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Two kinds of breather solitary wave and rogue wave solutions for the (3+1)-dimensional Kadomtsev-Petviashvili equation

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Abstract

In this paper, the (3+1)-dimensional Kadomtsev-Petviashvili equation is investigated. Two kinds of periodic breather solitary wave and rogue wave solutions are obtained by using the two-wave method and the homoclinic breather limit approach with the aid of Maple. Deflection of rogue wave varying with the seed solution u_0 is investigated.

Keywords: (3+1)-dimensional Kadomtsev-Petviashvili equation, homoclinic breather limit approach, two-wave method, rational breather solutions, rogue wave.
2010 MSC: 35G20, 35C05, 35C07.

1. Introduction

Rogue waves are called monster waves or extreme waves in the ocean, which are catastrophic natural physical phenomena [1, 2, 9]. The study of rogue waves has many important applications in some fields since they can signal fascinating stories (thunderstorms, earthquakes, and hurricanes). For a quite long time, rogue waves are thought to be mysterious since they appear from nowhere and disappear without a trace [2]. Generally speaking, they develop due to the interaction of the nonlinearity and dispersion in the wave propagation. In fact, rogue waves can be regarded as a special type of solitary waves and have drawn much attention in some fields of nonlinear science [4, 8, 11, 12]. Recently, Akhmediev et al. [1] presented explicit forms of the rational solutions to describe them by the deformed Darboux transformation. Yan [18] and Ma et al. [10] investigated the nonautonomous rogue waves in one-dimensional and three-dimensional generalized nonlinear Schrödinger equations with variable coefficients by the similarity transformation and direct ansatz. More recently, Dai et al. [7] discussed their propagation behaviors in a variable coefficient higher-order nonlinear Schrödinger equation by a similarity transformation connected with the constant coefficient Hirota equation.

Now we consider the following (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation

$$u_{xt} + u_{xxxx} + 3(u^2)_{xx} - u_{yy} - u_{zz} = 0, \tag{1.1}$$

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where u is a polynomial in its arguments, $u : R_x \times R_y \times R_z \times R_t \to R$. It is understood that this equation has been used widely in describing dynamics of solitons and nonlinear waves in the fields of plasmas and superfluids. As a matter of fact, when u is z-independent, Eq. (1.1) is completely integrable. Therefore, certain solutions can be derived in various approaches, for instance, the inverse scattering transformation and Hirota's bilinear method. When Eq. (1.1) is non-integrable, difficulties become obvious in order to get exact solutions of the equation. It is worthy to note that Wang and Lou have revealed some special type exact solutions in their report [13]. Also, after applying a generalized variable-coefficient algebraic method [3] to the (3+1)-dimensional KP equation, Bai et al. successfully constructed several new families of exact solutions with interesting potentials for future physical applications. The bilinear Bäcklund transformation and some new explicit solutions of Eq. (1.1) are also derived in Wu's recent work [14]. But to our best knowledge, rational breather solutions to the (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation (1.1) have not been reported in previous literatures.

In this work, a novel approach of seeking rogue wave solution, the homoclinic breather limit approach [5, 15], is proposed. By using the homoclinic breather limit approach and two-wave method [6, 16, 17], we obtain two kinds of breather solitary wave and rogue wave solutions. Furthermore, we also investigate differently mechanical features of these wave solutions [19, 20].

2. Two kinds of periodic breather solitary wave solutions

In this section, the two kinds of periodic breather solitary wave solutions are constructed by using two-wave method and extended homoclinic test technique as well as the bilinear method.

Setting $\xi = x + t$ in Eq. (1.1), we get

$$u_{\xi\xi} + u_{\xi\xi\xi\xi} + 3(u^2)_{\xi\xi} - u_{yy} - u_{zz} = 0. \tag{2.1}$$

It is easy to see that Eq. (2.1) has a seed solution u_0 which is an arbitrary constant.

By using Painlevé test we can assume the solution of Eq. (2.1) as follows

$$u(\xi, y, z) = u_0 + 2(\ln f)_{\xi\xi}, \tag{2.2}$$

where $f(\xi, y, z)$ is unknown real function. Substituting Eq. (2.2) into Eq. (2.1) we obtain the following bilinear form

$$(D_\xi^4 + (1 + 6u_0)D_\xi^2 - D_y^2 - D_z^2) f \cdot f = 0, \tag{2.3}$$

where $D_y^2 f \cdot f = 2f_{yy}f - 2f_y^2, D_\xi^4 f \cdot f = 2(f_{\xi\xi\xi\xi}f - 4f_{\xi\xi\xi}f_\xi + 3f_{\xi\xi}^2)$. With regard to Eq. (2.3), using the homoclinic test technique we can seek the solution in the form

$$f = e^{(-w_1(\xi - ay))} + \delta_1 \cos(w(\xi + by + bz)) + \delta_2 e^{(w_1(\xi - ay))}, \tag{2.4}$$

where $a, b, w, w_1, \delta_1, \delta_2$ are real constants to be determined. Substituting Eq. (2.4) into Eq. (2.3) and equating all the coefficients of different powers of $e^{w_1(\xi - at)}$, $e^{-w_1(\xi - at)}$, $\sin(w(\xi + by + bz))$, $\cos(w(\xi + by + bz))$ and the constant term to zero, we can obtain a set of algebraic equations for $a, b, w, w_1, \delta_1, \delta_2$. Solving the system with the aid of Maple, we get the following results

$$\begin{cases} -\delta_1(-w^4 - (1 + 6u_0)w_1^2 - 2w^2b^2 + w_1^2a^2 + 6w_1^2w^2 + (1 + 6u_0)w^2 - w_1^4) = 0, \\ 16\delta_2 w_1^4 + 2\delta_1^2 w^2 b^2 + 4\delta_1^2 w^4 - (1 + 6u_0)\delta_1^2 w^2 + 4(1 + 6u_0)\delta_2 w_1^2 - 4\delta_2 w_1^2 a^2 = 0, \\ -2w_1\delta_1 w(-2w^2 + 2w_1^2 + (1 + 6u_0) + ba) = 0. \end{cases} \tag{2.5}$$

Solving the above Eqs. (2.5) and taking $w_1 = w$ yields

$$w = \pm\frac{\sqrt{2b^2 - a^2}}{2}, \ \delta_1 = \pm 2\sqrt{\frac{(2a^2 - (1 + 6u_0) - 2b^2)\delta_2}{4b^2 - (1 + 6u_0) - a^2}}, \ ba = -1 - 6u_0, \tag{2.6}$$

where a, b, w, δ_2 are real constants to be determined.

Substituting Eq. (2.5) and Eq. (2.6) into Eq. (2.4), and choosing $u_0 \neq -\frac{1}{6}$ and $\delta_2 > 0$, we have

$$\begin{aligned} f_1(\xi, y, z) &= 2\sqrt{\delta_2}\cosh(-w(\xi - ay) - \frac{1}{2}\ln(\delta_2)) + h_1\cos(w(\xi + by + bz)), \\ f_2(\xi, y, z) &= 2\sqrt{\delta_2}\cosh(-w(\xi - ay) - \frac{1}{2}\ln(\delta_2)) - h_1\cos(w(\xi + by + bz)), \end{aligned} \tag{2.7}$$

where $h_1 = 2\sqrt{\frac{(2a^2-(1+6u_0)-2b^2)\delta_2}{4b^2-(1+6u_0)-a^2}}, w = \pm\frac{\sqrt{2b^2-a^2}}{2}, a, b \in \mathbb{R}$.

Substituting Eq. (2.7) into Eq. (2.2) yields the periodic breather soliton solutions of Eq. (2.2) as follows, respectively.

$$\begin{aligned} u_1(\xi, y, z) &= u_0 + \frac{(m_0 - 2m_1\sinh(-w(\xi - ay) - \frac{1}{2}\ln(\delta_2))\sin(w(\xi + by + bz)))}{(\cosh(-w(\xi - ay) - \frac{1}{2}\ln(\delta_2)) + m_1\cos(w(\xi + by + bz)))^2}, \\ u_2(\xi, y, z) &= u_0 + \frac{2w^2(m_0 + 2m_1\sinh(-w(\xi - ay) - \frac{1}{2}\ln(\delta_2))\sin(w(\xi + by + bz)))}{(\cosh(-w(\xi - ay) - \frac{1}{2}\ln(\delta_2)) - m_1\cos(w(\xi + by + bz)))^2}, \end{aligned}$$

where $m_1 = \sqrt{\frac{2a^2-(1+6u_0)-2b^2}{4b^2-(1+6u_0)-a^2}}, m_0 = \frac{3(2b^2-a^2)}{4b^2-1-6u_0-a^2}, w = \pm\frac{\sqrt{2b^2-a^2}}{2}, a, b \in \mathbb{R}$.

Substituting $\xi = x + t$ into $u_2(\xi, y, z)$ and letting $\delta_2 = 1$, yields the periodic breather soliton solutions of the (3+1)-D KP equation as follows (see Fig. 1 (a), (b))

$$u_2^{(1)}(x, y, z, t) = u_0 + \frac{2w^2(m_0 - 2m_1\sinh(w(x - ay + t))\sin(w(x + bz + by + t)))}{(\cosh(w(x - ay + t)) - m_1\cos(w(x + bz + by + t)))^2},$$

where $m_1 = \sqrt{\frac{a^2-(1+6u_0)-4w^2}{8w^2-(1+6u_0)+a^2}}, m_0 = \frac{12w^2}{8w^2-(1+6u_0)+a^2}$.

The solution $u_2^{(1)}(x, y, z, t)$ is a periodic breather soliton which has period $\frac{2\pi}{w}$, and the forward-direction (or backward-direction) wave shows periodic breather feature as trajectory along the straight line $x = ay - t$; meanwhile, it takes on soliton feature as trajectory along the straight line $x = -(bz + by + t)$ for (3+1)-D KP equation. Especially, this wave shows both breather and periodic feature to space variable t (see Fig. 1). It is obvious $u_1(\xi, y, z)$ and $u_2(\xi, y, z)$ have the same structure and behavior with $u_2^{(1)}(x, y, z, t)$.

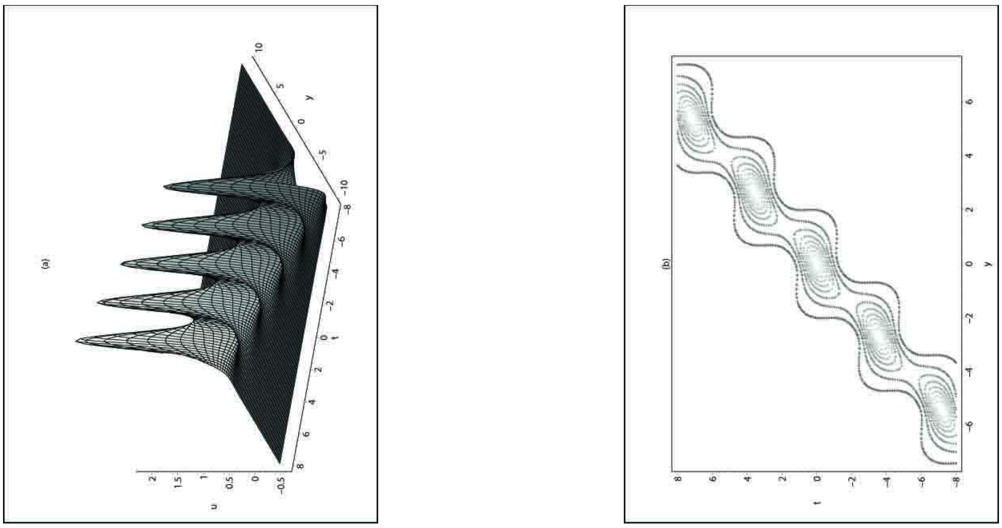

Figure 1: (a) The figure of $u_2^{(1)}(x, y, z, t)$ as $a = \frac{4}{3}, u_0 = -\frac{1}{2}, w = 0.82, x = z = 0$. (b) Plot of contours for $u_2^{(1)}(x, y, z, t)$ as $a = \frac{4}{3}, u_0 = -\frac{1}{2}, w = 0.82, x = z = 0$.

Being similar to the above way, we choose the extended homoclinic test function as follows

$$f(\xi, y, z) = e^{(-w_1(\xi+az+b_1y))} + \delta_1 \cos(w(\xi + az + b_2y)) + \delta_2 e^{(w_1(\xi+az+b_1y))}, \tag{2.8}$$

where $a, b_1, b_2, \delta_1, \delta_2, w, w_1$ are some free real constants. Substituting Eq. (2.8) into Eq. (2.2), and equating all the coefficients of different powers of $e^{(w_1(\xi+az+b_1y))}$, $e^{(-w_1(\xi+az+b_1y))}$, $\sin(w(\xi + az + b_2y))$, $\cos(w(\xi + az + b_2y))$ and the constant term to zero, we can obtain a set of algebraic equations for $a, b_i, \delta_i, w_i (i = 1, 2)$.

$$\begin{cases} -\delta_1(-w^2a^2 + 6w_1^2w^2 + (1 + 6u_0)w^2 - (1 + 6u_0)w_1^2 - w^2b_2^2 + w_1^2a^2 + w_1^2b_1^2 - w_1^4 - w^4) = 0, \\ 16\delta_2 w_1^4 - (1 + 6u_0)\delta_1^2w^2 + \delta_1^2w^2a^2 + \delta_1^2w^2b_2^2 + 4\delta_1^2w^4 - 4\delta_2w_1^2a^2 \\ \qquad - 4\delta_2w_1^2b_1^2 + 4(1 + 6u_0)\delta_2w_1^2 = 0, \\ -2w_1\delta_1w(-2w^2 + 2w_1^2 - a^2 + (1 + 6u_0) - b_1b_2) = 0. \end{cases}$$

Taking $w_1 = w$, and solving the system with the aid of Maple, we get the following results

$$w = \frac{1}{2}\sqrt{b_2^2 - b_1^2}, \quad \delta_1 = \pm 2\sqrt{\frac{-(2b_1 + b_2)\delta_2}{b_1 + 2b_2}}, \quad b_1b_2 = -a^2 + 1 + 6u_0. \tag{2.9}$$

Substituting Eq. (2.9) into Eq. (2.8), and taking $\delta_2 > 0, b_2 \neq b_1$, we have

$$\begin{aligned} f_3(\xi, Y, z) &= 2\sqrt{\delta_2}\cosh(w(\xi + az + b_1y) + \frac{1}{2}\ln(\delta_2)) + h_2 \cos(w(\xi + az + b_2y)), \\ f_4(\xi, y, z) &= 2\sqrt{\delta_2}\cosh(w(\xi + az + b_1y) + \frac{1}{2}\ln(\delta_2)) - h_2 \cos(w(\xi + az + b_2y)), \end{aligned} \tag{2.10}$$

where $h_2 = 2\sqrt{\frac{-(2b_1+b_2)\delta_2}{b_1+2b_2}}, w = \frac{1}{2}\sqrt{b_2^2 - b_1^2}, b_1, b_2 \in R$. Substituting Eq. (2.10) into Eq. (2.6), and taking $b_1 \neq 0$, yields the periodic breather solutions of Eq. (2.2) as follows, respectively.

$$\begin{aligned} u_3(\xi, y, z) &= u_0 + \frac{2w^2(m_0 + 2m_1 \sinh(w(\xi + az + b_1y) + \frac{1}{2}\ln(\delta_2)) \sin(w(\xi + az + b_2y)))}{(\cosh(w(\xi + az + b_1y) + \frac{1}{2}\ln(\delta_2)) + m_1 \cos(w(\xi + az + b_2y)))^2}, \\ u_4(\xi, y, z) &= u_0 + \frac{2w^2(m_0 - 2m_1 \sinh(w(\xi + az + b_1y) + \frac{1}{2}\ln(\delta_2)) \sin(w(\xi + az + b_2y)))}{(\cosh(w(\xi + az + b_1y) + \frac{1}{2}\ln(\delta_2)) - m_1 \cos(w(\xi + az + b_2y)))^2}, \end{aligned}$$

where $m_0 = \frac{3(b_1^2-a^2+1+6u_0)}{b_1^2-2a^2+2+12u_0}, m_1 = \sqrt{-\frac{2b_1^2-a^2+1+6u_0}{b_1^2-2a^2+2+12u_0}}, w = \frac{1}{2}\sqrt{b_2^2 - b_1^2}, b_1, b_2 \in R$.

Substituting $\xi = x + t$ into $u_4(\xi, y, z)$ and letting $\delta_2 = 1$, yields another periodic breather soliton solutions of the (3+1)-D KP equation as follows

$$u_4^{(1)}(x, y, z, t) = u_0 + \frac{2w^2(m_0 - 2m_1 \sinh(w(x + az + b_1y + t)) \sin(w(x + az + b_2y + t)))}{(\cosh(w(x + az + b_1y + t)) - m_1 \cos(w(x + az + b_2y + t)))^2},$$

where $m_0 = \frac{12w^2}{8w^2+a^2-(1+6u_0)+b_1^2}, m_1 = \sqrt{\frac{a^2+b_1^2-(1+6u_0)-4w^2}{8w^2-(1+6u_0)+a^2+b_1^2}}$.

The solution $u_4^{(1)}(x, y, z, t)$ is also a periodic breather soliton. It is generated by the interaction between the soliton of variable $X = w(x + az + b_1y + t)$ and the periodic wave of variable $Y = w(x + az + b_2y + t)$ (see Fig. 2 (a), (b)). It is obvious that $u_3(\xi, y, z)$ and $u_4(\xi, y, z)$ have the same structure and behavior of with $u_4^{(1)}(x, y, z, t)$.

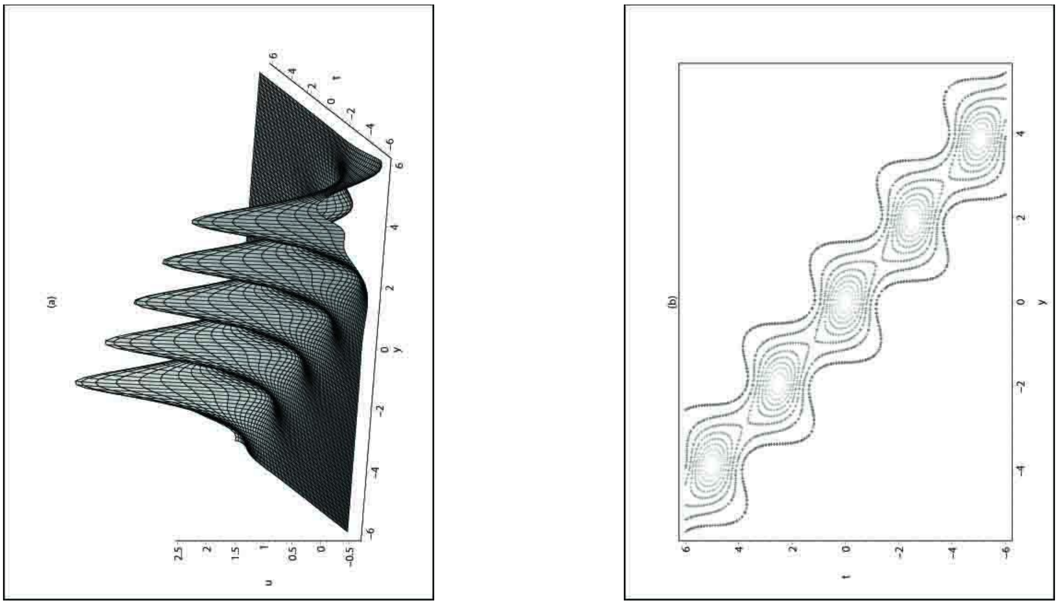

Figure 2: (a) The figure of $u_4^{(1)}(x,y,z,t)$ as $a=1, u_0=-\frac{1}{2}, b_1=1.3, w=0.9, \delta_2=1, x=z=0$. (b) Plot of contours for $u_4^{(1)}(x,y,z,t)$ as $a=1, u_0=-\frac{1}{2}, b_1=1.3, w=0.9, \delta_2=1, x=z=0$.

3. Rogue waves solutions

In this section, let the period of periodic wave go to infinity in homoclinic breather wave solution $u_2^{(1)}(x,y,z,t)$ and $u_4^{(1)}(x,y,z,t)$, we can obtain a rational breather-wave solutions of (3+1)-D Kadomtsev-Petviashvili equation, respectively, and they are just the rogue wave solutions.

Now we consider a limit behavior of $u_2^{(1)}$ as the period $\frac{2\pi}{w}$ of periodic wave $\cos(w(x+bz+by+t))$ goes to infinity, i.e., $w\to 0$. By computing, we obtain the following result

$$U_{\text{rogue wave}} = u_0 + \frac{16\left(\frac{6}{a^2-1-6u_0} - (x-ay+t)\left(x-\frac{(1+6u_0)y}{a}-\frac{(1+6u_0)z}{a}+t\right)\right)}{\left((x-ay+t)^2+\left(x-\frac{(1+6u_0)y}{a}-\frac{(1+6u_0)z}{a}+t\right)^2+\frac{12}{a^2-1-6u_0}\right)^2},$$

here $m_1\to 1$, $a^2>1+6u_0$, and $2b^2=a^2$ as $w\to 0$.

$U_{\text{rogue wave}}$ contains two waves with different velocities and directions. It is easy to see that $U_{\text{rogue wave}}$ is a rational solution of Eq. (1.1). Moreover, we can show that $U_{\text{rogue wave}}$ is also a breather-type solution. In fact, $U\to u_0$ for fixed x as $y=z=c$ and $t\to\infty$. So, U is not only a rational breather solution but also a rogue wave solution which has two to three times amplitude higher than its surrounding waves and generally forms in a short time (see Fig. 3). It is a new discovery that the rogue wave solutions can come from the breather solitary wave solution for Eq. (1.1). One may think that whether the energy collection and superposition of breather solitary wave in many periods lead to a rogue wave or not.

Similarly, we consider a limit behavior of $u_4^{(1)}$ as the period $\frac{2\pi}{w}$ of periodic wave $\cos(w(x+az+b_2y+t))$ goes to infinity, i.e., $w\to 0$. By computing, we obtain the rational breather wave solution of (3+1)-dimensional KP equation, and it is just a rogue wave solution as follows (see Fig. 4)

$$U_{\text{rogue wave}} = u_0 + \frac{8\left(\frac{12}{a^2-1-6u_0+b_1^2} - 2(x+az+b_1y+t)\left(x+az-\frac{(a^2-1-6u_0)y}{b_1}+t\right)\right)}{\left(\frac{12}{a^2-1-6u_0+b_1^2}+(x+az+b_1y+t)^2+\left(x+az-\frac{(a^2-1-6u_0)y}{b_1}+t\right)^2\right)^2},$$

here $m_1\to 1$, $a^2+b_1^2>1+6u_0$, and $b_2=-b_1$ as $w\to 0$.

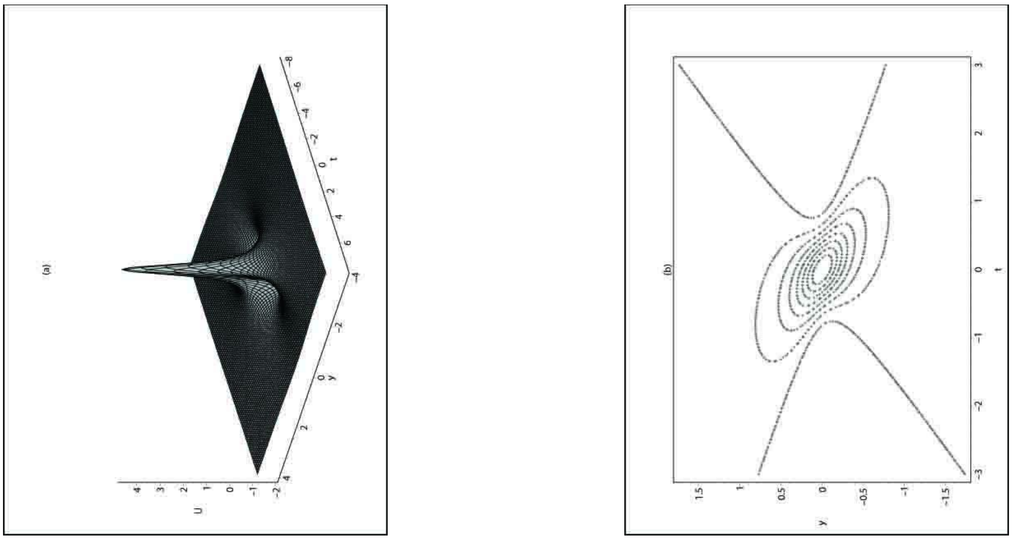

Figure 3: (a) The figure of $U^1_{\text{rogue wave}}$ as $a = 2.1, u_0 = -1.2, x = z = 0$. (b) Plot of contours for $U^1_{\text{rogue wave}}$ as $a = 2.1, u_0 = -1.2, x = z = 0$.

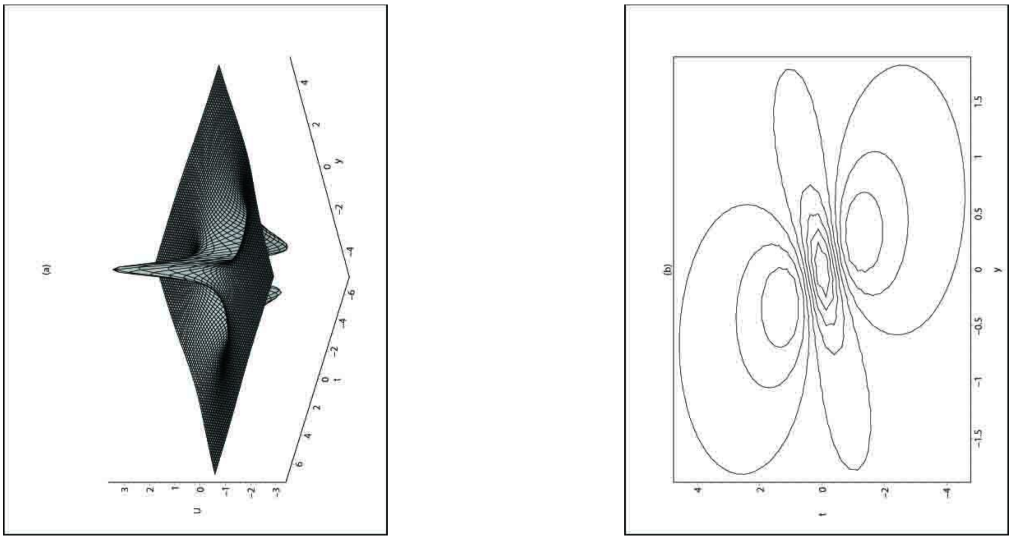

Figure 4: (a) The figure of $U^1_{\text{rogue wave}}$ as $a = 1, b_1 = 1.8, u_0 = -\frac{1}{2}, x = z = 0$. (b) Plot of contours for $U^1_{\text{rogue wave}}$ as $a = 1, b_1 = 1.8, u_0 = -\frac{1}{2}, x = z = 0$.

4. Conclusion

In the current work, we proposed a new method for seeking rogue wave, the homoclinic breather limit approach. Applying this approach to the (3+1)-dimensional Kadomtsev-Petviashvili equation, we obtained two kinds of breather solitary and rational breather solutions. Furthermore, rational breather solution obtained here is just a rogue wave solution of the (3+1)-dimensional KP equation. This result shows that the homoclinic breather limit approach combined with some other techniques is effective and promising for constructing rogue wave solution of nonlinear evolution equations.

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