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Fuzzy fixed point theorems for multivalued fuzzy contractions in b-metric spaces

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Abstract

In this paper, we introduce the new concept of multivalued fuzzy contraction mappings in b-metric spaces and establish the existence of α-fuzzy fixed point theorems in b-metric spaces which can be utilized to derive Nadler's fixed point theorem in the framework of b-metric spaces. Moreover, we provide examples to support our main result.

Keywords: b-metric spaces, fuzzy fixed point, fuzzy mappings, fuzzy set.
2010 MSC: 47H10, 54H25.

1. Introduction

A study of fixed point for a multivalued (set-valued) mappings was originally initiated by von Neumann [25]. The development of geometric fixed point theory for multivalued mapping was initiated with the work of Nadler [24]. He combined the ideas of multivalued mapping and Lipschitz mapping and used the concept of Hausdorff metric to establish the multivalued contraction principle, usually referred as Nadler's contraction mapping principle. Several researches were conducted on the generalizations of the concept of Nadler's contraction mapping principle.

In 1981, Heilpern [20] obtained a fixed point theorem for fuzzy contraction mappings, which is a generalization of the fixed point theorem for multivalued mappings of Nadler's contraction principle. Subsequently, several authors generalized and studied the existence of fixed points and common fixed points of fuzzy mappings satisfying a contractive type condition (see [1, 2, 5, 6, 13, 18, 19, 21, 23, 26, 27, 28, 29, 32]).

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On the other hand, Czerwik [15] introduced the concept of b-metric spaces, as a generalization of metric spaces and proved the contraction mapping principle in b-metric spaces. Since then, a number of authors investigated fixed point theorems for single-valued and miltivalued mappings in b-metric spaces (see [3, 4, 7, 8, 9, 10, 11, 12, 14, 16, 17, 22, 31] and references therein).

To the best of our knowledge, there is no result so far concerning the existence of α-fuzzy fixed point for fuzzy contraction mappings in b-metric spaces. The object of this paper is to prove the existence of α-fuzzy fixed point theorems for fuzzy mappings in complete b-metric space and we also give illustrative examples to support our main results. Finally, we showed some relation of multivalued mappings and fuzzy mappings, which can be utilized to derive fixed point for multivalued mappings.

2. Preliminaries

The following notations of a b-metric space is given by Czerwik [15] (see also [7, 16, 17]).

Definition 2.1. Let X be a nonempty set and the functional $d : X \times X \to [0, \infty)$ satisfies:

(b1) $d(x, y) = 0$ if and only if $x = y$;
(b2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(b3) there exists a real number $s \geq 1$ such that $d(x, z) \leq s[d(x, y) + d(y, z)]$ for all $x, y, z \in X$.

Then d is called a b-metric on X and a pair (X, d) is called a b-metric space with coefficient s.

Remark 2.2. If we take $s = 1$ in above definition then b-metric spaces turns into ordinary metric spaces. Hence, the class of b-metric spaces is larger than the class of metric spaces.

The following examples of b-metric on X was given in [7, 8, 9, 15, 16, 30].

Example 2.3. The set $l_p(\mathbb{R})$ with $0 < p < 1$, where $l_p(\mathbb{R}) := \{\{x_n\} \subset \mathbb{R} \mid \sum_{n=1}^{\infty} |x_n|^p < \infty\}$, together with the functional $d : l_p(\mathbb{R}) \times l_p(\mathbb{R}) \to [0, \infty)$,

$$d(x, y) := (\sum_{n=1}^{\infty} |x_n - y_n|^p)^{\frac{1}{p}},$$

(where $x = \{x_n\}, y = \{y_n\} \in l_p(\mathbb{R})$) is a b-metric space with coefficient $s = 2^{\frac{1}{p}} > 1$. Notice that the above result holds for the general case $l_p(X)$ with $0 < p < 1$, where X is a Banach space.

Example 2.4. Let X be a set with the cardinal $card(X) \geq 3$. Suppose that $X = X_1 \cup X_2$ is a partition of X such that $card(X_1) \geq 2$. Let $s > 1$ be arbitrary. Then, the functional $d : X \times X \to [0, \infty)$ defined by:

$$d(x, y) := \begin{cases} 0, & x = y \\ 2s, & x, y \in X_1 \\ 1, & \text{otherwise}, \end{cases}$$

is a b-metric on X with coefficient $s > 1$.

Example 2.5. Let $X = \{a, b, c\}$ and $d(a, b) = d(b, a) = d(b, c) = d(c, b) = 1$ and $d(a, c) = d(c, a) = m \geq 2$. Then,

$$d(x, y) = \frac{m}{2}\left[d(x, z) + d(z, y)\right]$$

for all $x, y, z \in X$. If $m > 2$, the ordinary triangle inequality does not hold.

Definition 2.6 (Boriceanu[9])**.** Let (X, d) be a b-metric space. Then a sequence $\{x_n\}$ in X is called:

(a) Convergent if and only if there exists $x \in X$ such that $d(x_n, x) \to 0$ as $n \to \infty$.
(b) Cauchy if and only if $d(x_n, x_m) \to 0$ as $m, n \to \infty$.

(c) Complete if and only if every Cauchy sequence is convergent.

Let (X, d) be a b-metric space, denote $CB(X)$ be the collection of nonempty closed bounded subsets of X and by $CL(X)$ the class of all nonempty closed subsets of X. For $x \in X$ and $A, B \in CL(X)$, we define

$$d(x, A) = \inf_{a \in A} d(x, a),$$

$$\delta(A, B) = \sup\{d(a, B) : a \in A\}.$$

Then the generalized Hausdorff b-metric H on $CL(X)$ inducted by d is defined as

$$H(A, B) = \begin{cases} \max\{\delta(A, B), \delta(B, A)\}, & \text{if the maximum exists;} \\ +\infty, & \text{otherwise,} \end{cases}$$

for all $A, B \in CL(X)$.

Let (X, d) be a b-metric space. We cite the following lemmas from Czerwik[15, 16, 17] and Singh *et al.*[30].

Lemma 2.7. *Let (X, d) be a b-metric space. For any $A, B, C \in CL(X)$ and any $x, y \in X$, we have the following:*

(i) $d(x, B) \leq d(x, b)$ *for all* $b \in B$*;*

(ii) $d(x, B) \leq H(A, B)$ *for all* $x \in A$*;*

(iii) $\delta(A, B) \leq H(A, B)$*;*

(iv) $H(A, A) = 0$*;*

(v) $H(A, B) = H(B, A)$*,*

(vi) $H(A, C) \leq s(H(A, B) + H(B, C))$*;*

(vii) $d(x, A) \leq s(d(x, y) + d(y, A))$.

Lemma 2.8. *Let (X, d) be a b-metric space. For $A \in CL(X)$ and $x \in X$, then we have*

$$d(x, A) = 0 \Longleftrightarrow x \in \overline{A} = A,$$

where $\overline{A}$ denotes the closure of the set A.

Lemma 2.9. *Let (X, d) be a b-metric space. For $A, B \in CL(X)$ and $q > 1$. Then, for all $a \in A$, there exists $b \in B$ such that*

$$d(a, b) \leq qH(A, B).$$

Let Ψ_b be a set of strictly increasing functions in b-metric space, $\psi : [0, \infty) \to [0, \infty)$ such that $\sum_{n=0}^{\infty} s^n \psi^n(t) < +\infty$ for each $t > 0$, where ψ^n is n-th iterate of ψ. It is known that for each $\psi \in \Psi_b$, we have $\psi(t) < t$ for all $t > 0$ and $\psi(0) = 0$ for $t = 0$.

Now we introduce a notion of fuzzy set, fuzzy mappings and α-fuzzy fixed point in b-metic space.

Let (X, d) be a b-metric space. A fuzzy set in X is a function with domain X and values in $[0, 1]$. Let $\mathcal{F}(X)$ stands for the collection of all fuzzy sets in X, then the function value $A(x)$ is called the grade of membership of x in A. If X is endowed with a topology, for $\alpha \in [0, 1]$, the α-level set of A is denoted by $[A]_\alpha$ and is defined as follows:

$$[A]_\alpha = \{x : A(x) \geq \alpha\}; \quad \alpha \in (0, 1], \tag{2.1}$$
$$[A]_0 = \overline{\{x : A(x) > 0\}}.$$

where $\overline{B}$ denotes the closure of B in X.

For $A, B \in \mathcal{F}(X)$, a fuzzy set A is said to be more accurate than a fuzzy set B (denoted by $A \subset B$) if and only if $Ax \leq Bx$ for each x in X, where $A(x)$ and $B(x)$ denote the membership function of A and B, respectively. Now, for $x \in X$, $A, B \in \mathcal{F}(X)$, $\alpha \in [0, 1]$ and $[A]_\alpha, [B]_\alpha \in CB(X)$, we define

$$d(x, S) = \inf\{d(x, a); a \in S\},$$

$$p_\alpha(x, A) = \inf\{d(x, a); a \in [A]_\alpha\},$$

$$p_\alpha(A, B) = \inf\{d(a, b); a \in [A]_\alpha, b \in [B]_\alpha\},$$

$$p(A, B) = \sup_\alpha p_\alpha(A, B),$$

$$H\big([A]_\alpha, [B]_\alpha\big) = \max\Big\{\sup_{a\in[A]_\alpha} d(a, [B]_\alpha), \sup_{b\in[B]_\alpha} d(b, [A]_\alpha)\Big\},$$

Remark 2.10. The function $H : CL(X) \times CL(X) \to \mathcal{F}(X)$ is a generalized Hausdorff fuzzy b-metric, that is, $H(A, B) = +\infty$ if $\max\{(A, B), (B, A)\}$ do not exists.

Definition 2.11. Let X be a nonempty set and Y be a b-metric space. A mapping T is said to be a *fuzzy mapping* if T is a mapping from the set X into $\mathcal{F}(Y)$.

Remark 2.12. The function value $(Tx)(y)$ is the grade of membership of y in Tx.

Definition 2.13. Let (X, d) be a b-metric space and T be a fuzzy mapping from X into $\mathcal{F}(X)$. A point z in X is called an *α-fuzzy fixed point* of T if $z \in [Tz]_{\alpha(z)}$.

3. α-fuzzy fixed point in b-metric spaces

In this section, we state and prove the existence result of an α-fuzzy fixed point theorem for a fuzzy mapping in the framework of a b-metric space as follows.

Theorem 3.1. *Let (X, d) be a complete b-metric space with coefficient $s \geq 1$, let $T : X \to \mathcal{F}(X)$, $\alpha : X \to (0, 1]$ such that $[Tx]_{\alpha(x)}$ is a nonempty closed subsets of X for all $x \in X$ and $\psi \in \Psi_b$, such that*

$$H([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq \psi(d(x, y)), \tag{3.1}$$

for all $x, y \in X$. Then T has an α-fuzzy fixed point.

Proof. Let x_0 be an arbitrary point in X. Suppose that there exists $x_1 \in [Tx_0]_{\alpha(x_0)}$. Since, $[Tx_1]_{\alpha(x_1)}$ is a nonempty closed subsets of X. Clearly, if $x_0 = x_1$ or $x_1 \in [Tx_1]_{\alpha(x_1)}$, so x_1 is an α-fuzzy fixed point of T. Hence, the proof is completed. Thus, throughout the proof, we assume that $x_0 \neq x_1$ and $x_1 \notin [Tx_1]_{\alpha(x_1)}$. Hence $d(x_1, [Tx_1]_{\alpha(x_1)}) > 0$, by condition (3.1) and $\psi \in \Psi_b$, we have

$$\begin{aligned} 0 < d(x_1, [Tx_1]_{\alpha(x_1)}) &\leq H([Tx_0]_{\alpha(x_0)}, [Tx_1]_{\alpha(x_1)}) \\ &\leq \psi(d(x_0, x_1)) \\ &< \psi(rd(x_0, x_1)) \end{aligned}$$

where $r > 1$ is a real number. Since, $[Tx_1]_{\alpha(x_1)}$ is a nonempty closed subsets of X. Suppose that there exists $x_2 \in [Tx_1]_{\alpha(x_1)}$ and $x_1 \neq x_2$ such that

$$0 < d(x_1, x_2) \leq \psi(d(x_0, x_1)) < \psi(rd(x_0, x_1)).$$

Since, $[Tx_2]_{\alpha(x_2)}$ is a nonempty closed subsets of X. We assume that $x_2 \notin [Tx_2]_{\alpha(x_2)}$, then $d(x_2, [Tx_2]_{\alpha(x_2)}) > 0$, by condition (3.1) and $\psi \in \Psi_b$, we also have

$$\begin{aligned} 0 < d(x_2, [Tx_2]_{\alpha(x_2)}) &\leq H([Tx_1]_{\alpha(x_1)}, [Tx_2]_{\alpha(x_2)}) \\ &\leq \psi(d(x_1, x_2)) \\ &< \psi^2(rd(x_0, x_1)). \end{aligned}$$

Suppose that there exists $x_3 \in [Tx_2]_{\alpha(x_2)}$ and $x_2 \neq x_3$ such that

$$0 < d(x_2, x_3) \leq \psi(d(x_1, x_2)) < \psi^2(rd(x_0, x_1)).$$

By induction, we can construct the sequence $\{x_n\}$ in X. Such that $x_n \notin [Tx_n]_{\alpha(x_n)}$, $x_{n+1} \in [Tx_n]_{\alpha(x_n)}$ and

$$\begin{aligned} 0 < d(x_n, [Tx_n]_{\alpha(x_n)}) &\leq d(x_n, x_{n+1}) \\ &\leq \psi(d(x_{n-1}, x_n)) \\ &< \psi^n(rd(x_0, x_1)). \end{aligned}$$

for all $n \in \mathbb{N}$. For $m, n \in \mathbb{N}$ with $m > n$, we have

$$\begin{aligned} d(x_n, x_m) &\leq sd(x_n, x_{n+1}) + s^2 d(x_{n+1}, x_{n+2}) + \cdots + s^{m-n-2} d(x_{m-3}, x_{m-2}) \\ &\quad + s^{m-n-1} d(x_{m-2}, x_{m-1}) + s^{m-n} d(x_{m-1}, x_m) \\ &\leq s\psi^n(rd(x_0, x_1)) + s^2\psi^{n+1}(rd(x_0, x_1)) + \cdots + s^{m-n}\psi^{m-1}(rd(x_0, x_1)) \\ &= \frac{1}{s^{n-1}}[s^n\psi^n(rd(x_0, x_1)) + s^{n+1}\psi^{n+1}(rd(x_0, x_1)) + \cdots + s^{m-1}\psi^{m-1}(rd(x_0, x_1))]. \end{aligned}$$

Since $\psi \in \Psi_b$, we know that the series $\sum_{i=0}^{\infty} s^i\psi^i(rd(x_0, x_1))$ converges. So $\{x_n\}$ is a Cauchy sequence in X. By the completeness of X, there exists $x^* \in X$ such that $\lim_{n\to\infty} x_n = x^*$. Now we claim that $x^* \in [Tx^*]_{\alpha(x^*)}$. By condition $(b3)$ of b-metric space, we have

$$\begin{aligned} d(x^*, [Tx^*]_{\alpha(z)}) &\leq s[d(x^*, x_{n+1}) + d(x_{n+1}, [Tx^*]_{\alpha(x^*)})] \\ &\leq s[d(x^*, x_{n+1}) + H([Tx_n]_{\alpha(x_n)}, [Tx^*]_{\alpha(x^*)})] \\ &\leq s[d(x^*, x_{n+1}) + \psi(d(x_n, x^*))]. \end{aligned}$$

Letting $n \to \infty$, and $\psi(0) = 0$, we have $d(x^*, [Tx^*]_{\alpha(x^*)}) = 0$. Since, $[Tx^*]_{\alpha(x^*)}$ is closed we obtain that $x^* \in [Tx^*]_{\alpha(x^*)}$. Therefore, x^* is α-fuzzy fixed point of T. This completes the proof. □

By substituting $\psi(t) = ct$ where $c \in (0, 1)$, in Theorem 3.1, we get the following.

Corollary 3.2. *Let (X, d) be a complete b-metric space with coefficient $s \geq 1$, let $T : X \to \mathcal{F}(X)$, $\alpha : X \to (0, 1]$ such that $[Tx]_{\alpha(x)}$ is a nonempty closed subsets of X, for all $x \in X$ such that*

$$H([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq kd(x, y) \tag{3.2}$$

for all $x, y \in X$, where $0 < k < 1$. Assume that $k < \frac{1}{s}$, then T has an α-fuzzy fixed point.

Remark 3.3. If we set $s = 1$ in Corollary 3.2 (it corresponds to the case of metric spaces) and $[Tx]_{\alpha(x)} \in CB(X)$, we get the following result.

Corollary 3.4. *[20] Let (X,d) be a complete metric space, T be a fuzzy mapping from X to $\mathcal{F}(X)$ and $\alpha : X \to (0,1]$ be a mapping such that $[Tx]_{\alpha(x)}$ is a nonempty closed bounded subsets of X, for all $x \in X$ such that*

$$H([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq kd(x,y) \tag{3.3}$$

for all $x, y \in X$, where $0 \leq k < 1$, then T has an α-fuzzy fixed point.

Next, we give some examples to support the validity of our result.

Example 3.5. Let $X = \{0,1,2\}$ and define metric $d : X \times X \to \mathbb{R}$ by

$$d(x,y) = \begin{cases} 0 & , x = y \\ \dfrac{1}{6} & , x \neq y \text{ and } x, y \in \{0,1\} \\ \dfrac{1}{2} & , x \neq y \text{ and } x, y \in \{0,2\} \\ 1 & , x \neq y \text{ and } x, y \in \{1,2\}. \end{cases}$$

It is easy to see that (X,d) is a complete b-metric space with coefficient $s = \frac{3}{2}$, which the ordinary triangle inequality does not hold. Define fuzzy mapping $T : X \to \mathcal{F}(X)$ by

$$(T0)(t) = (T1)(t) \begin{cases} \dfrac{1}{2} & , t = 0 \\ 0 & , t = 1, 2, \end{cases}$$

$$(T2)(t) = \begin{cases} 0 & , t = 0, 2 \\ \dfrac{1}{2} & , t = 1. \end{cases}$$

Define $\alpha : X \to (0,1]$ by $\alpha(x) = \dfrac{1}{2}$ for all $x \in X$. Now we obtain that

$$[Tx]_{\frac{1}{2}} = \begin{cases} \{0\} & , x = 0, 1 \\ \{1\} & , x = 2. \end{cases}$$

For $x, y \in X$, we get

$$H([T0]_{\frac{1}{2}}, [T2]_{\frac{1}{2}}) = H([T1]_{\frac{1}{2}}, [T2]_{\frac{1}{2}}) = H(\{0\}, \{1\}) = \frac{1}{6}.$$

Define $\psi : [0,\infty) \to [0,\infty)$ by $\psi(t) = \dfrac{1}{3}t$ for all $t > 0$. Thus, we have

$$H([T0]_{\frac{1}{2}}, [T1]_{\frac{1}{2}}) = 0 < \frac{1}{2}d(1,0),$$

$$H([T0]_{\frac{1}{2}}, [T2]_{\frac{1}{2}}) = \frac{1}{6} = (\frac{1}{3})(\frac{1}{2}) = \frac{1}{3}d(0,2),$$

and

$$H([T1]_{\frac{1}{2}}, [T2]_{\frac{1}{2}}) = \frac{1}{6} < (\frac{1}{3})(1) = \frac{1}{3}d(1,2),$$

for all $x, y \in X$. Therefore all conditions of Theorem 3.1 hold and there exists a point $0 \in X$ such that $0 \in [T0]_{\frac{1}{2}}$ is an α-fuzzy fixed point of T.

Therefore, Corollary 3.4 and the results of fuzzy mappings in metric space cannot be applied for this example.

Example 3.6. Let $X = [0, 1]$ and $d : X \times X \to [0, \infty)$ as $d(x, y) = |x - y|^2$ for all $x, y \in X$. Then (X, d) is a complete b-metric space with coefficient $s = 2$, but it is not usual metric space.
Let us define $T : X \to \mathcal{F}(X)$ by

$$(Tx)(t) = \begin{cases} 0 & , 0 \le t < \frac{2}{3} \\ \dfrac{2}{3} & , \frac{2}{3} \le t \le \frac{2(x+1)}{3} \\ \dfrac{2}{5} & , \frac{2(x+1)}{3} < t \le 1, \end{cases}$$

Define $\alpha : X \to (0, 1]$ by $\alpha(x) = \dfrac{2}{3}$. We observe that

$$[Tx]_{\frac{2}{3}} = \Big[\frac{2}{3}, \frac{2(x+1)}{3}\Big]$$

for all $x \in X$. Thus, $[Tx]_{\alpha(x)}$ is a nonempty closed subsets of X. Consider $H([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) = \frac{4}{9}(x-y)^2 = \frac{4}{9}d(x, y)$, where $\psi(t) = \frac{4}{9}t$. Therefore all conditions of Theorem 3.1 hold and thus there exists $\frac{2}{3} \in X$ is an α-fuzzy fixed point of T. Note that Corollary 3.4 and the results of fuzzy mappings in usual metric space cannot be applied for this example but Theorem 3.1 and Corollary 3.2 are applicable.

Here, we study some relation of multivalued mappings and fuzzy mappings. Indeed, we indicate that Corollary 3.2 can be utilized to derive fixed point for mutivalued mapping.

Corollary 3.7. *Let (X, d) be a complete b-metric space (with coefficient $s \ge 1$) and $S : X \to CL(X)$ be multivalued mapping such that*

$$H(Sx, Sy) \le kd(x, y), \tag{3.4}$$

for all $x, y \in X$, where $0 < k < 1$. Assume that $k < \frac{1}{s}$, then there exists $u \in X$ such that $u \in Su$.

Proof. Let $\alpha : X \to (0, 1]$ be an arbitrary mapping and $T : X \to \mathcal{F}(X)$ defined by

$$(Tx)(t) = \begin{cases} \alpha(x) & , t \in Sx \\ 0 & , t \notin Sx. \end{cases}$$

By a routine calculation, we obtain that

$$[Tx]_{\alpha(x)} = \{t : (Tx)(t) \ge \alpha(x)\} = Sx.$$

Now condition (3.4) become condition (3.1). Therefore, Corollary 3.2 can be applied to obtain $u \in X$ such that $u \in [Tu]_{\alpha(u)} = Su$. This implies that multivalued mapping S have a fixed point. This completes the proof. □

Corollary 3.8. *[17] Let (X, d) be a complete b-metric space (with coefficient $s \ge 1$) and $S : X \to CB(X)$ be multivalued mapping such that*

$$H(Sx, Sy) \le kd(x, y), \tag{3.5}$$

for all $x, y \in X$, where $0 < k < 1$. Assume that $k < \frac{1}{s}$, then there exists $u \in X$ such that $u \in Su$.

Remark 3.9. If we set $s = 1$ in Corollary 3.8 (it corresponds to the case of metric spaces), we find theorem of Nadler [24]. Hence, Corollary 3.8 is an extension of the result of Nadler [24].

Corollary 3.10. *[24] Let (X, d) be a complete metric space and $S : X \to CB(X)$ be multivalued mapping such that*

$$H(Sx, Sy) \le kd(x, y), \tag{3.6}$$

for all $x, y \in X$, where $0 < k < 1$. Then there exists $u \in X$ such that $u \in Su$.

4. Conclusions

In the present work we introduced a new concept of fuzzy mappings in complete b-metric spaces. Also, we derived the existence of α-fuzzy fixed point theorems for fuzzy mappings in complete b-metric space and we also give illustrative examples to support our main result, showing that while existing results in usual metric space and ordinary metric space are not applicable, our result is. Finally, we showed some relation of multivalued mappings and fuzzy mappings, which can be utilized to derive fixed point for multivalued mappings.

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References

[1] M. Abbas, B. Damjanović, R. Lazović, *Fuzzy common fixed point theorems for generalized contractive mappings*, Appl. Math. Lett., **23** (2010), 1326–1330. 1

[2] J. Ahmad, A. Azam, S. Romaguera, *On locally contractive fuzzy set valued mappings*, J. Inequalities Appl., **2014** (2014), 10 pages. 1

[3] H. Aydi, M-F. Bota, E. Karapinar, S. Mitrović, *A fixed point theorem for set-valued quasi-contractions in b-metric spaces*, Fixed Point Theory Appl, **2012** (2012), 8 pages. 1

[4] H. Aydi, M-F. Bota, E. Karapinar, S. Moradi, *A common fixed point for weak ϕ-contractions on b-metric spaces*, Fixed Point Theory, **13** (2012), 337–346. 1

[5] A. Azam, M. Arshad, I. Beg, *Fixed points of fuzzy contractive and fuzzy locally contractive maps*, Chaos, Solitons and Fractals, **4** (2009), 2836–2841. 1

[6] A. Azam, I. Beg, *Common fixed points of fuzzy maps*, Mathematical and Computer Modelling **49** (2009), 1331–1336. 1

[7] I. A. Bakhtin, *The contraction mapping principle in quasimetric spaces*, Funct. Anal., Unianowsk Gos. Ped. Inst., **30** (1989), 26–37. 1, 2, 2

[8] V. Berinde, *Generalized contractions in quasimetric spaces*, Seminar on Fixed Point Theory, Preprint, **3** (1993), 3–9. 1, 2

[9] M. Boriceanu, M. Bota and A.Petru, *Multivalued fractals in b-metric spaces*, Central European J. Math., **8** (2010), 367–377. 1, 2, 2.6

[10] M. Boriceanu, A. Petruşel, I. A. Rus, *Fixed point theorems for some multivalued generalized contractions in b-metric spaces*, Int. J. Math. Stat., **6** (2010), 65–76. 1

[11] M. Boriceanu, *Strict fixed point theorems for multivalued operators in b-metric spaces*, Int. J. Mod. Math., **4** (2009), 285–301. 1

[12] M. Boriceanu, *Fixed point theory for multivalued generalized contraction on a set with two b-metrics*, Studia Univ. Babeş-Bolyai, Mathematica, **3** (2009), 3–14. 1

[13] R. K. Bose, M. K. Roychowdhury, *Fixed point theorems for generalized weakly contractive mappings*, Surv. Math. Appl., **4** (2009), 215–238. 1

[14] M. Bota, E. Karapnar, O.Mleşniţe, *Ulam-Hyers stability results for fixed point problems via $\alpha - \psi$-contractive mapping in (b)-metric space*, Abstract and Applied Analysis, **2013** (2013), Article ID 825293, 6 pages. 1

[15] S. Czerwik, *Contraction meppings in b-metric spaces*, Acta Mathematica et Informatica Universitatis Ostraviensis, **1** (1993), 5–11. 1, 2, 2, 2

[16] S. Czerwik, K. Dlutek, S. L. Singh, *Round-off stability of iteration procedures for operators in b-metric spaces*, J. Natur. Phys. Sci. **11** (1997), 87–94. 1, 2, 2, 2

[17] S. Czerwik, *Nonlinear set-valued contraction mappings in b-metric spaces*, Atti Sem. Mat. Univ. Modena, **46** (1998), 263–276. 1, 2, 2, 3.8

[18] V. D. Estruch, A. Vidal, *A note on fixed fuzzy points for fuzzy mappings*, Rend Istit Univ Trieste, **32** (2001), 39–45. 1

[19] M. Frigon, D. O'Regan, *Fuzzy contractive maps and fuzzy fixed points*, Fuzzy Sets and Systems, **129** (2002), 39–45. 1

[20] S. Heilpern, *Fuzzy mappings and fixed point theorems*, J. Math. Anal. Appl., **83** (1981), 566–569. 1, 3.4

[21] M. A. Kutbi, J. Ahmad, A. Azam, N. Hussain, *On fuzzy fixed points for fuzzy maps with generalized weak property*, J. Appl. Math., **2014** (2014), Article ID 549504, 12 pages. 1

[22] M. A. Kutbi, E. Karapinar, J. Ahmad, A. Azam, *Some fixed point results for multi-valued mappings in b-metric spaces*, J. Inequalities Appl., **2014** (2014), 11 pages. 1

[23] B. S. Lee, S. J. Cho, *A fixed point theorem for contractive type fuzzy mappings*, Fuzzy Sets and Systems, **61** (1994), 309–312. 1

[24] S. B. Nadler, *Multivalued contraction mapping*, Pacific J. Math., **30** (1969), 475–488. 1, 3.9, 3.10

[25] J. von Neumann, *Zur theorie der gesellschaftsspiele*, Math. Annalen, **100** (1928), 295–320. 1

[26] S. Phiangsungnoen, W. Sintunavarat, P. Kumam, *Common α-fuzzy fixed point theorems for fuzzy mappings via $\beta_{\mathcal{F}}$-admissible pair*, J. Intelligent and Fuzzy Systems, **27** (2014), 2463–2472. 1

[27] S. Phiangsungnoen, W. Sintunavarat, P. Kumam, *Fuzzy fixed point theorems in Hausdorff fuzzy metric spaces*, J. Inequalities Appl., **2014** (2014), 10 pages. 1

[28] S. Phiangsungnoen, W. Sintunavarat, P. Kumam, *Fuzzy fixed point theorems for fuzzy mappings via β-admissible with applications*, J. Uncertainty Anal, Appl,, **2** (2014), 17 pages. 1

[29] B. Singh, M. S. Chauhan, *Fixed points of associated multimaps of fuzzy maps*, Fuzzy Sets and Systems, **110** (2000), 131–134. 1

[30] S. L. Singh, B. Prasad, *Some coincidence theorems and stability of iterative procedures*, Comput. Math. Appl., **55** (2008), 2512–2520. 2, 2

[31] W. Sintunavarat, S. Plubtieng, P. Katchang, *Fixed point result and applications on b-metric space endowed with an arbitrary binary relation*, Fixed Point Theory and Applications, **2013** (2013), 13 pages. 1

[32] D. Turkoglu, B. E. Rhoades, *A fixed fuzzy point for fuzzy mapping in complete metric spaces*, Math. Commun., **10** (2005), 115–121.