Event-triggered H_∞ controller design for networked Takagi-Sugeno systems with uncertainties and time delay

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Abstract

This paper considers an event-triggered communication scheme for a class of networked Takagi-Sugeno (T-S) fuzzy systems with uncertainties and time delay. By the parallel distributed compensation fuzzy control rules, a new type of closed-loop nonlinear networked control systems (NCSs) with an interval time delay, uncertainties and event-triggered communication strategy is modeled as a class of networked T-S fuzzy systems. In order to deal with the integral items and convert the coupling time-varying matrix inequalities into a class of decoupling matrix inequalities, a new delay-dependent stabilization criterion is presented firstly. Secondly, some novel criteria for the asymptotic stability analysis and control synthesis of event-triggered networked T-S fuzzy systems with time delay and uncertainties are established in terms of linear matrix inequalities (LMIs). Finally, a numerical example is given to illustrate the effectiveness of the proposed method. ©2017 All rights reserved.

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1. Introduction

Networked control systems (NCSs) are control systems in which controller, sensors and actuators are distributed and connected to a controlled plant [26]. While compared with the traditional control systems with point-to-point connection, the main advantages of NCSs are low cost, increased systems flexibility, reduced weight and power requirements. However, the insertion of a communication network causes some challenging issues such as networked-induced delays, packet dropouts, sensor saturation, fading channels and incomplete information, which may lead to performance degradation or even instability of an NCSs [4, 6, 13–17, 20, 22, 24, 25, 27].

Generally, networked-induced delays, packet dropouts and disorder are mainly caused by the limited network bandwidth. Communication bandwidth is a scarce resource, if the signal transmission only occurs

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when the relevant information from the sensor to the controller needs to be transmitted, then not only all of the transmitted signals desired performance, but also the inefficient information is possibly avoided in the transmission. Thus, some network resources can be released to other communication tasks in need [1]. As to the limited network bandwidth, two following methods are usually used to reduce the network communication load. The first method is used to reduce the number of the released data. Event-triggering communication scheme has been proved to be an efficient way to reduce the transmitted data in the networks [7, 9, 10, 12, 19, 28–30]. The second method is to reduce the size of the transmitted data. It considers the event-triggered communication scheme and quantization designs for NCSs in [3].

In order to save the limited network resources, some techniques and results have been employed in the literatures. For controlling a discrete time distributed system over a network, Yook et al. in [21] outlined a deterministic communication framework for reducing the communication burden. Recently, Wang and Lemmon [11] proposed an event-triggered communication scheme for NCSs, where a subsystem broadcasts its state information to its neighbors only when the subsystem's local state error exceeds a specified threshold. However, the common assumption in the above mentioned work is that the controller must be known as prior. If the controller is not predetermined, the above mentioned methods are no longer valid. Some existing methods for T-S fuzzy systems [2, 8, 18] have employed the model transformation technique or bounding inequality [5, 23] for bounding integral terms. Therefore, the T-S fuzzy-model-based approach has shown its advantages in using a small number of rules to model higher-order nonlinear systems.

In this paper, we will develop a novel adaptive event-triggered communication scheme to save the limited network resources and H_∞ controller design method for the T-S fuzzy system with time delays. Different from some existing ones, the T-S fuzzy system in this paper is modeled as a delay system considering the network-stated delays and adaptive event-triggering scheme. Using Lyapunov-Krasovskii function approach, some new sufficient conditions in terms of LMIs are obtained such that guarantee the asymptotic stability of the closed-loop NCSs systems. The explicit expression of H_∞ controller is also derived for the networked T-S fuzzy systems with event-triggered scheme, network-induced delays and uncertainties. Finally, a simulation example is provided to illustrate the effectiveness of the proposed method.

2. System framework

Consider the following uncertain T-S fuzzy system with time-delay. The i th rule of the system is expressed as follows:

R^i : if $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} , ..., and $\theta_g(t)$ is M_{ig} , then

$$\begin{cases} \dot{x}(t) = \bar{A}_i x(t) + A_{di} x(t - \tau_d) + \bar{B}_i u(t) + B_{\omega i} \omega(t), \\ z(t) = C_i x(t) + D_i(t), \end{cases} \quad (2.1)$$

where $i = 1, 2, \dots, r$, r is the number of inferred rules, and $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are state vector and input vector, respectively. $\theta_j(t)$ ($j = 1, 2, \dots, g$) is the premise variables, and M_{ij} ($i = 1, 2, \dots, r; j = 1, 2, \dots, g$) are the fuzzy sets. Denote $\theta(t) = [\theta_1(t), \dots, \theta_g(t)]^T$, and assume that $\theta(t)$ is either given or a function of $x(t)$ and does not depend on $u(t)$. $\omega(t)$ denotes exogenous disturbance signal, $z(t) \in \mathbb{R}^p$ denotes system output. $A_i, A_{di}, B_i, B_{\omega i}, C_i$ and D_i are constant matrices with compatible dimensions. $\bar{A}_i = A_i + \Delta A_i(t)$, $\bar{B}_i = B_i + \Delta B_i(t)$, $\Delta A_i(t)$ and $\Delta B_i(t)$ are time-varying matrices assumed to satisfy the following admissible condition:

$$\begin{bmatrix} \Delta A_i(t) & \Delta B_i(t) \end{bmatrix} = G_i F_i(t) \begin{bmatrix} E_{ai} & E_{bi} \end{bmatrix},$$

where G_i, E_{ai} , and E_{bi} are constant matrices with appropriate dimensions and $F_i(t)$ is an unknown time-varying matrix satisfying

$$F_i^T(t) F_i(t) \leq I. \quad (2.2)$$

The dynamics of the T-S fuzzy system (2.1) is defined as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r u_i(\theta(t)) [\bar{A}_i x(t) + A_{di} x(t-\tau_d) + \bar{B}_i u(t) + B_{\omega i} \omega(t)], \\ z(t) = \sum_{i=1}^r u_i(\theta(t)) [C_i x(t) + D_i u(t)], \end{cases} \quad (2.3)$$

where $u_i(\theta(t)) = \frac{h_i(\theta(t))}{\sum_{i=1}^r h_i(\theta(t))}$ and $h_i(\theta(t)) = \prod_{j=1}^g M_{ij}(\theta_j(t))$, $M_{ij}(\theta_j(t))$ is the membership value of $\theta_j(t)$ in M_{ij} .

In this paper, it is assumed that $u_i(\theta(t)) \geq 0$, $\sum_{i=1}^r u_i(\theta(t)) > 0$ for all $t > 0$. The fuzzy weighting functions $u_i(\theta(t))$ satisfy $u_i(\theta(t)) \geq 0$ and $\sum_{i=1}^r u_i(\theta(t)) = 1$.

Before further proceeding, the following assumptions are made, which are common in NCSs research in the literatures.

Assumption 2.1. Communication delays from the sensor to the controller and from the controller to the actuator, and the computational and waiting delays are lumped together as $\eta_k \in [\eta_m, \eta_M]$, where η_m and η_M are the lower and upper bounds of η_k , respectively.

Assumption 2.2. The holding time of a zero order hold (ZOH) at the actuator is $t \in \Omega = [t_k h + \eta_k, t_{k+1} h + \eta_{k+1})$, where η_k is the transmission delay, $t_k h + \eta_k$ is the instant when the control signal reaches the ZOH.

Considering the limited bandwidth of the communication network, we propose an event-triggered communication scheme for the NCSs. The event-triggered communication scheme can be expressed as

$$t_{k+1} h = t_k h + \min\{e^T(i_k h) \Phi e(i_k h) > \delta x^T(t_k h) \Phi x(t_k h)\}, \quad (2.4)$$

where $\Phi > 0$ is a weighting matrix. The threshold error between the current sampled data and the latest transmission one is defined as

$$e(i_k h) = x(i_k h) - x(t_k h),$$

where $i_k h = t_k h + n h$, $n \in \mathbb{N}$.

Remark 2.3. Under the communication scheme (2.4) and the above assumption, we divided the ZOH $t \in \Omega$ into $\Omega_{n,k} = [t_k h + n h + \eta_k, t_k h + n h + h + \eta_{k+1})$, where $n = 0, \dots, t_{k+1} - t_k - 1$.

Define $\eta(t) = t - i_k h$, $t \in \Omega_{n,k}$. Then the transmitted state $x(t_k h)$ can be written as

$$x(t_k h) = x(t - \eta(t)) - e_k(i_k h), t \in \Omega_{n,k}. \quad (2.5)$$

It is clear that $\eta(t)$ is a piecewise-linear function satisfying

$$\eta_m \leq \eta_k \leq \eta(t) \leq h + \max\{\eta_k, \eta_{k+1}\} \leq h + \eta_M, t \in \Omega_{n,k}.$$

Let $\tau_1 = \eta_m$, $\tau_2 = h + \eta_M$. In this section, a T-S fuzzy model controller will be designed to stabilize the T-S fuzzy system (2.3). The i th controller rule is

R^i : if $\theta_1(t_k h)$ is M_{i1} , $\theta_2(t_k h)$ is $M_{i2}, \dots$, and $\theta_g(t_k h)$ is M_{ig} , then

$$u(t) = K_j x(t_k h), t \in [t_k h + \eta_k, t_{k+1} h + \eta_{k+1}), \quad (2.6)$$

where K_j ($j = 1, 2, \dots, r$) are controller gains to be determined.

Combining (2.5) and (2.6) together, the defuzzified output of the controller is

$$u(t) = \sum_{j=1}^r h_j(\theta(t_k h)) K_j (x(t - \eta(t)) - e_k(i_k h)), t \in \Omega_{n,k}. \quad (2.7)$$

Combining (2.7) together with (2.2), the closed-loop fuzzy system can be described as

$$\begin{cases} \dot{x}(t) = \tilde{A}_i x(t) + \tilde{A}_{di} x(t - \tau_d) + \tilde{B}_i x(t - \eta(t)) - \tilde{B}_i e_k(i_k h) + \tilde{B}_{\omega i} \omega(t), \\ z(t) = \tilde{C}_i x(t) + \tilde{K}_j \tilde{D}_i x(t - \eta(t)) - \tilde{K}_j \tilde{D}_i e_k(i_k h), \end{cases} \quad (2.8)$$

where

$$\begin{aligned} \tilde{A}_i &= \sum_{i=1}^r u_i(\theta(t)) [A_i + \Delta A_i(t)], & \tilde{A}_{di} &= \sum_{i=1}^r u_i(\theta(t)) A_{di}, & \bar{A}_i &= A_i + \Delta A_i(t), \\ \tilde{B}_i &= \sum_{i=1}^r u_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t_k h)) [B_i + \Delta B_i(t)] K_j, & \tilde{B}_{\omega i} &= \sum_{i=1}^r u_i(\theta(t)) B_{\omega i}, & \tilde{B}_{\omega i} &= \sum_{i=1}^r u_i(\theta(t)) B_{\omega i}, \\ \tilde{D}_i &= \sum_{i=1}^r u_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t_k h)) D_i, & \tilde{C}_i &= \sum_{i=1}^r u_i(\theta(t)) C_i, \end{aligned}$$

where the initial condition of the state is $x(t) = \Phi(t)$, $\Phi(0) = x_0$, $t \in [-\tau_2, -\tau_1]$.

Remark 2.4. From the above analysis, it is derived that

$$e^T(i_k h) \Phi e(i_k h) \leq \delta \rho^T(i_k h) \Phi \rho(i_k h), \quad (2.9)$$

where $\rho(i_k h) = x(t - \tau(t)) - e(i_k h)$, $t \in \Omega_{n,k}$, which will be employed in the stability analysis and controller synthesis of the system (2.8) later.

Lemma 2.5 ([22]). *For any matrices V_1, V_2, V_3 with appropriate dimensions, and $\eta(t)$ is time-varying delay with lower and upper delay bounds η_1 and η_2 , the condition of*

$$V_1 + (\eta_2 - \eta(t))V_2 + (\eta(t) - \eta_1)V_3 < 0$$

holds if and only if

$$V_1 + (\eta_2 - \eta_1)V_2 < 0, \quad V_1 + (\eta_2 - \eta_1)V_3 < 0.$$

Lemma 2.6 ([7]). *For a given symmetric matrix Ω_1 and any real matrices Ω_2, Ω_3 with appropriate dimensions $\Omega_1 + \text{sym}\{\Omega_2 \Delta \Omega_3\} < 0$ for all $\Delta \in \Theta$, where $\Theta = \{\Delta = \text{diag}(\Delta_1, \dots, \Delta_k, \xi_1 I, \dots, \xi_l I) : \|\Delta\| \leq 1, \Delta_i \in \mathbb{R}^{n_i \times n_i}, i = 1, \dots, k, \xi_j \in \mathbb{R}, j = 1, \dots, l, k, l \in \mathbb{Z}^+\}$ if and only if there exists an $L \in \zeta$ such that*

$$\begin{bmatrix} \Omega_1 + \Omega_3^T L \Omega & * \\ \Omega_2 & -L \end{bmatrix} < 0$$

holds, where $\zeta = \{\text{diag}(s_1 I, \dots, s_k I, s_1, \dots, s_l) : 0 < s_i \in \mathbb{R}, 0 < s_j \in \mathbb{R}^{n_i \times n_i}, k, l \in \mathbb{Z}^+\}$.

Definition 2.7. The closed-loop system (2.8) is said to be asymptotically stable with an H_∞ disturbance attenuation level γ such that

- (1) system (2.8) is asymptotically stable with $\omega(t) = 0$;
- (2) under zero initial condition, $\|z(t)\|_2 < \gamma \|\omega(t)\|_2$, for any nonzero $\omega(t) \in \zeta_2[0, \infty)$ and a prescribed $\gamma > 0$.

3. Main results

3.1. Stability analysis

Theorem 3.1. *For given scalars $\tau_1, \tau_2, \tau_d, \delta, \gamma$ and a matrix $\Phi > 0$, the equilibrium of the system (2.8) is asymptotically stable, if there exist symmetric matrices $P > 0, Q_i > 0, R_i > 0$ ($i = 1, 2, 3$), $W_j > 0$ ($j = 1, 2$) and matrices M with appropriate dimensions such that the following matrix inequalities hold for $i, j = 1, 2, \dots, r, i \leq j$,*

$$\Pi_1^{ij} + \Pi_1^{ji} < 0, \quad (3.1)$$

$$\Pi_2^{ij} + \Pi_2^{ji} < 0, \quad (3.2)$$

$$\begin{bmatrix} R_2 & * \\ U & R_2 \end{bmatrix} > 0, \quad (3.3)$$

where

$$\begin{aligned} \Pi_1^{ij} &= \begin{bmatrix} \Phi_1^{ij} & * & * & * & * \\ \tau_1^2 R_1 \Gamma_1^{ij} & \Xi_1^{ij} & * & * & * \\ (\tau_2 - \tau_1)^2 R_2 \Gamma_1^{ij} & 0 & \Xi_2^{ij} & * & * \\ \tau_d^2 R_3 \Gamma_1^{ij} & 0 & 0 & \Xi_3^{ij} & * \\ (\tau_2 - \tau_1) W_2 \Gamma_1^{ij} & 0 & 0 & 0 & \Xi_4^{ij} \end{bmatrix}, & \Pi_2^{ij} &= \begin{bmatrix} \Phi_2^{ij} & * & * & * & * \\ \tau_1^2 R_1 \Gamma_1^{ij} & \Xi_1^{ij} & * & * & * \\ (\tau_2 - \tau_1)^2 R_2 \Gamma_1^{ij} & 0 & \Xi_2^{ij} & * & * \\ \tau_d^2 R_3 \Gamma_1^{ij} & 0 & 0 & \Xi_3^{ij} & * \\ (\tau_2 - \tau_1) M^T & 0 & 0 & 0 & \Xi_4^{ij} \end{bmatrix}, \\ \Pi^{ij} &= \begin{bmatrix} \Pi_{11}^{ij} & * \\ \Pi_{12}^{ij} & \Pi_{13}^{ij} \end{bmatrix}, & \Pi_{11}^{ij} &= \begin{bmatrix} \Phi_3^{ij} & * & * \\ A_{di}^T P + R_3 & \Phi_4^{ij} & * \\ K_j^T \bar{B}_i^T P + W_1 & 0 & \Phi_5^{ij} \end{bmatrix}, \\ \Pi_{12}^{ij} &= \begin{bmatrix} -K_j^T \bar{B}_i^T P & 0 & -\delta \Phi^T \\ B_{\omega i}^T P & 0 & 0 \\ R & 0 & R_2 - U^T \\ 0 & 0 & R_2 - U \end{bmatrix}, & \Pi_{13}^{ij} &= \begin{bmatrix} \Phi_6^{ij} & * & * & * \\ 0 & -\gamma^2 I & * & * \\ 0 & 0 & \Phi_7^{ij} & * \\ 0 & 0 & U & \Phi_8^{ij} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \Phi_1^{ij} &= \Pi^{ij} + 2M\Gamma_2^{ij} + (\Gamma_3^{ij})^T \Gamma_3^{ij} + \tau_1 M W_2^{-1} M^T + (\tau_2 - \tau_1)(\Gamma_1^{ij})^T W_1^T \Gamma_2^{ij} + (\tau_2 - \tau_1)(\Gamma_2^{ij})^T W_1 \Gamma_1^{ij}, \\ \Phi_2^{ij} &= \Pi^{ij} + 2M\Gamma_2^{ij} + (\Gamma_3^{ij})^T \Gamma_3^{ij} + \tau_1 M W_2^{-1} M^T, \quad \Xi_1^{ij} = -\tau_1^2 R_1, \quad \Xi_2^{ij} = -(\tau_2 - \tau_1)^2 R_2, \quad \Xi_3^{ij} = -\tau_d^2 R_3, \\ \Xi_4^{ij} &= -(\tau_2 - \tau_1) W_2, \quad \Phi_3^{ij} = P \bar{A}_i + \bar{A}_i^T P + Q_1 + Q_3 - R_1 - R_3 - W_1, \quad \Phi_4^{ij} = -Q_3 - R_3, \\ \Phi_8^{ij} &= -Q_2 - R_2, \quad \Phi_5^{ij} = -2R_2 + U^T + U - W_1 + \delta \Phi, \quad \Phi_6^{ij} = (\delta - 1)\Phi, \quad \Phi_7^{ij} = Q_2 - Q_1 - R_1 - R_2, \\ \Gamma_1^{ij} &= [\bar{A}_i \ A_{di} \ \bar{B}_i K_j \ -\bar{B}_i K_j \ B_{wi} \ 0 \ 0], \quad \Gamma_2^{ij} = [I \ 0 \ -I \ 0 \ 0 \ 0 \ 0], \quad \Gamma_3^{ij} = [C_i \ 0 \ D_i K_j \ -D_i K_j \ 0 \ 0 \ 0]. \end{aligned}$$

Proof. Construct a Lyapunov functional candidate for system (2.8) as

$$V(t, x(t)) = \sum_{i=1}^4 V_i(t, x(t)), \quad (3.4)$$

where

$$\begin{aligned} V_1(t, x(t)) &= x^T(t) P x(t), \\ V_2(t, x(t)) &= \int_{t-\tau_1}^t x^T(v) Q_1 x(v) dv + \int_{t-\tau_2}^{t-\tau_1} x^T(v) Q_2 x(v) dv + \int_{t-\tau_d}^{t-\tau_1} x^T(v) Q_3 x(v) dv, \\ V_3(t, x(t)) &= \tau_1 \int_{t-\tau_1}^t \int_s^t \dot{x}^T(v) R_1 \dot{x}(v) dv ds + (\tau_2 - \tau_1) \int_{t-\tau_2}^{t-\tau_1} \int_s^t \dot{x}(v) R_2 \dot{x}(v) dv ds \\ &\quad + \tau_d \int_{t-\tau_d}^t \int_s^t \dot{x}^T(v) R_3 \dot{x}(v) dv ds, \\ V_4(t, x(t)) &= (\tau_2 - \eta(t)) \int_{t-\eta(t)}^t \dot{x}^T(t) W_2 \dot{x}(v) dv + (\tau_2 - \eta(t)) [x(t) - x(t - \eta(t))]^T W_1 [x(t) - x(t - \eta(t))]. \end{aligned}$$

Taking the derivation of $V(t, x(t))$ for $t \in [t_k h + \eta_k, t_{k+1} h + \eta_{k+1})$ and by adding and subtracting the term $e^T(i_k h) \Phi e(i_k h)$, we have

$$\dot{V}(t, x(t)) = \sum_{i=1}^4 \dot{V}_i(t, x(t)), \quad (3.5)$$

where

$$\begin{aligned}\dot{V}_1(t, x(t)) &= 2x^T(t)P\dot{x}(t), \\ \dot{V}_2(t, x(t)) &= x^T(t)[Q_1 + Q_3]x(t) + x^T(t - \tau_1)[Q_2 - Q_1]x(t - \tau_1) - x^T(t - \tau_2)Q_2x(t - \tau_2) \\ &\quad - x^T(t - \tau_d)Q_3x(t - \tau_d), \\ \dot{V}_3(t, x(t)) &= \tau_1^2 \dot{x}^T(t)R_1\dot{x}(t) - \tau_1 \int_{t-\tau_1}^t \dot{x}^T(s)R_1\dot{x}(s)ds + (\tau_2 - \tau_1)^2 \dot{x}^T(t)R_2\dot{x}(t) + \tau_d^2 \dot{x}^T(t)R_3\dot{x}(t) \\ &\quad - (\tau_2 - \tau_1) \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)R_2\dot{x}(s)ds - \tau_d \int_{t-\tau_d}^t \dot{x}^T(s)R_3\dot{x}(s)ds, \\ \dot{V}_4(t, x(t)) &= (\tau_2 - \eta(t))\dot{x}(t)W_2\dot{x}(t) - \int_{t-\eta(t)}^t \dot{x}(v)W_2\dot{x}(v)dv - [x(t) - x(t - \eta(t))]^T W_1[x(t) \\ &\quad - x(t - \eta(t))] + 2(\tau_2 - \eta(t))[x(t) - x(t - \eta(t))]^T W_1\dot{x}(t).\end{aligned}$$

Using Jessen's inequality and reciprocally convex approach to deal with integral items in (3.3), we obtain

$$-\tau_1 \int_{t-\tau_1}^t \dot{x}^T(s)R_1\dot{x}(s)ds \leq -\xi_1^T(t)\Pi_1\xi_1(t), \quad (3.6)$$

$$-(\tau_2 - \tau_1) \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)R_2\dot{x}(s)ds \leq -\xi_2^T(t)\Pi_2\xi_2(t), \quad (3.7)$$

where

$$\xi_1^T(t) = [x^T(t) \ x^T(t - \tau_1)], \quad \xi_2^T(t) = [x^T(t - \tau_1) \ x^T(t - \tau(t)) \ x^T(t - \tau_2)]$$

and

$$\begin{aligned}\Pi_1 &= \begin{bmatrix} R_1 & * \\ -R_1 & R_1 \end{bmatrix}, \\ \Pi_2 &= \begin{bmatrix} R_2 & * & * \\ U - R_2 & 2R_2 - U^T - U & * \\ -U & U - R_2 & R_2 \end{bmatrix}.\end{aligned}$$

Using the Newton-Leibniz formula and the relationship, we have

$$2\varepsilon^T(t)M[x(t) - x(t - \eta(t)) - \int_{t-\eta(t)}^t \dot{x}(s)ds] = 0, \quad (3.8)$$

where

$$\varepsilon^T(t) = [x^T(t), x^T(t - \tau_d), x^T(t - \eta(t)), e_k^T(i_k h), \omega^T(t), x^T(t - \tau_1), x^T(t - \tau_2)].$$

From the basic inequality $-2ab \leq \rho a^T + b^T \rho^{-1}b$, $\rho > 0$, there always exists real matrix $W_2 > 0$ such that

$$-2\varepsilon^T(t)M \int_{t-\eta(t)}^t \dot{x}(s)ds \leq \eta(t)\varepsilon^T(t)MW_2^{-1}M\varepsilon(t) + \int_{t-\eta(t)}^t \dot{x}(s)W_2\dot{x}(s)ds.$$

Combining (3.4), (3.5), (3.6), (3.7), (3.8), and (2.9), we obtain

$$\dot{V}(t, x(t)) \leq \sum_{i=1}^r \sum_{j=1}^r u_i(\theta(t))u_j(\theta(t))\varepsilon^T(t)(\Sigma^{ij} + \Sigma^{ji})\varepsilon(t) + \gamma^2 \omega^T(t)\omega(t) - z^T(t)z(t), \quad (3.9)$$

where

$$\Sigma^{ij} = \Pi^{ij} + \Sigma_1^{ij} + (\tau_2 - \eta(t))\Sigma_2^{ij} + \tau_1 MW_2^{-1}M^T + (\eta(t) - \tau_1)MW_2^{-1}M^T + (\Gamma_3^{ij})^T \Gamma_3^{ij} + 2M\Gamma_2^{ij},$$

$$\Sigma_1^{ij} = (\Gamma_1^{ij})^T [\tau_1^2 R_1 + (\tau_2 - \tau_1)^2 R_2 + \tau_d^2 R_3] \Gamma_1^{ij}, \quad \Sigma_2^{ij} = (\Gamma_1^{ij})^T W_2 \Gamma_1^{ij} + 2(\Gamma_2^{ij})^T W_1 \Gamma_1^{ij}$$

By Schur complement, (3.1) and (3.2) imply that

$$\varepsilon^T(t)(\Sigma^{ij} + \Sigma^{ji})\varepsilon(t) < 0.$$

Then it can be concluded from (3.9) and (3.5) that

$$\dot{V}(t, x(t)) \leq \gamma^2 \omega^T(t) \omega(t) - z^T(t) z(t). \quad (3.10)$$

Integrating both sides of (3.10) from t_0 to t and letting $t \rightarrow \infty$, we can get the following inequality with the zero initial condition:

$$\int_{t_0}^{\infty} z^T(s) z(s) ds \leq \int_{t_0}^{\infty} \gamma^2 \omega^T(s) \omega(s) ds,$$

which implies that $\|z(t)\| < \gamma \|\omega(t)\|_2$. Next we can prove the stability of system (2.8). Combining (3.1) and (3.2) and using Schur complement, when $\omega(t) = 0$, we have $\dot{V}(t, x(t)) < 0$. This completes the proof. $\square$

Since $\bar{A}_i$ and $\bar{B}_i$ contain uncertainties in (3.1) and (3.2), Theorem 3.1 cannot be determined directly for the stability of system (2.8). The following results of Theorem 3.2 will provide sufficient criterion for the closed-loop system (2.8) with uncertainties to be robustly asymptotically stable.

Theorem 3.2. *Under the event-triggered communication scheme (2.4), the system (2.8) is robustly asymptotically stable with an H_∞ performance index γ for the disturbance attention, for some given positive scalars τ_1, τ_2, δ and matrix $\Phi > 0$, if there exist real symmetric matrices $P > 0$, $Q_i > 0$, $R_i > 0$ ($i = 1, 2, 3$), $W_j > 0$ ($j = 1, 2$) and matrices M with appropriate dimension, and scalars $\varepsilon_{ij} > 0$, such that the following matrix inequalities hold for $i, j = 1, 2, \dots, r, i \leq j$,*

$$\Pi_3^{ij} + \Pi_3^{ji} < 0, \quad (3.11)$$

$$\Pi_4^{ij} + \Pi_4^{ji} < 0, \quad (3.12)$$

where

$$\begin{aligned} \Pi_3^{ij} &= \begin{bmatrix} \Phi_{11}^{ij} & * & * & * \\ \Phi_{21}^{ij} & \Phi_{22}^{ij} & * & * \\ E_{ij} & 0 & -\varepsilon_{ij} I & * \\ \varepsilon_{ij} H_{ij} & \varepsilon_{ij} \Phi_{42}^{ij} & 0 & -\varepsilon_{ij} I \end{bmatrix}, \\ \Pi_4^{ij} &= \begin{bmatrix} \Theta_{11}^{ij} & * & * & * \\ \Theta_{21}^{ij} & \Phi_{22}^{ij} & * & * \\ E_{ij} & 0 & -\varepsilon_{ij} I & * \\ \varepsilon_{ij} M_{ij} & \varepsilon_{ij} \Theta_{42}^{ij} & 0 & -\varepsilon_{ij} I \end{bmatrix}, \\ \Pi_5^{ij} &= \begin{bmatrix} \Pi_{51}^{ij} & * \\ \Pi_{52}^{ij} & \Pi_{53}^{ij} \end{bmatrix}, \\ \Pi_{51}^{ij} &= \begin{bmatrix} \Phi_{31}^{ij} & * & * \\ A_{di}^T P + R_3 & \Phi_4^{ij} & * \\ K_j^T B_i^T P + W_1 & 0 & \Phi_5^{ij} \end{bmatrix}, \\ \Pi_{52}^{ij} &= \begin{bmatrix} -K_j^T B_i^T P & 0 & -\delta \Phi^T \\ B_{\omega i}^T P & 0 & 0 \\ R_1 & 0 & R_2 - U^T \\ 0 & 0 & R_2 - U \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
\Phi_{31}^{ij} &= PA_i + A_i^T P + Q_1 + Q_3 - R_1 - R_3 - W_1, \\
\Phi_{11}^{ij} &= \Pi_5^{ij} + 2M\Gamma_2^{ij} + (\Gamma_3^{ij})^T \Gamma_3^{ij} + \tau_1 MW_2^{-1} M^T + (\tau_2 - \tau_1)(\Gamma_{11}^{ij})^T W_1 \Gamma_2^{ij} + (\tau_2 - \tau_1)(\Gamma_2^{ij})^T W_1 \Gamma_{11}^{ij}, \\
\Theta_{11}^{ij} &= \Pi_5^{ij} + 2M\Gamma_2^{ij} + (\Gamma_3^{ij})^T \Gamma_3^{ij} + \tau_1 MW_2^{-1} M^T, \\
\Phi_{31}^{ij} &= PA_i + A_i^T P + Q_1 + Q_3 - R_1 - R_3 - W_1, \\
\Phi_{21}^{ij} &= \text{col}\{\tau_1^2 R_1 \Gamma_{11}^{ij}, (\tau_2 - \tau_1)^2 R_2 \Gamma_{11}^{ij}, \tau_d^2 R_3 \Gamma_{11}^{ij}, (\tau_2 - \tau_1) W_2 \Gamma_{11}^{ij}\}, \\
\Phi_{22}^{ij} &= \text{diag}\{-\tau_1^2 R_1, -(\tau_2 - \tau_1)^2 R_2, -\tau_d^2 R_3, -(\tau_2 - \tau_1) W_2\}, \\
\Phi_{42}^{ij} &= [\tau_1^2 G_i^T R_1, (\tau_2 - \tau_1)^2 G_i^T R_2, \tau_d^2 G_i^T R_3, (\tau_2 - \tau_1) G_i^T W_2], \\
\Theta_{21}^{ij} &= \text{col}\{\tau_1^2 R_1 \Gamma_{11}^{ij}, (\tau_2 - \tau_1)^2 R_2 \Gamma_{11}^{ij}, \tau_d^2 R_3 \Gamma_{11}^{ij}, (\tau_2 - \tau_1) M^T\}, \\
\Theta_{42}^{ij} &= [\tau_1^2 G_i^T R_1, (\tau_2 - \tau_1)^2 G_i^T R_2, \tau_d^2 G_i^T R_3, 0], \\
\Gamma_{11}^{ij} &= [A_i \ A_{di} \ B_i K_j \ -B_i K_j \ B_{wi} \ 0 \ 0], \\
E_{ij} &= [E_{ai} \ 0 \ E_{bi} K_j \ -E_{bi} K_j \ 0 \ 0 \ 0], \\
H_{ij} &= [G_i^T P + (\tau_2 - \tau_1) G_i^T W_1 \ 0 \ -(\tau_2 - \tau_1) G_i^T W_1 \ 0 \ 0 \ 0], \\
M_{ij} &= [G_i^T P \ 0 \ 0 \ 0 \ 0 \ 0 \ 0].
\end{aligned}$$

Proof. Decomposing the resulting matrix inequalities (3.1) and (3.2) into nominal and uncertainties parts lead to

$$L_{ij} = \tilde{L}_{ij} + D_i^T F_i(t) V_{ij} + V_{ij}^T F_i^T(t) D_i,$$

where $\tilde{L}_{ij}$ denotes the system (2.8), and $\tilde{L}_{ij} = L_{ij}$ subject to $\bar{A}_i$ and $\bar{B}_i$ in L_{ij} are replaced by A_i and B_i . It is clear that there exists $\varepsilon_{ij} > 0$, thus the following inequalities always hold:

$$\tilde{L}_{ij} + D_i^T F_i(t) V_{ij} + V_{ij}^T F_i^T(t) D_i \leq \tilde{L}_{ij} + D_i^T \varepsilon_{ij} V_{ij} + V_{ij}^T \varepsilon_{ij}^{-1} D_i,$$

Using Schur complement, one can obtain (3.11). Similarly, one can also obtain (3.12) from matrix inequalities (3.2). This completes the proof. $\square$

3.2. H_∞ controller design

Theorem 3.3. For some given positive constants τ_1, τ_2, δ and matrix $\Phi > 0$, under the event-triggered communication scheme (2.4), the system (2.8) is asymptotically stable with an H_∞ performance index γ and the controller feedback gains $K_j = Y_j X^{-1}$, if there exist real matrices $\tilde{P} > 0, \tilde{Q}_i > 0, \tilde{R}_i > 0$ ($i = 1, 2, 3$), $\tilde{W}_j > 0$ ($j = 1, 2$) and matrices $\tilde{M}$ with appropriate dimensions such that the following LMIs hold for $i, j = 1, 2, \dots, r, i \leq j$,

$$\tilde{\Pi}_3^{ij} + \tilde{\Pi}_3^{ji} < 0, \quad (3.13)$$

$$\tilde{\Pi}_4^{ij} + \tilde{\Pi}_4^{ji} < 0, \quad (3.14)$$

$$\begin{bmatrix} \tilde{R}_2 & * \\ \tilde{U} & \tilde{R}_2 \end{bmatrix} > 0,$$

where

$$\begin{aligned}
\tilde{\Pi}_3^{ij} &= \begin{bmatrix} \tilde{\Phi}_{11}^{ij} & * & * & * \\ \tilde{\Phi}_{21}^{ij} & \tilde{\Phi}_{22}^{ij} & * & * \\ \tilde{E}_{ij} & 0 & -\varepsilon_{ij} I & * \\ \varepsilon_{ij} \tilde{H}_{ij} & \varepsilon_{ij} \tilde{\Phi}_{42}^{ij} & 0 & -\varepsilon_{ij} I \end{bmatrix}, \\
\tilde{\Pi}_4^{ij} &= \begin{bmatrix} \tilde{\Phi}_{11}^{ij} & * & * & * \\ \tilde{\Theta}_{21}^{ij} & \tilde{\Theta}_{22}^{ij} & * & * \\ \tilde{E}_{ij} & 0 & -\varepsilon_{ij} I & * \\ \varepsilon_{ij} \tilde{M}_{ij} & \varepsilon_{ij} \tilde{\Theta}_{42}^{ij} & 0 & -\varepsilon_{ij} I \end{bmatrix}, \quad \tilde{\Pi}_5^{ij} = \begin{bmatrix} \tilde{\Pi}_{51}^{ij} & * \\ \tilde{\Pi}_{52}^{ij} & \tilde{\Pi}_{53}^{ij} \end{bmatrix},
\end{aligned}$$

$$\tilde{\Pi}_{51}^{ij} = \begin{bmatrix} \tilde{\Phi}_{31}^{ij} & * & * \\ XA_{di}^T + \tilde{R}_3 & \tilde{\Phi}_{41}^{ij} & * \\ Y_j^T B_i^T + \tilde{W}_1 & 0 & \tilde{\Phi}_{51}^{ij} \end{bmatrix},$$

$$\tilde{\Pi}_{52}^{ij} = \begin{bmatrix} -Y_j^T B_i^T & 0 & -\delta \tilde{\Phi}^T \\ B_{wi}^T & 0 & 0 \\ \tilde{R}_1 & 0 & \tilde{R}_2 - \tilde{U}^T \\ 0 & 0 & \tilde{R}_2 - \tilde{U} \end{bmatrix},$$

$$\tilde{\Pi}_{53}^{ij} = \begin{bmatrix} \tilde{\Phi}_{61}^{ij} & * & * & * \\ 0 & -\gamma^2 I & * & * \\ 0 & 0 & \tilde{\Phi}_{71}^{ij} & * \\ 0 & 0 & \tilde{U} & \tilde{\Phi}_{81}^{ij} \end{bmatrix},$$

$$\tilde{\Phi}_{11}^{ij} = \Pi_5^{ij} + 2\tilde{M}\Gamma_2^{ij}, \quad \tilde{\Phi}_{41}^{ij} = -\tilde{Q}_3 - \tilde{R}_3, \quad \tilde{\Phi}_{31}^{ij} = A_i X + XA_i^T + \tilde{Q}_1 + \tilde{Q}_3 - \tilde{R}_1 - \tilde{R}_3 - \tilde{W}_1,$$

$$\tilde{\Phi}_{51}^{ij} = -2\tilde{R}_2 + \tilde{U}^T + \tilde{U} - \tilde{W}_1 + \delta \tilde{\Phi}, \quad \tilde{\Phi}_{81}^{ij} = -\tilde{Q}_2 - \tilde{R}_2, \quad \tilde{\Phi}_{61}^{ij} = (\delta - 1)\tilde{\Phi}, \quad \tilde{\Phi}_{71}^{ij} = \tilde{Q}_2 - \tilde{Q}_1 - \tilde{R}_1 - \tilde{R}_2$$

$$\tilde{\Phi}_{21}^{ij} = \text{col}\{\tau_1^2 \tilde{f}_2^{ij}, (\tau_2 - \tau_1)^2 \tilde{f}_2^{ij}, \tau_d^2 \tilde{f}_2^{ij}, (\tau_2 - \tau_1) \tilde{f}_2^{ij}, \tilde{f}_3^{ij}, \tau_1 \tilde{M}^T, \tilde{f}_2^{ij}, (\tau_2 - \tau_1) \Gamma_2^{ij} \tilde{W}_1\},$$

$$\tilde{\Phi}_{22}^{ij} = -\text{diag}\{-\tau_1^2 X \tilde{R}_1^{-1} X, -(\tau_2 - \tau_1)^2 X \tilde{R}_2^{-1} X, -\tau_d^2 X \tilde{R}_3^{-1} X, -(\tau_2 - \tau_1) X \tilde{W}_2^{-1} X, I, \tau_1 \tilde{W}_2, \alpha X \alpha^{-1} X\},$$

$$\tilde{\Theta}_{21}^{ij} = \text{col}\{\tau_1^2 \tilde{f}_2^{ij}, (\tau_2 - \tau_1)^2 \tilde{f}_2^{ij}, \tau_d^2 \tilde{f}_2^{ij}, (\tau_2 - \tau_1) \tilde{M}, \tilde{f}_3^{ij}, \tau_1 \tilde{M}^T\},$$

$$\tilde{\Theta}_{22}^{ij} = -\text{diag}\{-\tau_1^2 X \tilde{R}_1^{-1} X, -(\tau_2 - \tau_1)^2 X \tilde{R}_2^{-1} X, -\tau_d^2 X \tilde{R}_3^{-1} X, -(\tau_2 - \tau_1) \tilde{W}_2, I, \tau_1 \tilde{W}_2\},$$

$$\tilde{\Phi}_{42}^{ij} = [\tau_1^2 G_i^T, (\tau_2 - \tau_1)^2 G_i^T, \tau_d^2 G_i^T, (\tau_2 - \tau_1) G_i^T],$$

$$\tilde{\Theta}_{42}^{ij} = [\tau_1^2 G_i^T, (\tau_2 - \tau_1)^2 G_i^T, \tau_d^2 G_i^T, 0],$$

$$\tilde{f}_2^{ij} = [A_i X \quad A_{di} X \quad B_i Y_j \quad -B_i Y_j \quad B_{wi} \quad 0 \quad 0],$$

$$\tilde{f}_3^{ij} = [C_i X \quad 0 \quad D_i Y_j \quad -D_i Y_j \quad 0 \quad 0 \quad 0],$$

$$\tilde{E}_{ij} = [E_{ai} X \quad 0 \quad E_{bi} Y_j \quad -E_{bi} Y_j \quad 0 \quad 0 \quad 0],$$

$$\tilde{H}_{ij} = [G_i^T + (\tau_2 - \tau_1) G_i^T W_1 \quad 0 \quad -(\tau_2 - \tau_1) G_i^T W_1 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$M_{ij} = [G_i^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0].$$

Proof. Letting $X = P^{-1}$, $XQ_1X = \tilde{Q}_1$, $XQ_2X = \tilde{Q}_2$, $XQ_3X = \tilde{Q}_3$, $XR_1X = \tilde{R}_1$, $XR_2X = \tilde{R}_2$, $XR_3X = \tilde{R}_3$, $XW_1X = \tilde{W}_1$, $XW_2X = \tilde{W}_2$, $XM\tilde{X} = \tilde{M}$, $\tilde{\Phi} = X\Phi X$, and $Y = KX$, subsequently, pre- and post-multiplying both sides of Π_3^{ij} and Π_3^{ji} with $\text{diag}\{X, X, X, X, I, X, X, R_1^{-1}, R_2^{-1}, R_3^{-1}, W_2^{-1}, I, X, I, I\}$ and its transpose, Π_4^{ij} and Π_4^{ji} with $\text{diag}\{X, X, X, X, I, X, X, R_1^{-1}, R_2^{-1}, R_3^{-1}, X, I, X, I, I\}$ and its transpose. Then, pre-multiplying matrix $\text{diag}\{X, X\}$ and post-multiplying matrix $\text{diag}\{X^T, X^T\}$ to (3.3), and thus (3.13) and (3.14) are readily derived from (3.11) and (3.12) by the Schur complement. This completes the proof. $\square$

Remark 3.4. Due to the nonlinear items $X\tilde{R}_1^{-1}X$, $X\tilde{R}_2^{-1}X$, $X\tilde{R}_3^{-1}X$ and $X\tilde{W}_2^{-1}X$, the matrix inequalities (3.13) and (3.14) cannot be directly solved by Matlab LMI Toolbox. However, the nonlinear minimization based on the cone complementary linearization (CCL) algorithm can be adopted to solve this non-convex problem.

4. Numerical example

Consider the following uncertain nonlinear system with time delay:

$$\begin{cases} \dot{x}_1(t) = -a \frac{v^T}{L t_0} x_1(t) - b \frac{v^T}{L t_0} x_1(t - \tau_d) + \frac{v^T}{l t_0} u(t) + \omega(t), \\ \dot{x}_2(t) = a \frac{v^T}{L t_0} x_1(t) + b \frac{v^T}{L t_0} x_1(t - \tau_d), \\ \dot{x}_3(t) = \frac{v^T}{t_0} \sin[x_2(t) + a \frac{v^T}{L} x_1(t) + b \frac{v^T}{2L} x_1(t - \tau_d)], \end{cases} \quad (4.1)$$

where $a + b = 1$, $a = 0.7$, $v = -1$, $T = 2$, $t_0 = 0.5$, $L = 5.6$, $l = 2.8$.

And the following fuzzy models are used for fuzzy controller design:

$$\begin{cases} R^1 : \text{IF } \theta(t) = x_2(t) + a \frac{vT}{L} x_1(t) + b \frac{vT}{2L} x_1(t - \tau_d) \text{ is } 0, \\ \text{THEN } \dot{x}(t) = A_1 x(t) + A_{d1} x(t - \tau_d) + B_1 u(t) + B_{\omega 1} \omega(t), \\ R^2 : \text{IF } \theta(t) = x_2(t) + a \frac{vT}{L} x_1(t) + b \frac{vT}{2L} x_1(t - \tau_d) \text{ is } \pm\pi, \\ \text{THEN } \dot{x}(t) = A_2 x(t) + A_{d2} x(t - \tau_d) + B_2 u(t) + B_{\omega 2} \omega(t), \end{cases}$$

where

$$A_1 = \begin{bmatrix} -a \frac{vT}{Lt_0} & 0 & 0 \\ a \frac{vT}{Lt_0} & 0 & 0 \\ a \frac{v^2 T^2}{Lt_0} & \frac{vT}{t_0} & 0 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -b \frac{vT}{Lt_0} & 0 & 0 \\ b \frac{vT}{Lt_0} & 0 & 0 \\ b \frac{v^2 T^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -a \frac{vT}{Lt_0} & 0 & 0 \\ a \frac{vT}{Lt_0} & 0 & 0 \\ a \frac{dv^2 T^2}{Lt_0} & \frac{vT}{t_0} & 0 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -b \frac{vT}{Lt_0} & 0 & 0 \\ b \frac{vT}{Lt_0} & 0 & 0 \\ b \frac{dv^2 T^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} \frac{vT}{Lt_0} & 0 & 0 \end{bmatrix}^T, \quad B_{\omega 1} = B_{\omega 2} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T.$$

Figure 1: The release instants and release interval.

Figure 2: The state response of system.

In (4.1), we set $d = \frac{10t_0}{\pi}$ and employ the following membership function

$$\begin{cases} u_1(\theta(t)) = (1 - \frac{1}{1+\exp(\beta)}) \times (\frac{1}{1+\exp(\beta)}), \\ u_2(\theta(t)) = 1 - u_1(\theta(t)), \end{cases}$$

where $\beta = -3\theta(t) + \pi$. Then the controller output is chosen as $z_1(t) = z_2(t) = [0.1, 0.1, 0.01]x(t)$, and the disturbance input is assumed to be $\omega(t) = \sin(0.1t) \exp(-0.1t)$.

Define state constant delay $\tau_d = 0.1s$ and give $\tau_1 = 0.01, \tau_2 = 0.2, \delta = 0.05$,

$$\Phi = \begin{bmatrix} 10 & 10 & 20 \\ 10 & 20 & 20 \\ 10 & 20 & 30 \end{bmatrix},$$

while guaranteeing asymptotic stability of system (4.1) H_∞ performance index $\gamma = 2$. We can obtain the controller feedback gains as follows:

$$K_1 = [-1.4356 \quad -0.4298 \quad 0.0049], \quad K_2 = [-1.3493 \quad -0.6235 \quad 0.0078].$$

Taking the sampling period $h = 0.5s$, $\tau_d = 0.1s$, the release instants and release intervals of system (4.1) are shown in Fig. 1 with initial state conditions of $[-0.5\pi, -0.75\pi, -5]$, $K_1 = [-1.4356, -0.4298, 0.0049]$, $K_2 = [-1.3493, -0.6235, 0.0078]$. Fig. 2 depicts the state response of $x_1(t), x_2(t)$ and $x_3(t)$ under the above designed fuzzy control $0.01 < \eta(t) < 0.2$. It can be seen that with the event-triggered H_∞ control, the closed-loop system (4.1) is asymptotically stable with state time delay τ_d .

5. Conclusions

In order to reduce the communication load in the network, this paper provides an event-triggering scheme for T-S fuzzy systems with time-delay and uncertainties. Moreover, employing the networked T-S fuzzy model and an event-triggered communication scheme, the stabilization criterion is obtained in terms of matrix inequalities, and the maximum allowable delay and the feedback gain can be obtained simultaneously by solving an optimization problem. A numerical example has been given to show the effectiveness of the proposed approach.

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