J. Nonlinear Sci. Appl. 3 (2010), no. 1, 12–20

The Journal of Nonlinear Science and Applications
http://www.tjnsa.com

α-OPEN SETS AND α-BICONTINUITY IN DITOPOLOGICAL TEXTURE SPACES

ŞENOL DOST

Communicated by S. Jafari
This paper is dedicated to my wife Meliha's 35th birthday

Abstract. In this paper we generalize the notion of α-open and α-closed sets to ditopological texture spaces. We also introduce the concepts of α-bicontinuous difunctions and discuss their properties in detail.

1. Introduction

Textures were introduced by L. M. Brown as a point-set for the study of fuzzy sets, but they have since proved useful as a framework in which to discuss complement free mathematical concepts. In this section we recall some basic notions regarding textures and ditopologies, and an adequate introduction to the theory and the motivation for its study may be obtained from [1, 2, 3, 4, 5, 6, 7].

If S is a set, a *texturing* $\mathcal{S}$ of S is a point-separating, complete, completely distributive lattice containing S and $\emptyset$, and for which arbitrary meets coincide with intersections, and finite joins with unions. The pair $(S, \mathcal{S})$ is then called a *texture space* or shortly, *texture*.

For a texture $(S, \mathcal{S})$, most properties are conveniently defined in terms of the *p-sets* $P_s = \bigcap\{A \in \mathcal{S} \mid s \in A\}$ and, as a dually, the *q-sets*, $Q_s = \bigvee\{A \in \mathcal{S} \mid s \notin A\}$. For $A \in \mathcal{S}$ the *core* $A^\flat$ of A is defined by $A^\flat = \{s \in S \mid A \not\subseteq Q_s\}$.

The following are some basic examples of textures we will need later on.

Examples 1.1. (1) If X is a set and $\mathcal{P}(X)$ the powerset of X, then $(X, \mathcal{P}(X))$ is the *discrete texture* on X. For $x \in X$, $P_x = \{x\}$ and $Q_x = X \setminus \{x\}$.

Date: Revised: 09 July 2009.
2000 *Mathematics Subject Classification.* 54A05, 54C10.
Key words and phrases. Texture, Ditopology, α-open, α-closed, α-bicontinuity.

(2) Setting $\mathbb{I} = [0,1]$, $\mathcal{I} = \{[0,r), [0,r] \mid r \in \mathbb{I}\}$ gives the *unit interval texture* $(\mathbb{I}, \mathcal{I})$. For $r \in \mathbb{I}$, $P_r = [0,r]$ and $Q_r = [0,r)$.

A *ditopology* on a texture $(S, \mathcal{S})$ is a pair (τ, κ) of subsets of $\mathcal{S}$, where the set of *open sets* τ satisfies

(1) $S, \emptyset \in \tau$,
(2) $G_1, G_2 \in \tau \Longrightarrow G_1 \cap G_2 \in \tau$ and
(3) $G_i \in \tau,\ i \in I \Longrightarrow \bigvee_i G_i \in \tau$,

and the set of *closed sets* κ satisfies

(1) $S, \emptyset \in \kappa$,
(2) $K_1, K_2 \in \kappa \Longrightarrow K_1 \cup K_2 \in \kappa$ and
(3) $K_i \in \kappa,\ i \in I \Longrightarrow \bigcap K_i \in \kappa$.

Hence a ditopology is essentially a "topology" for which there is no *a priori* relation between the open and closed sets. For $A \in \mathcal{S}$ we define the *closure* and the *interior* of A under (τ, κ) by the equalities

$$clA = \bigcap\{K \in \kappa \mid A \subseteq K\} \text{ and } intA = \bigvee\{G \in \tau \mid G \subseteq A\}.$$

A texturing $\mathcal{S}$ need not be closed under the operation of taking the set complement. Now, suppose that $(S, \mathcal{S})$ has a complementation σ, that is an involution $\sigma : \mathcal{S} \to \mathcal{S}$ satisfying $A, B \in \mathcal{S}, A \subseteq B \Longrightarrow \sigma(B) \subseteq \sigma(A)$. Then if τ and κ are related by $\kappa = \sigma[\tau]$ we say that (τ, κ) is a *complemented ditopology* on $(S, \mathcal{S}, \sigma)$. In this case we have $\sigma(clA) = int(\sigma(A))$ and $\sigma(intA) = cl(\sigma(A))$.

We denote by $O(S, \mathcal{S}, \tau, \kappa)$, or when there can be no confusion by $O(S)$, the set of open sets in $\mathcal{S}$. Likewise, $C(S, \mathcal{S}, \tau, \kappa)$, $C(S)$ will denote the set of closed sets.

One of the most useful notions of (ditopological) texture spaces is that of *difunction*. A difunction is a special type of direlation [4]. For a difunction $(f, F) : (S, \mathcal{S}) \to (T, \mathcal{T})$ we will have cause to use the *inverse image* $f^{\leftarrow}B$ and *inverse co-image* $F^{\leftarrow}B$, $B \in \mathcal{T}$, which are equal; and the *image* $f^{\rightarrow}A$ and *co-image* $F^{\rightarrow}A$, $A \in \mathcal{S}$, which are usually not.

Now we consider complemented textures $(S_j, \mathcal{S}_j, \sigma_j)$, $j = 1, 2$ and the difunction $(f, F) : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$. The complement of the difunction (f, F) is denoted by $(f, F)'$ [4]. If $(f, F) = (f, F)'$ then the difunction (f, F) is called complemented.

The difunction $(f, F) : (S, \mathcal{S}, \tau_S, \kappa_S) \to (T, \mathcal{T}, \tau_T, \kappa_T)$ is called *continuous* if $B \in \tau_T \Longrightarrow F^{\leftarrow}B \in \tau_S$, *cocontinuous* if $B \in \kappa_T \Longrightarrow f^{\leftarrow}B \in \kappa_S$ and *bicontinuous* if it is both.

For complemented difunctions these two properties are equivalent.

Finally, we also recall from [8, 9, 11] the classes of ditopological texture spaces and difunctions.

Definition 1.2. For a ditopological texture space $(S, \mathcal{S}, \tau, \kappa)$:

(1) $A \in \mathcal{S}$ is called pre-open (resp. semi-open, β-open) if $A \subseteq intclA$ (resp. $A \subseteq clintA, A \subseteq clintclA$). $B \in \mathcal{S}$ is called pre-closed (resp. semi-closed, β-closed) if $clintB \subseteq B$ (resp. $intclB \subseteq B, intclintB \subseteq B$)

(2) A difunction $(f,F):(S,\mathcal{S},\tau_S,\kappa_S)\to(T,\mathcal{T},\tau_T,\kappa_T)$ is called pre-continuous (resp. semi-continuous, β-continuous) if $F^{\leftarrow}(G)\in PO(S)$ (resp. $F^{\leftarrow}(G)\in SO(S), F^{\leftarrow}(G)\in \beta O(S)$) for every $G\in O(T)$. It is called pre-cocontinuous (resp. semi-cocontinuous, β-cocontinuous) if $F^{\leftarrow}(K)\in PC(S)$ (resp. $F^{\leftarrow}(K)$
$\in SC(S), F^{\leftarrow}(K)\in \beta C(S)$) for every $K\in C(T)$

We denote by $PO(S,\mathcal{S},\tau,\kappa)$ ($\beta O(S,\mathcal{S},\tau,\kappa)$), more simply by $PO(S)$ ($\beta O(S)$) , the set of pre-open sets (β-open sets) in $\mathcal{S}$. Likewise, $PC(S,\mathcal{S},\tau,\kappa)$ ($\beta C(S,\mathcal{S},\tau,\kappa)$), $PC(S)$ ($\beta C(S)$) will denote the set of pre-closed (β-closed sets) sets.

2. α-open and α-closed sets

We begin by recalling [13] that a subset A of a topological space X is called α-open if $A\subseteq intclintA$. Dually, A is α-closed if $X\setminus A$ is α-open, equivalently if it satisfies $clintclA\subseteq A$. This leads to the following analogous concepts in a ditopological texture space.

Definition 2.1. Let $(S,\mathcal{S},\tau,\kappa)$ be ditopological texture space and $A\in\mathcal{S}$.

(1) If $A\subseteq intclintA$ then A is α-open.
(2) If $clintclA\subseteq A$ then A is α-closed.

We denote by $O_\alpha(S,\mathcal{S},\tau,\kappa)$, or when there can be no confusion by $O_\alpha(S)$, the set of α-open sets in $\mathcal{S}$. Likewise, $C_\alpha(S,\mathcal{S},\tau,\kappa)$, or $C_\alpha(S)$ will denote the set of α-closed sets.

Proposition 2.2. *For a given ditopological texture space $(S,\mathcal{S},\tau,\kappa)$:*

(1) $O(S)\subseteq O_\alpha(S)$ *and* $C(S)\subseteq C_\alpha(S)$
(2) *Arbitrary join of α-open sets is α-open.*
(3) *Arbitrary intersection of α-closed sets is α-closed.*

Proof. (1) Let $G\in O(S)$. Since $intG=G$ we have $G\subseteq intclintG$. Thus $G\in O_\alpha(S)$. Secondly, let $K\in C(S)$. Since $clK=K$ we have $clintclK\subseteq K$ and so $K\in C_\alpha(S)$.

(2) Let $\{A_j\}_{j\in J}$ be a family of α-open sets. Then for each $j\in J$, $A_j\subseteq intclintA_j$. Now, $\bigvee A_j\subseteq\bigvee intclintA_j\subseteq int\bigvee clintA_j=intcl\bigvee intA_j=intclint\bigvee A_j$. Hence $\bigvee A_j$ is a α-open set.

The result (3) is dual of (2). □

Remark 2.3.

(1) It is obvious that every α-open (α-closed) set is semi-open (semi-closed) and pre-open (pre-closed) set. Thus we have the following diagram:

$$\begin{tikzcd}
 & & \begin{array}{c}\text{semi-open}\\ \text{(semi-closed)}\end{array} \arrow[dr] & \\
\begin{array}{c}\text{open}\\ \text{(closed)}\end{array} \arrow[r] & \alpha - \begin{array}{c}\text{open}\\ \text{(closed)}\end{array} \arrow[ur] \arrow[dr] & & \beta - \begin{array}{c}\text{open}\\ \text{(closed)}\end{array} \\
 & & \begin{array}{c}\text{pre-open}\\ \text{(pre-closed)}\end{array} \arrow[ur] &
\end{tikzcd}$$

(2) It is well known that from every fuzzy topological space can be obtained a complemented ditopological texture space [2, 5]. Hence, we see from [10] that every fuzzy C-set [12] is a C-set in the sense of ditopological texture space.

Generally there is no relation between the α-open and α-closed sets, but for a complemented ditopological space we have the following result.

Proposition 2.4. *For a complemented ditopological space $(S, \mathcal{S}, \sigma, \tau, \kappa)$:*

$$A \in \mathcal{S} \quad \textit{is} \quad \alpha - \textit{open if and only if} \quad \sigma(A) \textit{ is} \quad \alpha - \textit{closed.}$$

Proof. For $A \in \mathcal{S}$, since $\sigma(intA) = cl(\sigma(A))$ and $\sigma(clA) = int(\sigma(A))$ the proof is trivial. □

Examples 2.5. (1) If $(X, \mathcal{T})$ is a topological space then $(X, \mathcal{P}(X), \pi_X, \mathcal{T}, \mathcal{T}^c)$ is a complemented ditopological texture space. Here $\pi_X(Y) = X \setminus Y$ for $Y \subseteq X$ is the usual complementation on $(X, \mathcal{P}(X))$ and $\mathcal{T}^c = \{\pi_X(G) \mid G \in \mathcal{T}\}$. Clearly the α-open, α-closed sets in $(X, \mathcal{T})$ correspond precisely to the α-open, α-closed respectively, in $(X, \mathcal{P}(X), \pi_X, \mathcal{T}, \mathcal{T}^c)$.

(2) For the unit interval texture $(\mathbb{I}, \mathcal{I})$ of Examples 1.1.(2), let ι be the complementation $\iota([0, r)) = [0, 1 - r]$, $\iota([0, r]) = [0, 1 - r)$, and $(\tau_{\mathbb{I}}, \kappa_{\mathbb{I}})$ the standard complemented ditopology given by

$$\tau_{\mathbb{I}} = \{[0, r) \mid r \in \mathbb{I}\} \cup \{\mathbb{I}\}, \quad \kappa_{\mathbb{I}} = \{[0, r] \mid r \in \mathbb{I}\} \cup \{\emptyset\}.$$

For this space we obtain $O_\alpha(S) = C_\alpha(S) = \mathcal{I}$.

Definition 2.6. Let $(S, \mathcal{S}, \tau, \kappa)$ be a ditopological texture space. For $A \in \mathcal{S}$, we define:

(1) The α-closure $cl_\alpha A$ of A under (τ, κ) by the equality $cl_\alpha(A) = \bigcap\{B \mid B \in C_\alpha(S) \text{ and } A \subseteq B\}$.
(2) The α-interior $int_\alpha A$ of A under (τ, κ) by the equality $int_\alpha(A) = \bigvee\{B \mid B \in O_\alpha(S) \text{ and } B \subseteq A\}$.

From the Proposition 2.2, it is obtained, $int_\alpha(A) \in O_\alpha(S)$, $cl_\alpha(A) \in C_\alpha(S)$, while $A \in O_\alpha(S) \iff A = int_\alpha(A)$ and $A \in C_\alpha(S) \iff A = cl_\alpha(A)$.

Clearly, $int_\alpha(A)$ is the greatest α-open set which is contained in A and $cl_\alpha(A)$ is α- closed set which contains A and we have, $A \subseteq cl_\alpha(A) \subseteq clA$ and $intA \subseteq int_\alpha(A) \subseteq A$.

The following statements are obtained easily from the definitions:

Proposition 2.7. *The following are satisfied.*

(1) $cl_\alpha(\emptyset) = \emptyset$
(2) $cl_\alpha(A)$ *is* $\alpha-$*closed, for all* $A \in \mathcal{S}$.
(3) *If* $A \subseteq B$ *then* $cl_\alpha(A) \subseteq cl_\alpha(B)$, *for every* $A, B \in \mathcal{S}$.
(4) $cl_\alpha(cl_\alpha(A)) = cl_\alpha(A)$

We now wish to generalize the notions of neighborhood and coneighborhood [6, Definition 2.1]. The following definiton would be seem to be appropriate.

Definition 2.8. Let (τ, κ) be a ditopology on $(S, \mathcal{S})$.

(1) If $s \in S^\flat$, a α-neighbourhood of s is a set $N \in \mathcal{S}$ for which there exists $G \in O_\alpha(S)$ satisfying $P_s \subseteq G \subseteq N \not\subseteq Q_s$.
(2) If $s \in S$, a α-coneighbourhood of s is a set $M \in \mathcal{S}$ for which there exists $K \in C_\alpha(S)$ satisfying $P_s \not\subseteq M \subseteq K \subseteq Q_s$.

We denote the set of α-nhds (α-conhds) of s by $\eta_{O_\alpha}(s)$ $(\mu_{C_\alpha}(s))$, respectively. These concepts are useful tool for semi bicontinuity.

3. α-BICONTINUITY

We recall that a function between fuzzy topological spaces is called fuzzy α-continuous [14] if the inverse image of each fuzzy open set is fuzzy α-open. This leads to the following concepts for a difunction between ditopological texture spaces.

Definition 3.1. Let $(S_j, \mathcal{S}_j, \tau_j, \kappa_j)$, $j = 1, 2$, be ditopological texture spaces and $(f, F) : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ a difunction.

(1) It is called α-continuous, if $F^{\leftarrow}(G) \in O_\alpha(S_1)$, for every $G \in O(S_2)$.
(2) It is called α-cocontinuous, if $f^{\leftarrow}(K) \in C_\alpha(S_1)$, for every $K \in C(S_2)$.
(3) It is called α-bicontinuous, if it is α-continuous and α-cocontinuous.

Now we give characterization of the α-bicontinuity by the concept α-interior and α-closure.

Proposition 3.2. *Let* $(f, F) : (S_1, \mathcal{S}_1, \tau_1, \kappa_1) \to (S_2, \mathcal{S}_2, \tau_2, \kappa_2)$ *be a difunction.*

(1) *The following are equivalent:*
  (a) (f, F) *is* α-*continuous.*
  (b) $int(F^{\to}A) \subseteq F^{\to}(int_\alpha A)$, $\forall\, A \in \mathcal{S}_1$.
  (c) $f^{\leftarrow}(intB) \subseteq int_\alpha(f^{\leftarrow}B)$, $\forall\, B \in \mathcal{S}_2$.
(2) *The following are equivalent:*
  (a) (f, F) *is* α-*cocontinuous.*
  (b) $f^{\to}(cl_\alpha A) \subseteq cl(f^{\to}A)$, $\forall\, A \in \mathcal{S}_1$.
  (c) $cl_\alpha(F^{\leftarrow}B) \subseteq F^{\leftarrow}(clB)$, $\forall\, B \in \mathcal{S}_1$.

Proof. We prove (1), leaving the dual proof of (2) to the interested reader.

(a)$\Longrightarrow$(b) Take $A \in \mathcal{S}_1$. From [4, Theorem 2.24 (2 a)] and the definition of interior,

$$f^{\leftarrow} int(F^{\rightarrow}A) \subseteq f^{\leftarrow}(F^{\rightarrow}A) \subseteq A.$$

Since inverse image and coimage under a difunction is equal, $f^{\leftarrow} int(F^{\rightarrow}A) = F^{\leftarrow} int(F^{\rightarrow}A)$. Thus, $f^{\leftarrow} int(F^{\rightarrow}A) \in O_{\alpha}(S_1)$, by α-continuity. Hence $f^{\leftarrow} int(F^{\rightarrow}A) \subseteq int_{\alpha}(A)$ and applying [4, Theorem 2.4 (2 b)] gives

$$int(F^{\rightarrow}A) \subseteq F^{\rightarrow}\big(f^{\leftarrow} int(F^{\rightarrow}A)\big) \subseteq F^{\rightarrow}(int_{\alpha}A),$$

which is the required inclusion.

(b)$\Longrightarrow$(c). Let $B \in \mathcal{S}_2$. Applying inclusion (b) to $A = f^{\leftarrow}B$ and using [4, Theorem 2.4 (2 b)] gives

$$int(B) \subseteq int(F^{\rightarrow}(f^{\leftarrow}B)) \subseteq F^{\rightarrow}(int_{\alpha}(f^{\leftarrow}B)).$$

Hence, we have $f^{\leftarrow}(intB) \subseteq f^{\leftarrow}\big(F^{\rightarrow} int_{\alpha}(f^{\leftarrow}B)\big) \subseteq int_{\alpha}(f^{\leftarrow}B)$ by [4, Theorem 2.24 (2 a)].

(c)$\Longrightarrow$(a). Applying (c) for $B \in O(S_2)$ gives

$$f^{\leftarrow}B = f^{\leftarrow}(int(B)) \subseteq int_{\alpha}(f^{\leftarrow}B),$$

so $F^{\leftarrow}B = f^{\leftarrow}B = int_{\alpha}(f^{\leftarrow}B) \in SO(S_1)$. Hence, (f, F) is continuous. $\square$

The semi bicontinuity can be characterized by the concept α-neighborhood and α-coneighborhood.

Theorem 3.3. *Let $(S_j, \mathcal{S}_j, \tau_j, \kappa_j)$, $j = 1, 2$, ditopology and $(f, F) : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ be a difunction.*

(1) *The following are equivalent:*
   (i) (f, F) is α-continuous.
   (ii) Given $s \in S_1^{\flat}$, $N \in \eta(f^{\rightarrow}(P_s)) \Longrightarrow \exists N^* \in \eta_{\alpha O}(s)$ so that $f^{\rightarrow}(N^*) \subseteq N$.
   (iii) Given $s \in S_1^{\flat}$, $N \in \eta(f^{\rightarrow}(P_s)) \cap \tau_2 \Longrightarrow \exists N^* \in \eta_{\alpha O}(s)$ so that $f^{\rightarrow}(N^*) \subseteq N$.
(2) *The following are equivalent:*
   (i) (f, F) is α-cocontinuous.
   (ii) Given $s \in S_1^{\flat}$, $M \in \mu(F^{\rightarrow}(Q_s)) \Longrightarrow \exists M^* \in \mu_{\alpha C}(s)$ so that $M \subseteq F^{\rightarrow}(M^*)$.
   (iii) Given $s \in S_1^{\flat}$, $M \in \mu(F^{\rightarrow}(Q_s)) \cap \kappa_2 \Longrightarrow \exists M^* \in \mu_{\alpha C}(s)$ so that $M \subseteq F^{\rightarrow}(M^*)$.

Proof. It can be easily proved as in [6, Theorem 2.6]. $\square$

In addition to the above theorem;

Theorem 3.4. *Let $(S_j, \mathcal{S}_j, \sigma_j, \tau_j, \kappa_j)$, $j = 1, 2$, complemented ditopology and $(f, F) : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ be complemented difunction.*

(1) *The following are equivalent:*

(i) (f, F) *is* α*-continuous.*

(iv) (f, F) *is* α*-cocontinuous.*

(v) $f^{\to}(clintclA) \subseteq cl(f^{\to}(A))$ *for each* $A \in \mathcal{S}_1$.

(vi) $clintcl(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}(clB)$ *for each* $B \in \mathcal{S}_2$.

(2) *The following are equivalent:*

(i) (f, F) *is* α*-cocontinuous.*

(iv) (f, F) *is* α*-continuous.*

(v) $int(F^{\to}(A)) \subseteq F^{\to}(intclintA)$ *for each* $A \in \mathcal{S}_1$.

(vi) $F^{\leftarrow}(intB) \subseteq intclint(F^{\leftarrow}(B))$ *for each* $B \in \mathcal{S}_2$.

Proof. We prove (1), leaving the essentially dual proof of (2) to the interested reader.

(i) $\Longrightarrow$ *(iv)* Since (f, F) is complemented, $(F', f') = (f, F)$. From [4, Lemma 2.20], $\sigma_1((f')^{\leftarrow}(B)) = f^{\leftarrow}(\sigma_2(B))$ and $\sigma_1((F')^{\leftarrow}(B)) = F^{\leftarrow}(\sigma_2(B))$ for all $B \in \mathcal{S}_2$. Hence the proof is clear from these equalities.

(iv) $\Longrightarrow$ *(v)* Let $A \in \mathcal{S}_1$. Then $clf^{\to}(A)$ is closed set in $(S_2, \mathcal{S}_2)$. From (iv) $f^{\leftarrow}(clf^{\to}(A))$ is α-closed in $(S_1, \mathcal{S}_1)$. Hence we have $clintclA \subseteq clintclf^{\leftarrow}(clf^{\to}A) \subseteq f^{\leftarrow}cl(f^{\to}A)$. So then $f^{\to}(clintclA) \subseteq cl(f^{\to}A)$.

(v) $\Longrightarrow$ *(vi)* Let $B \in \mathcal{S}_2$. Then $f^{\leftarrow}(B) \in \mathcal{S}_1$ and by hypothesis we have $f^{\to}(clintcl(f^{\leftarrow}$
$(B)) \subseteq cl(f^{\to}(f^{\leftarrow}(B)))$. Thus, $f^{\to}(clintcl(f^{\leftarrow}(B))) \subseteq clB$ and $clintcl(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}(f^{\to}(clintcl(f^{\leftarrow}(B)))) \subseteq f^{\leftarrow}(clB)$.

(vi) $\Longrightarrow$ *(i)* Let $G \in O(S_2)$ and $K = \sigma_2(G)$. By *(vi)* $f^{\to}(clintcl(f^{\leftarrow}(K))) \subseteq clK = K$, since $K \in C(S_2)$. That is $clintcl(f^{\leftarrow}(\sigma_2(G))) \subseteq f^{\leftarrow}(\sigma_2(G))$ or by [4, Lemma 2.20], $clintcl(\sigma_1((f')^{\leftarrow}G)) \subseteq \sigma_1((f')^{\leftarrow}G)$. Thus $(f')^{\leftarrow}(G) \subseteq intclint(f')^{\leftarrow}(G)$ and we have $F^{\leftarrow}(G) \subseteq intclintF^{\leftarrow}(G)$ since (f, F) is complemented. Hence $F^{\leftarrow}(G)$ is α-open set. $\square$

Corollary 3.5. *Let* $(f, F) : (S_1, \mathcal{S}_1, \tau_1, \kappa_1) \to (S_2, \mathcal{S}_2, \tau_2, \kappa_2)$ *be a difunction.*

(1) *If* (f, F) *is* α*-continuous then:*
  (a) $f^{\to}(clA) \subseteq cl(f^{\to}(A))$, *for every* $A \in PO(S_1)$.
  (b) $cl(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}(clB)$, *for every* $B \in O(S_2)$.

(2) *If* (f, F) *is* α*-cocontinuous then:*
  (a) $int(F^{\to}(A)) \subseteq F^{\to}(intA)$ *for every* $A \in PC(S_1)$.
  (b) $F^{\leftarrow}(intB) \subseteq int(F^{\leftarrow}B)$, *for every* $B \in C(S_2)$.

Proof. We prove (1), leaving the essentially dual proof of (2) to the interested reader.

(a) Let $A \in PO(S_1)$. Then $clA \subseteq clintclA$ and so $f^{\to}(clA) \subseteq f^{\to}(clintclA)$. From Theorem 3.4(1)-(iv), we have, $f^{\to}(clA) \subseteq cl(f^{\to}(A))$.

(b)Let $B \in O(S_2)$. From the assumption, $f^{\leftarrow}(B)$ is α-open and by Remark 2.3, $f^{\leftarrow}(B) \in PO(S_1)$. Hence, $f^{\leftarrow}(B) \subseteq intcl(f^{\leftarrow}(B))$ and so $cl(f^{\leftarrow}(B)) \subseteq clintcl(f^{\leftarrow}(B))$. From Theorem 3.4(1)-(v), we have $cl(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}(clB)$.

□

Remark 3.6. Since every open set is α-open, every bicontinuous difunction is α-bicontinuous. On the other hand, it is clear that every α-continuous (α-cocontinuous) difunction is semi-continuous (semi-cocontinuous) and pre-continuous (pre-cocontinuous). Thus we have the following diagram:

$$\begin{tikzcd} & & \text{semi-continuity (semi-cocontinuity)} \arrow[dr] & \\ \text{continuity (cocontinuity)} \arrow[r] & \alpha - \text{continuity (cocontinuity)} \arrow[ur] \arrow[dr] & & \beta - \text{continuity (cocontinuity)} \\ & & \text{pre-continuity (pre-cocontinuity)} \arrow[ur] & \end{tikzcd}$$

Proposition 3.7. *$(f, F) : (S_1, \mathcal{S}_1, \tau_1, \kappa_1) \to (S_2, \mathcal{S}_2, \tau_2, \kappa_2)$ be a difunction.*

(1) *If (f, F) is pre-continuous and semi-continuous then (f, F) is α-continuous.*
(2) *If (f, F) is pre-cocontinuous and semi-cocontinuous then (f, F) is α-cocontinuous.*

Proof. We prove (1), leaving the essentially dual proof of (2) to the interested reader. Let $B \in O(S_2)$. Then $f^{\leftarrow}(B)$ is pre-open and semi-open. Hence we have $f^{\leftarrow}(B) \subseteq intcl(f^{\leftarrow}(B))$ and $f^{\leftarrow}(B) \subseteq clint(f^{\leftarrow}(B))$ and

$$f^{\leftarrow}(B) \subseteq intcl(clint(f^{\leftarrow}(B))) = intclint(f^{\leftarrow}(B)).$$

So (f, F) is α-continuous difunction.

□

References

[1] L. M. Brown, M. Diker, Ditopological texture spaces and intuitionistic sets, Fuzzy sets and systems 98, (1998), 217–224.

[2] L. M. Brown, R. Ertürk, Fuzzy Sets as Texture Spaces, I. Representation Theorems, Fuzzy Sets and Systems 110 (2) (2000), 227–236.

[3] L. M. Brown, R. Ertürk, Fuzzy sets as texture spaces, II. Subtextures and quotient textures, Fuzzy Sets and Systems 110 (2) (2000), 237–245.

[4] L. M. Brown, R. Ertürk, and Ş. Dost, Ditopological texture spaces and fuzzy topology, I. Basic Concepts, Fuzzy Sets and Systems 147 (2) (2004), 171–199. 3

[5] L. M. Brown, R. Ertürk, and Ş. Dost, Ditopological texture spaces and fuzzy topology, II. Topological Considerations, Fuzzy Sets and Systems 147 (2) (2004), 201–231.

[6] L. M. Brown, R. Ertürk, and Ş. Dost, Ditopological texture spaces and fuzzy topology, III. Separation Axioms, Fuzzy Sets and Systems 157 (14) (2006), 1886–1912.

[7] M. Demirci, *Textures and C-spaces*, Fuzzy Sets and Systems 158 (11) (2007), 1237–1245.

[8] Ş. Dost, L. M. Brown, and R. Ertürk, β-open and β-closed sets in ditopological setting, submitted.

[9] Ş. Dost, Semi-open and semi-closed sets in ditopological texture space, submitted.

[10] Ş. Dost, C-sets and C-bicontinuity in ditopological texture space, preprint.

[11] M. M. Gohar, Compactness in ditopological texture spaces,(PhD Thesis, Hacettepe University, 2002).

[12] S. Jafari, Viswanathan K., Rajamani, M., Krishnaprakash, S. On decomposition of fuzzy A-continuity, The J. Nonlinear Sci. Appl. (4) 2008 236-240.

[13] O. Njastad, On some classes of nearly open sets, Pacific J. Math. 15, (1965), 961-970

[14] M. K. Singal, N. Rajvanshi, Fuzzy alpha-sets and alpha-continuous maps, Fuzzy Sets and Systems 48 (1992), 383–390.

Hacettepe University, Department of Secondary Science and Mathematics Education, 06800 Beytepe, Ankara, Turkey.

E-mail address: dost@hacettepe.edu.tr