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Common fixed point of four self maps on dislocated metric spaces

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Abstract

The purpose of this paper is to generalize and to unify common fixed theorems of Bennani et al. [S. Bennani, H. Bourijal, S. Mhanna, D. El Moutawakil, J. Nonlinear Sci Appl., 8 (2015), 86–92] on dislocated metric spaces.

Keywords: Coincidence point, common fixed point, dislocated metric spaces.
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1. Introduction and Preliminaries

In 2000, Hitzler [7] introduced the concept of dislocated metric spaces as a generalization of metric spaces and presented variants of Banach contraction principle in such spaces. Amini-Haradi [2] re-introduced the dislocated space under the name of a metric-like space and proved some fixed theorems in this space. Very recently, Bennani et al. [4] established two new common fixed point theorems for four self maps on dislocated metric spaces, which improved the results of Panthi and Jha [9] without any continuity requirement.

In this paper, we establish a common fixed point theorem satisfying a contraction of Ćirić type in dislocated metric spaces. Our result unifies and generalizes the two theorems of Bennani et al. [4]. As further application, we give some common fixed theorems in 0-complete weak partial metric spaces.

Now, let us recall some basic concepts and facts about dislocated metric spaces.

Definition 1.1. [2, 7] A mapping $d : X \times X \to [0, +\infty)$, where X is a nonempty set, is said to be a dislocated metric (in short, a d-metric) on X if the following conditions hold:

(d1) $d(x, y) = d(y, x)$,
(d2) if $d(x, y) = d(y, x) = 0$ then $x = y$,

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(d3) $d(x, y) \leq d(x, z) + d(z, y)$,

for all $x, y, z \in X$. The pair (X, d) is called a dislocated metric space (in short, a d-metric space).

It is clear that a d-metric space (X, d) satisfies all of the conditions of a metric except that $d(x, x)$ may be positive for $x \in X$.

Definition 1.2. [7] Let (X, d) be a d-metric space. Then

(1) A sequence $\{x_n\}$ in X converges to $x \in X$ if and only if $\lim_{n\to+\infty} d(x, x_n) = 0$.
(2) A sequence $\{x_n\}$ in X is called a Cauchy sequence if and only if $d(x_n, x_m) \to 0$ as $m, n \to +\infty$.
(3) The space (X, d) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges.

Remark 1.3. [4] It can be verified that in d-metric spaces, the following statements hold.

(i) A subsequence of a Cauchy sequence is a Cauchy sequence.
(ii) A Cauchy sequence which possesses a convergent subsequence, converges.
(iii) Limits of a convergent sequence are unique.
(iv) A d-metric d is continuous, that is, $\{x_n\}$ converges to x and $\{y_n\}$ converges to y imply $d(x_n, y_n) \to d(x, y)$ as $n \to \infty$.

Definition 1.4. Let A and S be self maps of a set X. If $Ax = Sx$ for some $x \in X$, then x is called a coincidence point of A and S. The pair A, S of self maps is weakly compatible if they commute at their coincidence points.

2. Main results

Now we can state our main results.

Theorem 2.1. *Let (X, d) be a d-metric space and let A, B, T and S be four self maps on X such that $TX \subset AX$ and $SX \subset BX$. Suppose that there exists a real number λ with $\lambda \in [0, \frac{1}{2})$, satisfying for all $x, y \in X$*

$$d(Sx, Ty) \leq \lambda M(x, y), \tag{2.1}$$

where

$$M(x, y) = \max\{d(Ax, Ty), d(By, Sx), d(Ax, Sx), d(By, Ty), 2d(Ax, By)\}.$$

If the range of one of the mappings A, B, S and T is a complete subspace of X, then

(i) B and T have a coincidence point u,
(ii) A and S have a coincidence point v, and
(iii) $Av = Sv = Bu = Tu$.

Moreover, if the pairs $\{A, S\}$ and $\{B, T\}$ are weakly compatible, then A, B, S and T have a unique common fixed point y and $d(y, y) = 0$.

Proof. Let x_0 be an arbitrary point in X. Since $SX \subseteq BX$, there exists $x_1 \in X$ such that $Bx_1 = Sx_0$. Since $TX \subset AX$, there exists $x_2 \in X$ such that $Ax_2 = Tx_1$. Continuing this process, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X defined by

$$y_{2n} = Sx_{2n} = Bx_{2n+1},\ y_{2n+1} = Tx_{2n+1} = Ax_{2n+2},\ \forall n \in \mathbb{N}. \tag{2.2}$$

We claim that $\{y_n\}$ is a Cauchy sequence in (X, d).

First we prove that for each $n \geq 1$,

$$d(y_n, y_{n+1}) \leq \delta d(y_{n-1}, y_n), \tag{2.3}$$

where $\delta = \max\{\frac{\lambda}{1-\lambda}, 2\lambda\}$.

Using (2.1), we obtain

$$d(y_{2n}, y_{2n+1}) = d(Sx_{2n}, Tx_{2n+1}) \leq \lambda M(x_{2n}, x_{2n+1}),$$

where

$$\begin{aligned} & M(x_{2n}, x_{2n+1}) \\ = \;& \max\{d(Ax_{2n}, Tx_{2n+1}), d(Bx_{2n+1}, Sx_{2n}), d(Ax_{2n}, Sx_{2n}), d(Bx_{2n+1}, Tx_{2n+1}), 2d(Ax_{2n}, Bx_{2n+1})\} \\ = \;& \max\{d(y_{2n-1}, y_{2n+1}), d(y_{2n}, y_{2n}), d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), 2d(y_{2n-1}, y_{2n})\} \\ \leq \;& \max\{d(y_{2n-1}, y_{2n+1}), d(y_{2n}, y_{2n-1}) + d(y_{2n-1}, y_{2n}), d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), 2d(y_{2n-1}, y_{2n})\} \\ = \;& \max\{d(y_{2n-1}, y_{2n+1}), 2d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\}. \end{aligned}$$

Thus, we get the following three cases.

Case i. If $d(y_{2n}, y_{2n+1}) \leq \lambda d(y_{2n-1}, y_{2n+1})$, then

$$d(y_{2n}, y_{2n+1}) \leq \lambda d(y_{2n-1}, y_{2n}) + \lambda d(y_{2n}, y_{2n+1}).$$

From this , we deduce

$$d(y_{2n}, y_{2n+1}) \leq \frac{\lambda}{1-\lambda} d(y_{2n-1}, y_{2n}) \leq \delta d(y_{2n-1}, y_{2n}).$$

Case ii. If $d(y_{2n}, y_{2n+1}) \leq \lambda \cdot 2d(y_{2n-1}, y_{2n})$, then

$$d(y_{2n}, y_{2n+1}) \leq 2\lambda d(y_{2n-1}, y_{2n}) \leq \delta d(y_{2n-1}, y_{2n}).$$

Case iii. If $d(y_{2n}, y_{2n+1}) \leq \lambda d(y_{2n}, y_{2n+1})$, then by $\lambda \in [0, \frac{1}{2})$ we have $d(y_{2n}, y_{2n+1}) = 0 \leq \delta d(y_{2n-1}, y_{2n})$.

So, we have shown that $d(y_{2n}, y_{2n+1}) \leq \delta d(y_{2n-1}, y_{2n})$ for all $n \geq 1$. By similar arguments we can deduce that $d(y_{2n-1}, y_{2n}) \leq \delta d(y_{2n-2}, y_{2n-1})$ for all $n \geq 1$. Therefore, (2.3) holds for each n. That is , for $n \geq 1$ we have $d(y_n, y_{n+1}) \leq \delta d(y_{n-1}, y_n)$. From this, we see that for each $n \geq 1$

$$d(y_n, y_{n+1}) \leq \delta d(y_{n-1}, y_n) \leq \delta^2 d(y_{n-2}, y_{n-1}) \leq \cdots \leq \delta^n d(y_0, y_1). \tag{2.4}$$

By the triangle inequality, for $m > n$ we obtain

$$\begin{aligned} d(y_n, y_m) &\leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \cdots + d(y_{m-1}, y_m) \\ &\leq (\delta^n + \delta^{n+1} + \cdots + \delta^{m-1}) d(y_0, y_1) \\ &\leq \frac{\delta^n}{1-\delta} d(y_0, y_1). \end{aligned} \tag{2.5}$$

Notice that $\delta = \max\{\frac{\lambda}{1-\lambda}, 2\lambda\} \in [0, 1)$ because $\lambda \in [0, \frac{1}{2})$. From (2.5), it follows that $\{y_n\}$ is a Cauchy sequence. According to Remark 1.3(i), $\{Sx_{2n}\}$, $\{Bx_{2n+1}\}$, $\{Tx_{2n+1}\}$ and $\{Ax_{2n+2}\}$ are also Cauchy sequence.

Now we can suppose, without loss of generality, that SX is a complete subspace of X. Then the sequence $\{Sx_{2n}\}$ converges to some Sa such that $a \in X$. By Remark 1.3(ii), we see that $\{y_n\}$, $\{Bx_{2n+1}\}$, $\{Tx_{2n+1}\}$ and $\{Ax_{2n+2}\}$ converge to Sa. Since $SX \subset BX$, there exists $u \in X$ such that $Sa = Bu$. By the triangle inequality, we obtain $d(Bu, Bu) = d(Sa, Sa) \leq d(Sa, y_n) + d(y_n, Sa) = 2d(Sa, y_n)$. Since $d(Sa, y_n) \to 0$, we see $d(Bu, Bu) = 0$. We claim that $d(Bu, Tu) = 0$. Suppose, to the contrary, that $d(Bu, Tu) > 0$. By (d3) and (2.1) we get

$$d(Bu, Tu) \leq d(Bu, Sx_{2n}) + d(Sx_{2n}, Tu) \leq d(Bu, Sx_{2n}) + \lambda M(x_{2n}, u), \tag{2.6}$$

where

$$\begin{aligned} M(x_{2n}, u) &= \max\{d(Ax_{2n}, Tu), d(Bu, Sx_{2n}), d(Ax_{2n}, Sx_{2n}), d(Bu, Tu), 2d(Ax_{2n}, Bu)\} \\ &= \max\{d(y_{2n-1}, Tu), d(Bu, y_{2n}), d(y_{2n-1}, y_{2n}), d(Bu, Tu), 2d(y_{2n-1}, Bu)\}. \end{aligned}$$

Letting $n \to \infty$ in (2.6), by Remark 1.3(iv) we deduce that

$$\begin{aligned} d(Bu,Tu) &\leq d(Bu,Bu) + \lambda \max\{d(Bu,Tu), d(Bu,Bu), d(Bu,Bu), d(Bu,Tu), 2d(Bu,Bu)\} \\ &= \lambda d(Bu,Tu) < \frac{1}{2} d(Bu,Tu), \end{aligned}$$

which is a contradiction. Thus we obtain that $d(Bu,Tu) = 0$ and $Bu = Tu$, that is, u is a coincidence point of B and T. So we proved (i).

Since $TX \subset AX$, there exists $v \in X$ such that $Tu = Av$. We show that $Sv = Av$. Indeed, using (2.1) and $d(Bu,Bu)$=0, we have

$$d(Sv,Av) = d(Sv,Tu) \leq \lambda M(v,u),$$

where

$$\begin{aligned} M(v,u) &= \max\{d(Av,Tu), d(Bu,Sv), d(Av,Sv), d(Bu,Tu), 2d(Av,Bu)\} \\ &= \max\{d(Bu,Bu), d(Av,Sv), d(Av,Sv), d(Bu,Bu), 2d(Bu,Bu)\} \\ &= d(Av,Sv). \end{aligned}$$

From this and $\lambda \in [0, \frac{1}{2})$, we see that $d(Sv,Av) = 0$ and $Sv = Av$. Thus, v is a coincidence point of S and A. So we proved (ii). Since $Bu = Tu$, $Tu = Av$ and $Av = Sv$, we see that (iii) holds.

Now, we assume that the pairs $\{A,S\}$ and $\{B,T\}$ are weakly compatible. Then

$$AAv = ASv = SAv = SSv, \quad BBu = BTu = TBu = TTu.$$

Let $y = Bu = Tu = Av = Sv$. We show that y is a fixed point of T. From (2.1) and $d(y,y) = 0$, we obtain

$$d(y,Ty) = d(Sv,Ty) \leq \lambda M(v,y),$$

Where

$$\begin{aligned} M(v,y) &= \max\{d(Av,Ty), d(By,Sv), d(Av,Sv), d(By,Ty), 2d(Av,By)\} \\ &= \max\{d(y,Ty), d(Ty,y), d(y,y), d(Ty,Ty), 2d(y,Ty)\} \\ &= 2d(y,Ty). \end{aligned}$$

Since $2\lambda \in [0,1)$, we have $d(y,Ty) = 0$, which implies $Ty = y$, that is, y is a fixed point of T. It follows that $By = BBu = TTu = Ty = y$, which implies that y is a fixed point of B.

On the other hand, by (2.1) we get

$$d(Sy,y) = d(Sy,Ty) \leq \lambda M(y,y),$$

Where

$$\begin{aligned} M(y,y) &= \max\{d(Ay,Ty), d(By,Sy), d(Ay,Sy), d(By,Ty), 2d(Ay,By)\} \\ &= \max\{d(Sy,y), d(y,Sy), d(Sy,Sy), d(y,y), 2d(Sy,y)\} \\ &= 2d(Sy,y). \end{aligned}$$

Since $2\lambda \in [0,1)$, we see $d(Sy,y) = 0$, which implies $Sy = y$, that is, y is a fixed point of S. It follows that $Ay = ASv = SAv = Sy = y$, which implies that y is also a fixed point of A. Hence, we have shown that y is a common fixed point of S,T,A and B and $d(y,y) = d(y,Ty) = 0$.

Finally to prove the uniqueness, suppose that there exists $u,v \in X$ such that $Su = Tu = Au = Bu = u$ and $Sv = Tv = Av = Bv = v$. By (2.1), we get

$$d(u,v) = d(Su,Tv) \leq M(u,v),$$

where

$$\begin{aligned} M(u,v) &= \max\{d(Au,Tv), d(Bv,Su), D(Au,Su), d(Bv,Tv), 2d(Au,Bv)\} \\ &= \max\{d(u,v), d(v,u), d(u,u), d(v,v), 2d(u,v)\} \\ &= 2d(u,v). \end{aligned}$$

Since $2\lambda \in [0,1)$, it follows that $d(u,v)=0$ and $u=v$. □

From Theorem 2.1, we obtain the following corollary, which unifies the two theorems of Bennani [4].

Corollary 2.2. *Let (X,d) be a d-metric space and let A, B, T and S be four self maps on X such that $TX \subset AX$ and $SX \subset BX$. Suppose that there exist real numbers α, β, γ with $\alpha, \beta, \gamma \geq 0$ and $2\alpha + 2\beta + \frac{\gamma}{2} < \frac{1}{2}$, satisfying for all $x, y \in X$*

$$d(Sx,Ty) \leq \alpha(d(Ax,Ty) + d(By,Sx)) + \beta(d(Ax,Sx)) + d(By,Ty)) + \gamma d(Ax,By). \tag{2.7}$$

If the range of one of the mappings A, B, S and T is a complete subspace of X, then

(i) A and S have a coincidence point , and
(ii) B and T have a coincidence point.

Moreover, if the pairs $\{A,S\}$ and $\{B,T\}$ are weakly compatible, then A, B, S and T have a unique common fixed point y and $d(y,y)=0$.

Proof. Let $\lambda = 2\alpha + 2\beta + \frac{\gamma}{2}$. Since $\alpha, \beta, \gamma \geq 0$ and $2\alpha + 2\beta + \frac{\gamma}{2} < \frac{1}{2}$, we have $\lambda \in [0, \frac{1}{2})$. Using (2.7), for all $x, y \in X$ we see

$$\begin{aligned} d(Sx,Ty) &\leq \alpha(d(Ax,Ty) + d(By,Sx)) + \beta(d(Ax,Sx)) + d(By,Ty)) + \gamma d(Ax,By) \\ &\leq (2\alpha + 2\beta + \frac{\gamma}{2})M(x,y) \\ &= \lambda M(x,y) \end{aligned}$$

where $M(x,y) = \max\{d(Ax,Ty), d(By,Sx), d(Ax,Sx), d(By,Ty), 2d(Ax,By)\}$. From Theorem 2.1, the conclusion follows immediately. □

Remark 2.3. Let $\alpha, \beta, \gamma \geq 0$. If $\alpha + \beta + \gamma < \frac{1}{4}$, then

$$2\alpha + 2\beta + \frac{\gamma}{2} = \frac{1}{2}[(\alpha+\beta+\gamma) + 3(\alpha+\beta)] < \frac{1}{2} \cdot (\frac{1}{4} + 3 \cdot \frac{1}{4}) = \frac{1}{2}.$$

If $\gamma > 0$ and $\alpha + \beta + \gamma \leq \frac{1}{4}$, then $\alpha + \beta < \frac{1}{4}$ and

$$2\alpha + 2\beta + \frac{\gamma}{2} = \frac{1}{2}[(\alpha+\beta+\gamma) + 3(\alpha+\beta)] < \frac{1}{2} \cdot (\frac{1}{4} + 3 \cdot \frac{1}{4}) = \frac{1}{2}.$$

Consequently, Corollary 2.2 unifies the two theorems of Bennani et al. [4].

Using the same arguments as in Corollary 2.2, we obtain the following result.

Corollary 2.4. *Let (X,d) be a d-metric space and let A, B, T and S be four self maps on X such that $TX \subset AX$ and $SX \subset BX$. Suppose that there exist real numbers $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 with $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 \geq 0$ and $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \frac{\alpha_5}{2} < \frac{1}{2}$, satisfying for all $x, y \in X$*

$$d(Sx,Ty) \leq \alpha_1 d(Ax,Ty) + \alpha_2 d(By,Sx) + \alpha_3 d(Ax,Sx) + \alpha_4 d(By,Ty) + \alpha_5 d(Ax,By).$$

If the range of one of the mappings A, B, S and T is a complete subspace of X, then

(i) A and S have a coincidence point , and
(ii) B and T have a coincidence point.

Moreover, if the pairs $\{A, S\}$ and $\{B, T\}$ are weakly compatible, then A, B, S and T have a unique common fixed point y and $d(y, y) = 0$.

Putting $A = B$ and $S = T$ in Theorem 2.1, we obtain the following corollary.

Corollary 2.5. *Let (X, d) be a d-metric space and let A and T be two self maps on X such that $TX \subset AX$. Suppose that there exists a real number λ with $\lambda \in [0, \frac{1}{2})$, satisfying for all $x, y \in X$*

$$d(Tx, Ty) \leq \lambda M(x, y),$$

where

$$M(x, y) = \max\{d(Ax, Ty), d(Ay, Tx), d(Ax, Tx), d(Ay, Ty), 2d(Ax, Ay)\}.$$

If AX or TX is a complete subspace of X, then A and T have a coincidence point in X. Moreover, if the pair $\{A, T\}$ is weakly compatible, then A and T have a unique common fixed point y in X and $d(y, y) = 0$.

Putting $A = B = I_X$ in Theorem 2.1, where I_X is the identity mapping on X, we obtain the following corollary.

Corollary 2.6. *Let (X, d) be a d-metric space and let T and S be two self maps on X. Suppose that there exists a real number λ with $\lambda \in [0, \frac{1}{2})$, satisfying for all $x, y \in X$*

$$d(Sx, Ty) \leq \lambda M(x, y),$$

where

$$M(x, y) = \max\{d(x, Ty), d(y, Sx), d(x, Sx), d(y, Ty), 2d(x, y)\}.$$

Then S and T have a unique common fixed point y in X and $d(y, y) = 0$.

Putting $S = T = I_X$ in Theorem 2.1, where I_X is the identity mapping on X, we obtain the following corollary.

Corollary 2.7. *Let (X, d) be a complete d-metric space and let A and B be two surjective self maps on X. Suppose that there exists a real number λ with $\lambda \in [0, \frac{1}{2})$, satisfying for all $x, y \in X$*

$$d(x, y) \leq \lambda M(x, y),$$

where

$$M(x, y) = \max\{d(Ax, y), d(By, x), d(Ax, x), d(By, y), 2d(Ax, By)\}.$$

Then A and B have a unique common fixed point y in X and $d(y, y) = 0$.

3. Application to weak partial metric spaces

In 1992, Matthews [8] introduced the notion of partial metric space as a part of the study of denotational semantics of dataflow networks. Further, Matthews showed that the Banach Contraction principle is valid in partial metric spaces and can be applied in program verification. After that, many authors studied and generalized the results of Matthews (see, for example, [1, 3, 5, 6]).

First, let us shortly recall some definitions and facts about weak partial metric spaces. For more details, we can refer to [1, 6].

Definition 3.1. Let $p : X \times X \to [0, +\infty)$ be a function where X is nonempty set. If the function p satisfies the following conditions for all $x, y, z \in X$:

(pm1) $x = y$ if and only if $p(x, x) = p(y, y) = p(x, y)$,
(pm2) $p(x, x) \leqslant p(x, y)$,

(pm3) $p(x,y) = p(y,x)$,
(pm4) $p(x,y) \leqslant p(x,z) + p(z,y) - p(z,z)$,

then p is called a partial metric on X and the pair (X,p) is called a partial metric space. If the function p satisfies (pm1), (pm3) and (pm4), then p is called a weak partial metric on X and the pair (X,p) is called a weak partial metric space.

It is clear that the partial metric space is a weak partial metric space, but the converse may not be true; see [6, Example 12].

Definition 3.2. [1, 6] Let (X,p) be a weak partial metric space. Then:

(i) a sequence $\{x_n\}$ in X converges to $x \in X$ if and only if $p(x_n,x) \to p(x,x)$ as $n \to \infty$;
(ii) a sequence $\{x_n\}$ in X is called a Cauchy sequence if and only if $\lim_{m,n\to\infty} p(x_n,x_m)$ exists (and are finite);
(iii) a sequence $\{x_n\}$ in X is called a 0-Cauchy sequence if and only if $p(x_n,x_m) \to 0$ as $m,n \to \infty$;
(iv) the space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges;
(v) the space (X,p) is said to be 0-complete if every 0-Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$ such that $p(x,x) = 0$.

It is not hard to see that if (X,p) be a 0-complete weak partial metric space, then (X,p) is a complete dislocated metric space. Consequently, using Theorem 2.1 we obtain the following result.

Corollary 3.3. *Let (X,p) be a weak partial metric space and let A,B,T and S be four self maps on X such that $TX \subset AX$ and $SX \subset BX$. Suppose that there exists a real number λ with $\lambda \in [0,\frac{1}{2})$, satisfying for all $x,y \in X$*

$$p(Sx,Ty) \leq \lambda M(x,y),$$

where

$$M(x,y) = \max\{p(Ax,Ty), p(By,Sx), p(Ax,Sx), p(By,Ty), 2p(Ax,By)\}.$$

If the range of one of the mappings A,B,S and T is a complete subspace of X, then

(i) A and S have a coincidence point, and
(ii) B and T have a coincidence point.

Moreover, if the pairs $\{A,S\}$ and $\{B,T\}$ are weakly compatible, then A,B,S and T have a unique common fixed point.

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