \lambda_{\max }\left(B_{i}^{\top} B_{i}\right)} x(t)^{\top} x(t) \\
= & x(t)^{\top}\left(P_{i}^{2}+A_{i}^{\top} P_{i}+P_{i} A_{i}+\left(\alpha \lambda_{\max }\left(B_{i}^{\top} B_{i}\right)+2 \sqrt{\alpha g_{i} \lambda_{\max }\left(B_{i}^{\top} B_{i}\right)}+g_{i}\right) I\right) x(t) \\
= & x(t)^{\top}\left(P_{i}^{2}+A_{i}^{\top} P_{i}+P_{i} A_{i}+\left(\sqrt{\alpha \lambda_{\max }\left(B_{i}^{\top} B_{i}\right)}+\sqrt{g_{i}}\right)^{2} I\right) x(t)
\end{aligned}
$$

By Schur complement theorem [1] and condition (i), we get

$$
D^{+}(V(t)) \leqslant \eta_{i_{k}} V(t),
$$

which implies that

$$
\begin{equation*}
V(t) \leqslant V\left(\tau_{k}^{+}\right) e^{\eta_{i k}\left(t-\tau_{k}\right)}, \quad t \in\left(\tau_{k}, \tau_{k+1}\right] \tag{3.3}
\end{equation*}
$$

When $t=\tau_{k}$, by Lemmas 3.1,3.2, and the assumption about $\psi_{k}(t, x(t))$, we have

$$
\begin{align*}
& V\left(\tau_{k}^{+}\right)=\left(\left(I+D_{k}\right) x\left(\tau_{k}\right)+\psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right)\right)^{\top} P_{i}\left(\left(I+D_{k}\right) x\left(\tau_{k}\right)+\psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right)\right) \\
& =x\left(\tau_{k}\right)^{\top}\left(I+D_{k}\right)^{\top} P_{i}\left(I+D_{k}\right) x\left(\tau_{k}\right)+2 x\left(\tau_{k}\right)^{\top}\left(I+D_{k}\right)^{\top} P_{i} \psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right) \\
& +\psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right)^{\top} P_{i} \psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right) \\
& \leqslant \lambda_{\max }\left(P_{i-1}\left(I+D_{k}\right)^{\top} P_{i}\left(I+D_{k}\right)\right) x\left(\tau_{k}\right)^{\top} P_{i-1} x\left(\tau_{k}\right)+\psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right)^{\top} P_{i} \psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right) \\
& +2 x\left(\tau_{k}\right)^{\top}\left(I+D_{k}\right)^{\top} P_{i} \psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right) \\
& \leqslant \lambda_{\max }\left(P_{i-1}\left(I+D_{k}\right)^{\top} P_{i}\left(I+D_{k}\right)\right) \vee\left(\tau_{k}\right)+\psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right)^{\top} P_{i} \psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right) \\
& +2 \sqrt{x\left(\tau_{k}\right)^{\top}\left(I+D_{k}\right)^{\top} P_{i}\left(I+D_{k}\right) x\left(\tau_{k}\right) \psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right)^{\top} P_{i} \psi_{k}\left(\tau_{k}, x\left(\tau_{k}\right)\right)}  \tag{3.4}\\
& \leqslant \lambda_{\max }\left(P_{i-1}\left(I+D_{k}\right)^{\top} P_{i}\left(I+D_{k}\right)\right) V\left(\tau_{k}\right)+\rho_{k} \frac{\lambda_{\max }\left(P_{i}\right)}{\lambda_{\max }\left(P_{i-1}\right)} \times\left(\tau_{k}\right)^{\top} P_{i-1} \times\left(\tau_{k}\right) \\
& +2 \sqrt{\lambda_{\max }\left(P_{i-1}\left(I+D_{k}\right)^{\top} P_{i}\left(I+D_{k}\right)\right) \rho_{k} \frac{\lambda_{\max }\left(P_{i}\right)}{\lambda_{\max }\left(P_{i-1}\right)}} \times\left(\tau_{k}\right)^{\top} P_{i-1} \times\left(\tau_{k}\right) \\
& =\left(\sqrt{\lambda_{\max }\left(P_{i-1}\left(I+D_{k}\right)^{\top} P_{i}\left(I+D_{k}\right)\right)}+\sqrt{\frac{\lambda_{\max }\left(P_{i}\right)}{\lambda_{\max }\left(P_{i-1}\right)} \rho_{k}}\right)^{2} V\left(\tau_{k}\right) .
\end{align*}
$$

For $\mathrm{t} \in\left(\tau_{0}, \tau_{1}\right]$, by inequality (3.3), we have

$$
\mathrm{V}(\mathrm{t}) \leqslant \mathrm{V}\left(\tau_{0}^{+}\right) \mathrm{e}^{\mathfrak{\eta}_{\mathfrak{i}_{1}}\left(\mathrm{t}-\tau_{0}\right)}
$$

By inequality (3.4) and this last inequality, we have

$$
\begin{equation*}
V\left(\tau_{1}^{+}\right) \leqslant \beta_{1} V\left(\tau_{1}\right) \leqslant \beta_{1} V\left(\tau_{0}^{+}\right) e^{\eta_{\mathfrak{n}_{1}}\left(\tau_{1}-\tau_{0}\right)} \tag{3.5}
\end{equation*}
$$

For $t \in\left(\tau_{1}, \tau_{2}\right]$, it follows from (3.3) and (3.5) that

$$
V(t) \leqslant V\left(\tau_{1}^{+}\right) e^{\mathfrak{n}_{i_{2}}\left(t-\tau_{1}\right)} \leqslant \beta_{1} V\left(\tau_{0}^{+}\right) e^{\eta_{\mathfrak{i}_{1}}\left(\tau_{1}-\tau_{0}\right)} e^{\eta_{i_{2}}\left(t-\tau_{1}\right)} .
$$

By induction, for $t \in\left(\tau_{k}, \tau_{k+1}\right]$, we have

$$
V(t) \leqslant \beta_{1} \cdots \beta_{k} V\left(\tau_{0}^{+}\right) e^{\eta_{i_{1}}\left(\tau_{1}-\tau_{0}\right)} e^{\eta_{i_{2}}\left(\tau_{2}-\tau_{1}\right)} \cdots e^{\eta_{i_{k}}\left(\tau_{k}-\tau_{k-1}\right)} e^{\eta_{i_{k+1}}\left(t-\tau_{k}\right)}
$$

Since $\eta=\max \eta_{i_{k}}<0$ and $\ln \beta_{k}+\eta\left(\tau_{k+1}-\tau_{k}\right) \leqslant 0$, we get

$$
\begin{aligned}
V(t) & \leqslant \beta_{1} \cdots \beta_{k} V\left(\tau_{0}^{+}\right) e^{\eta_{i_{0}}\left(\tau_{1}-\tau_{0}\right)} e^{\eta_{i_{1}}\left(\tau_{2}-\tau_{1}\right)} \cdots e^{\eta_{i_{k}}\left(\tau_{k}-\tau_{k-1}\right)} e^{\eta_{i_{k}}\left(t-\tau_{k}\right)} \\
& \leqslant \beta_{1} \cdots \beta_{k} V\left(\tau_{0}^{+}\right) e^{\mathfrak{\eta}\left(\tau_{1}-\tau_{0}\right)} e^{\mathfrak{\eta}\left(\tau_{2}-\tau_{1}\right)} \cdots e^{\mathfrak{\eta}\left(\tau_{k}-\tau_{k-1}\right)} e^{\eta\left(t-\tau_{k}\right)} \\
& =\beta_{1} \cdots \beta_{k} V\left(\tau_{0}^{+}\right) e^{\eta\left(t-\tau_{0}\right)} \\
& =V\left(\tau_{0}^{+}\right) e^{\sum_{j=1}^{k} \beta_{j}} e^{\eta\left(t-\tau_{0}\right)} \\
& \leqslant V\left(\tau_{0}^{+}\right) e^{\sum_{j=1}^{k} \beta_{j}} e^{-\eta\left(\tau_{k}-\tau_{0}\right)} e^{\mathfrak{\eta}\left(t-\tau_{0}\right)} \\
& =V\left(\tau_{0}^{+}\right) e^{\ln \beta_{1}-\eta\left(\tau_{1}-\tau_{0}\right)} e^{\ln \beta_{1}-\eta\left(\tau_{2}-\tau_{1}\right)} \cdots e^{\ln \beta_{k}-\eta\left(\tau_{k}-\tau_{k-1}\right)} e^{\eta\left(t-\tau_{0}\right)} \\
& \leqslant V\left(\tau_{0}^{+}\right) e^{\eta\left(t-\tau_{0}\right)} .
\end{aligned}
$$

By Lemma 3.2 and this last inequality, we obtain

$$
\lambda_{\min }\left(P_{i}\right)\left\|x\left(t, \tau_{0}, x_{0}\right)\right\|^{2} \leqslant V(t) \leqslant V\left(\tau_{0}^{+}\right) e^{\eta\left(t-\tau_{0}\right)} \leqslant\left\|x_{0}\right\|^{2} \lambda_{\max }\left(P_{1}\right) e^{\eta\left(t-\tau_{0}\right)}
$$

That is

$$
\left\|x\left(t, \tau_{0}, x_{0}\right)\right\| \leqslant \sqrt{\frac{\lambda_{\max }\left(P_{1}\right)}{\lambda_{\min }\left(P_{i}\right)}}\left\|x_{0}\right\| e^{\frac{\eta}{2}\left(t-\tau_{0}\right)}
$$

This completes the proof.
Remark 3.4. Compared with the result shown by Xu and Teo [23], the form of our result is simpler and its computational cost is lower.
Theorem 3.5. Let the assumptions about $\omega(t), \phi_{i}(t, x(t)), \psi_{k}(t, x(t))$ be satisfied. If there exist matrices $L_{i}, W_{i}$ with $L_{i}$ is symmetric and positive definite such that the following conditions hold:
(i): $\left[\begin{array}{cc}I+\left(A_{i} L_{i}+C_{i} W_{i}\right)^{\top}+\left(A_{i} L_{i}+C_{i} W_{i}\right) & \sqrt{\sigma_{i} L_{i}} \\ \sqrt{\sigma_{i} L_{i}} & -I\end{array}\right]<0 ;$
(ii): $0<\sqrt{\lambda_{\max }\left(\mathrm{L}_{\mathrm{i}-1}\left(\mathrm{I}+\mathrm{D}_{\mathrm{k}}\right)^{\mathrm{T}} \mathrm{L}_{\mathrm{i}}^{-1}\left(\mathrm{I}+\mathrm{D}_{\mathrm{k}}\right)\right)}+\sqrt{\rho_{\mathrm{k}} \frac{\lambda_{\max }\left(\mathrm{L}_{\mathrm{i}}^{-1}\right)}{\lambda_{\max }\left(\mathrm{L}_{\mathrm{i}-1}^{-1}\right)}}<1$,
where

$$
\sigma_{i}=\left(\sqrt{\alpha \lambda_{\max }\left(B_{i}^{\top} B_{i}\right)}+\sqrt{g_{i}}\right)^{2}
$$

then the impulsive switched closed-loop system (2.2) is exponentially stable and we have the following switched linear feedback controllers:

$$
u(t)=F_{i} x(t), \quad F_{i}=W_{i} L_{i}^{-1}
$$

Proof. By Schur complement theorem [1], we know that condition (i) of Theorem 3.5 is equivalent to

$$
\begin{equation*}
I+\left(A_{i} L_{i}+C_{i} W_{i}\right)^{T}+\left(A_{i} L_{i}+C_{i} W_{i}\right)+\sigma_{i} L_{i}^{2}<0 \tag{3.6}
\end{equation*}
$$

Let

$$
P_{i}=L_{i}^{-1}, \quad F_{i}=W_{i} L_{i}^{-1}
$$

Multiplying $P_{i}$ from both sides of (3.6), we obtain

$$
P_{i}^{2}+P_{i}\left(A_{i} L+C_{i} W_{i}\right)^{\top} P_{i}+P_{i}\left(A_{i} L+C_{i} W_{i}\right) P_{i}+\sigma_{i} I \leqslant 0
$$

That is,

$$
P_{i}^{2}+\left(A_{i}+C_{i} F_{i}\right)^{\top} P_{i}+P_{i}\left(A_{i}+C_{i} F_{i}\right)+\sigma_{i} I \leqslant 0
$$

By using Schur complement theorem [1] again and this last inequality, we have

$$
\left[\begin{array}{cc}
\left(A_{i}+C_{i} F_{i}\right)^{\top} P_{i}+P_{i}\left(A_{i}+C_{i} F_{i}\right)+\sigma_{i} I & P_{i} \\
P_{i} & -I
\end{array}\right]<0
$$

Thus, condition (i) of Theorem 3.3 holds. Since

$$
0<\sqrt{\lambda_{\max }\left(\mathrm{L}_{\mathrm{i}-1}\left(\mathrm{I}+\mathrm{D}_{\mathrm{k}}\right)^{\mathrm{T}} \mathrm{~L}_{\mathrm{i}}^{-1}\left(\mathrm{I}+\mathrm{D}_{\mathrm{k}}\right)\right)}+\sqrt{\rho_{\mathrm{k}} \frac{\lambda_{\max }\left(\mathrm{L}_{i}^{-1}\right)}{\lambda_{\max }\left(\mathrm{L}_{\mathrm{i}-1}^{-1}\right)}}<1
$$

which implies

$$
\ln \beta_{k}+\eta\left(\tau_{k+1}-\tau_{k}\right) \leqslant 0
$$

and so condition (ii) of Theorem 3.3 also holds. Then the impulsive switched closed-loop system (2.2) is exponentially stable. This completes the proof.

## 4. A numerical example

In this section, we will illustrate the effectiveness of our result by using the same example which was shown in [23]. Consider the following nonlinear impulsive switched system with arbitrary switching laws and two switching status:

$$
\begin{gathered}
A_{1}=\left[\begin{array}{cc}
1 & 1 \\
0 & 1.5
\end{array}\right], \quad B_{1}=\left[\begin{array}{cc}
2.1 & 0.2 \\
-1.1 & 0
\end{array}\right], \quad C_{1}=\left[\begin{array}{cc}
1.6 & 1 \\
2 & 1.5
\end{array}\right], \quad \phi_{1}(t, x(t))=\left[\begin{array}{c}
\sin x_{1} \\
0
\end{array}\right], \\
A_{2}=\left[\begin{array}{ll}
2 & 1 \\
2 & 3
\end{array}\right], \quad B_{2}=\left[\begin{array}{cc}
1.5 & 1.3 \\
1.2 & 0
\end{array}\right], \quad C_{2}=\left[\begin{array}{cc}
1.8 & 0.8 \\
1 & 1.7
\end{array}\right], \quad \phi_{2}(t, x(t))=\left[\begin{array}{l}
\sin x_{1} \\
\sin x_{2}
\end{array}\right]
\end{gathered}
$$

and

$$
D_{k}=-\left[\begin{array}{cc}
0.58 & 0 \\
0 & 0.58
\end{array}\right], \quad \psi_{k}(t, x(t))=0.3\left[\begin{array}{c}
\sin x_{1} \\
\sin x_{2}
\end{array}\right], \quad \omega_{i}(t)=\left[\begin{array}{l}
x_{1} \sin 20 \pi t \\
x_{2} \sin 20 \pi t
\end{array}\right]
$$

Then, we can choose

$$
\alpha=g_{1}=g_{2}=1, \quad \rho_{k}=0.09
$$

and so

$$
\sigma_{1}=11.4060, \quad \sigma_{2}=10.3003
$$

Solving linear matrix inequalities in condition (i) of Theorem 3.5, we have

$$
\begin{aligned}
& \mathrm{L}_{1}=\left[\begin{array}{cc}
0.0435 & 0 \\
0 & 0.0435
\end{array}\right], \quad W_{1}=\left[\begin{array}{cc}
-3.2351 & 3.6644 \\
4.1145 & -5.6082
\end{array}\right], \\
& \mathrm{L}_{2}=\left[\begin{array}{cc}
0.0481 & 0 \\
0 & 0.0481
\end{array}\right], \quad W_{2}=\left[\begin{array}{cc}
-2.7226 & -3.6994 \\
4.7316 & 1.4918
\end{array}\right] .
\end{aligned}
$$

Simple calculations show that $\beta_{1}=0.4448<1$ and $\beta_{2}=0.5105<1$. All the conditions of Theorem 3.5 are satisfied, so the nonlinear impulsive switched system is exponentially stable.

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