Available online at www.isr-publications.com/jnsa
J. Nonlinear Sci. Appl., 10 (2017), 4137–4142
Research Article

ISSN: 2008-1898

Journal of Nonlinear Sciences and Applications

Journal Homepage: www.tjnsa.com - www.isr-publications.com/jnsa

Some identities of λ-Daehee polynomials

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Communicated by S.-H. Rim

Abstract

In this paper, we give some identities of λ-Daehee polynomials and investigate a new and interesting identities of λ-Daehee polynomial arising from the symmetry properties of the p-adic invariant integral on $\mathbb{Z}_p$.

Keywords: λ-Daehee polynomials, p-adic invariant integral on $\mathbb{Z}_p$.
2010 MSC: 11B83, 42A16.

1. Introduction

Let p be a fixed prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ will denote the ring of p-adic integers, the field of p-adic rational numbers and the completion of the algebraic closure of $\mathbb{Q}_p$. The p-adic norm $|\cdot|_p$ is normalized as $|p|_p = \frac{1}{p}$.

Let $f(x)$ be a uniformly differentiable function on $\mathbb{Z}_p$. Then the p-adic invariant integral on $\mathbb{Z}_p$ is defined by

$$I_0(f) = \int_{\mathbb{Z}_p} f(x)d\mu_0(x) = \lim_{N\to\infty} \frac{1}{p^N} \sum_{n=0}^{p^N-1} f(x), \quad \text{(see [15, 16])}. \tag{1.1}$$

From (1.1), we note that

$$I_0(f_1) - I_0(f) = f'(0), \tag{1.2}$$

where $f'(0) = \left.\frac{df(x)}{dx}\right|_{x=0}$ and $f_1(x) = f(x+1)$.

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doi:10.22436/jnsa.010.08.09

Received 2017-06-12

As is well-known, the Daehee polynomials are defined by

$$\frac{\log(1+t)}{t}(1+t)^x = \sum_{n=0}^{\infty} D_n(x)\frac{t^n}{n!}, \tag{1.3}$$

(see [1–3, 5–9, 14, 20, 21]). From (1.2), we note that

$$\int_{\mathbb{Z}_p}(1+t)^{x+y}d\mu_0(y) = \frac{\log(1+t)}{t}(1+t)^x = \sum_{n=0}^{\infty} D_n(x)\frac{t^n}{n!}, \tag{1.4}$$

where $|t|_p < p^{\frac{-1}{p-1}}$.

By (1.4), we easily get

$$\begin{aligned}\int_{\mathbb{Z}_p}(1+t)^{x+y}d\mu_0(y) &= \int_{\mathbb{Z}_p} e^{(x+y)\log(1+t)}d\mu_0(y)\\ &= \sum_{m=0}^{\infty}\int_{\mathbb{Z}_p}(x+y)^m d\mu_0(y)\frac{1}{m!}\big(\log(1+t)\big)^m\\ &= \sum_{n=0}^{\infty}\Big(\sum_{m=0}^{n} S_1(n,m)B_m(x)\Big)\frac{t^n}{n!},\end{aligned} \tag{1.5}$$

where $S_1(n,m)$ is the Stirling number of the first kind and $B_n(x)$ are the Bernoulli polynomials (see [17]).

From (1.4) and (1.5), we note that

$$D_n(x) = \sum_{m=0}^{n} S_1(n,m)B_m(x), \quad \text{(see [10, 12, 23])}.$$

Recently many researchers have studied symmetric identities of special polynomials (see [11, 13, 14, 16, 18, 19, 22]). In this paper, we give some identities of λ-Daehee polynomials and investigate a new and interesting identities of λ-Daehee polynomial arising from the symmetry properties of the p-adic invariant integral on $\mathbb{Z}_p$.

2. The λ-Daehee polynomials

In this section, we will investigate interesting identities of the λ-Daehee polynomials which are modified by the Daehee polynomials in (1.3).

The λ-Daehee polynomials are defined by the generating function to be

$$\frac{\lambda\log(1+t)}{(1+t)^{\lambda}-1}(1+t)^x = \sum_{n=0}^{\infty} D_n(x\mid\lambda)\frac{t^n}{n!}, \quad \text{(see [21])}, \tag{2.1}$$

when $x = 0$, $D_n(0\mid\lambda) = D_n(\lambda)$ are called λ-Daehee numbers.

For $|t|_p < p^{\frac{-1}{p-1}}$, by (1.2), we get

$$\int_{\mathbb{Z}_p}(1+t)^{\lambda y+x}d\mu_0(y) = \frac{\lambda\log(1+t)}{(1+t)^{\lambda}-1}(1+t)^x = \sum_{n=0}^{\infty} D_n(x\mid\lambda)\frac{t^n}{n!}.$$

From (2.1), we have

$$\begin{aligned}\sum_{n=0}^{\infty} D_n(x\mid\lambda)\frac{t^n}{n!} &= \frac{\lambda\log(1+t)}{(1+t)^{\lambda}-1}(1+t)^x\\ &= \sum_{m=0}^{\infty} B_m\Big(\frac{x}{\lambda}\Big)\lambda^m\frac{\big(\log(1+t)\big)^m}{m!}\\ &= \sum_{n=0}^{\infty}\Big(\sum_{m=0}^{n} B_m\Big(\frac{x}{\lambda}\Big)\lambda^m S_1(n,m)\Big)\frac{t^n}{n!}.\end{aligned} \tag{2.2}$$

Thus, by (2.2), we have the following theorem.

Theorem 2.1.

$$D_n(x\mid\lambda)=\sum_{m=0}^{n}B_m\left(\frac{x}{\lambda}\right)\lambda^m S_1(n,m),\quad (n\geqslant 0).$$

Recall that for $z\in\mathbb{R}$, the Harmonic polynomials $H_m(z)$ defined as follows

$$\sum_{n=0}^{\infty}H_n(z)t^n=\frac{-\ln(1-t)}{t(1-t)}(1-t)^z.$$

Observe that

$$\begin{aligned}\sum_{n=0}^{\infty}n!H_n(z)\frac{t^n}{n!}&=\frac{-\ln(1-t)}{t(1-t)}(1-t)^z\\&=\frac{1\cdot\ln(1+(-t))}{\left(1+(-t)\right)^1-1}\left(1+(-t)\right)^{z-1}\\&=\sum_{n=0}^{\infty}D_n(z-1\mid 1)(-1)^n\frac{t^n}{n!}.\end{aligned}\tag{2.3}$$

Thus, by (2.3), we have the following theorem.

Theorem 2.2. *For $n\in\mathbb{N}\cup\{0\}$, we have*

$$H_n(z)=D_n(z-1\mid 1)\frac{(-1)^n}{n!}.$$

3. Some identities of symmetry for λ-Daehee polynomials

In this paper, we give some new identities of symmetry for the λ-Daehee polynomials which are derived from our symmetric properties related to p-adic invariant integral on $\mathbb{Z}_p$. In addition, we investigate some new identities of symmetry for the λ-Daehee polynomial invariant under Dihedral group D_4 of degree 4 arising from the p-adic invariant integral on $\mathbb{Z}_p$.

In this section, we assume that $t\in\mathbb{Q}_p$ with $|t|_p<p^{\frac{-1}{p-1}}$. For $\lambda\in\mathbb{Z}_p$, let us take $f(x)=(1+t)^{\lambda x}$. Then, by (1.2), we get

$$\int_{\mathbb{Z}_p}(1+t)^{\lambda x}d\mu_0(x)=\frac{\lambda\log(1+t)}{(1+t)^\lambda-1}=\sum_{n=0}^{\infty}D_n(\lambda)\frac{t^n}{n!},$$

and

$$\int_{\mathbb{Z}_p}(1+t)^{\lambda y+x}d\mu_0(y)=\frac{\lambda\log(1+t)}{(1+t)^\lambda-1}(1+t)^x=\sum_{n=0}^{\infty}D_n(x\mid\lambda)\frac{t^n}{n!}.$$

As is well-known, the Bernoulli polynomials are defined by the p-adic invariant integral on $\mathbb{Z}_p$ as follows:

$$\int_{\mathbb{Z}_p}e^{(x+y)t}d\mu_0(y)=\frac{t}{e^t-1}e^{xt}=\sum_{n=0}^{\infty}B_n(x)\frac{t^n}{n!}.\tag{3.1}$$

From (3.1), we note that

$$\begin{aligned}\int_{\mathbb{Z}_p}e^{(y+\frac{x}{\lambda})t}d\mu_0(y)&=\sum_{m=0}^{\infty}D_m(x)\frac{1}{m!}(e^{\frac{1}{\lambda}t}-1)^m\\&=\sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}D_m(x)S_2(n,m)\lambda^{-n}\right)\frac{t^n}{n!},\end{aligned}\tag{3.2}$$

where $S_2(n, m)$ is the Stirling number of the second kind.

By (3.1) and (3.2), we get

$$\lambda^n B_n\left(\frac{x}{\lambda}\right) = \sum_{m=0}^{n} D_m(x) S_2(n, m), \quad (n \geqslant 0).$$

We assume that $w_1, w_2, w_3, w_4 \in \mathbb{N}$. From (1.1), we note that

$$\begin{aligned}
&\int_{\mathbb{Z}_p} (1+t)^{\lambda w_1 w_2 w_3 y + w_1 w_2 w_3 w_4 x + w_1 w_2 w_4 i + w_1 w_3 w_4 j + w_2 w_3 w_4 k} d\mu_0(y) \\
&= \lim_{N\to\infty} \frac{1}{p^N} \sum_{y=0}^{p^N-1} (1+t)^{\lambda w_1 w_2 w_3 y + w_1 w_2 w_3 w_4 x + w_1 w_2 w_4 i + w_1 w_3 w_4 j + w_2 w_3 w_4 k} \\
&= \lim_{N\to\infty} \frac{1}{w_4 p^N} \sum_{y=0}^{w_4 p^N-1} (1+t)^{\lambda w_1 w_2 w_3 y + w_1 w_2 w_3 w_4 x + w_1 w_2 w_4 i + w_1 w_3 w_4 j + w_2 w_3 w_4 k} \\
&= \frac{1}{w_4} \lim_{N\to\infty} \frac{1}{p^N} \sum_{l=0}^{w_4-1} \sum_{y=0}^{p^N-1} (1+t)^{\lambda w_1 w_2 w_3 (l + w_4 y) + w_1 w_2 w_3 w_4 x + w_1 w_2 w_4 i + w_1 w_3 w_4 j + w_2 w_3 w_4 k}.
\end{aligned} \tag{3.3}$$

Thus, by (3.3), we get

$$\begin{aligned}
&\frac{1}{w_1 w_2 w_3} \sum_{i=0}^{w_3-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_1-1} \int_{\mathbb{Z}_p} (1+t)^{\lambda w_1 w_2 w_3 y + w_1 w_2 w_3 w_4 x + w_1 w_2 w_4 i + w_1 w_3 w_4 j + w_2 w_3 w_4 k} d\mu_0(y) \\
&= \frac{1}{w_1 w_2 w_3 w_4} \lim_{N\to\infty} \frac{1}{p^N} \sum_{l=0}^{w_4-1} \sum_{i=0}^{w_3-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_1-1} \sum_{y=0}^{p^N-1} \\
&\quad \times (1+t)^{\lambda w_1 w_2 w_3 (l + w_4 y) + w_1 w_2 w_3 w_4 x + w_1 w_2 w_4 i + w_1 w_3 w_4 j + w_2 w_3 w_4 k}.
\end{aligned}$$

As this expression is invariant under any permutation $\sigma \in D_4$, we have the following theorem.

Theorem 3.1. *For $w_1, w_2, w_3, w_4 \in \mathbb{N}$, the following expressions*

$$\begin{aligned}
&\frac{1}{w_{\sigma(1)} w_{\sigma(2)} w_{\sigma(3)}} \sum_{i=0}^{w_{\sigma(3)}-1} \sum_{j=0}^{w_{\sigma(2)}-1} \sum_{k=0}^{w_{\sigma(1)}-1} \\
&\times \int_{\mathbb{Z}_p} (1+t)^{\lambda w_{\sigma(1)} w_{\sigma(2)} w_{\sigma(3)} y + w_{\sigma(1)} w_{\sigma(2)} w_{\sigma(3)} w_{\sigma(4)} x + w_{\sigma(1)} w_{\sigma(2)} w_{\sigma(4)} i + w_{\sigma(1)} w_{\sigma(3)} w_{\sigma(4)} j + w_{\sigma(2)} w_{\sigma(3)} w_{\sigma(4)} k} d\mu_0(y),
\end{aligned}$$

are the same for any $\sigma \in D_4$.

Now, we note that

$$\begin{aligned}
&\int_{\mathbb{Z}_p} (1+t)^{\lambda w_1 w_2 w_3 y + w_1 w_2 w_3 w_4 x + w_1 w_2 w_4 i + w_1 w_3 w_4 j + w_2 w_3 w_4 k} d\mu_0(y) \\
&= \sum_{n=0}^{\infty} D_n(w_1 w_2 w_3 w_4 x + w_1 w_2 w_4 i + w_1 w_3 w_4 j + w_2 w_3 w_4 k \mid \lambda w_1 w_2 w_3) \frac{t^n}{n!}.
\end{aligned} \tag{3.4}$$

Therefore, by Theorem 3.1 and (3.4), we obtain the following theorem.

Theorem 3.2. *For $n \geqslant 0$, $w_1, w_2, w_3, w_4 \in \mathbb{N}$, the following expressions*

$$\frac{1}{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}} \sum_{i=0}^{w_{\sigma(3)}-1} \sum_{j=0}^{w_{\sigma(2)}-1} \sum_{k=0}^{w_{\sigma(1)}-1} D_n(w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}x + w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(4)}i$$
$$+ w_{\sigma(1)}w_{\sigma(3)}w_{\sigma(4)}j + w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}k \mid \lambda\sigma(1)w_{\sigma(2)}w_{\sigma(3)}),$$

are the same for any $\sigma \in D_4$.

Now, we observe that

$$\begin{aligned}
&\frac{1}{w_1w_2w_3} \sum_{i=0}^{w_3-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_1-1} D_n(w_1w_2w_3w_4x + w_1w_2w_4i + w_1w_3w_4j + w_2w_3w_4k \mid \lambda w_1w_2w_3) \\
&= \frac{1}{w_2w_3w_4} \sum_{i=0}^{w_4-1} \sum_{j=0}^{w_3-1} \sum_{k=0}^{w_2-1} D_n(w_2w_3w_4w_1x + w_2w_3w_1i + w_2w_4w_1j + w_3w_4w_1k \mid \lambda w_2w_3w_4) \\
&= \frac{1}{w_3w_4w_1} \sum_{i=0}^{w_1-1} \sum_{j=0}^{w_4-1} \sum_{k=0}^{w_3-1} D_n(w_3w_4w_1w_2x + w_3w_4w_2i + w_3w_1w_2j + w_4w_1w_2k \mid \lambda w_3w_4w_1) \\
&= \frac{1}{w_4w_1w_2} \sum_{i=0}^{w_2-1} \sum_{j=0}^{w_1-1} \sum_{k=0}^{w_4-1} D_n(w_4w_1w_2w_3x + w_4w_1w_3i + w_4w_2w_3j + w_1w_2w_3k \mid \lambda w_4w_1w_2) \\
&= \frac{1}{w_2w_1w_4} \sum_{i=0}^{w_4-1} \sum_{j=0}^{w_1-1} \sum_{k=0}^{w_2-1} D_n(w_2w_1w_4w_3x + w_2w_1w_3i + w_2w_4w_3j + w_1w_4w_3k \mid \lambda w_2w_1w_4) \\
&= \frac{1}{w_4w_3w_2} \sum_{i=0}^{w_2-1} \sum_{j=0}^{w_3-1} \sum_{k=0}^{w_4-1} D_n(w_4w_3w_2w_1x + w_4w_3w_1i + w_4w_2w_1j + w_3w_2w_1k \mid \lambda w_4w_3w_2) \\
&= \frac{1}{w_3w_2w_1} \sum_{i=0}^{w_1-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_3-1} D_n(w_3w_2w_1w_4x + w_3w_2w_4i + w_3w_1w_4j + w_2w_1w_4k \mid \lambda w_3w_2w_1) \\
&= \frac{1}{w_1w_4w_3} \sum_{i=0}^{w_3-1} \sum_{j=0}^{w_4-1} \sum_{k=0}^{w_1-1} D_n(w_1w_4w_3w_2x + w_1w_4w_2i + w_1w_3w_2j + w_4w_3w_2k \mid \lambda w_1w_4w_3).
\end{aligned}$$

Therefore, we obtain the following theorem.

Theorem 3.3. *For $n \geqslant 0$, we have*

$$\begin{aligned}
&\frac{1}{w_1w_2w_3} \sum_{i=0}^{w_3-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_1-1} D_n(w_1w_2w_3w_4x + w_1w_2w_4i + w_1w_3w_4j + w_2w_3w_4k \mid \lambda w_1w_2w_3) \\
&= \frac{1}{w_2w_3w_4} \sum_{i=0}^{w_4-1} \sum_{j=0}^{w_3-1} \sum_{k=0}^{w_2-1} D_n(w_2w_3w_4w_1x + w_2w_3w_1i + w_2w_4w_1j + w_3w_4w_1k \mid \lambda w_2w_3w_4) \\
&= \frac{1}{w_3w_4w_1} \sum_{i=0}^{w_1-1} \sum_{j=0}^{w_4-1} \sum_{k=0}^{w_3-1} D_n(w_3w_4w_1w_2x + w_3w_4w_2i + w_3w_1w_2j + w_4w_1w_2k \mid \lambda w_3w_4w_1) \\
&= \frac{1}{w_4w_1w_2} \sum_{i=0}^{w_2-1} \sum_{j=0}^{w_1-1} \sum_{k=0}^{w_4-1} D_n(w_4w_1w_2w_3x + w_4w_1w_3i + w_4w_2w_3j + w_1w_2w_3k \mid \lambda w_4w_1w_2).
\end{aligned}$$

4. Conclusion

We consider the λ-Daehee polynomials such as various degenerate special polynomials over years: Koborov polynomials, λ-Bell polynomials, the degenerate Euler polynomials, the degenerate Bernoulli polynomials, the degenerate Genocchi polynomials and the Changhee polynomials have many applications in the mathematics and mathematical physics (see [4]). In Theorems 2.1 and 2.2, we gave some identities of related to the λ-Daehee polynomials. In Theorems 3.1, 3.2 and 3.3 we obtained new and novel symmetry properties related to λ-Daehee polynomials by using the symmetry properties of the p-adic invariant integral on $\mathbb{Z}_p$.

Acknowledgment

This paper was supported by Wonkwang University in 2017.

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