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Laplace discrete decomposition method for solving nonlinear Volterra-Fredholm integro-differential equations

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Abstract

In this article, a new modification of the Adomian Decomposition Method (ADM) that is called the Laplace Discrete Adomian Decomposition Method (LDADM) is applied to non-homogeneous nonlinear Volterra-Fredholm integro-differential equations. This method is based upon the Laplace Adomian decomposition method coupled with some quadrature rules of numerical integration. The performance of the proposed method is verified through absolute error measures between the approximated solutions and exact solutions. The series of experimental numerical results show that our proposed method performs in high accuracy and efficiency. The study highlights that the proposed method could be used to overcome the analytical approaches in solving nonlinear Volterra-Fredholm integro-differential equations.

Keywords: Volterra-Fredholm integro-differential equation, Adomian decomposition method, Laplace transform, absolute error, approximated solution.

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1. Introduction

The integro-differential equations have attracted much more interest of mathematicians and physicists which provides an efficiency for the description of many practical dynamical arising in engineering and scientific disciplines such as, physics, biology, electrochemistry, chemistry, economy, electromagnetic, control theory and viscoelasticity [2, 3, 5, 12, 13, 15, 16, 19, 21–23]. In recent years, many authors focus on the development of numerical and analytical techniques for fractional integro-differential equations.

The LADM is known for its rapid convergence in solution and also uses only little iteration as successfully applied in Kiymaz [30]. Furthermore, several modifications of the ADM and LADM methods can be seen in [6] with wide applications ranging from differential equations, partial differential equations, integral equations and integro-differential equations among others. It seems that the LADM method is

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always open for further modifications especially on discretizing the Adomian decomposition. In this paper, we aim at extending the Laplace Adomian decomposition method for finding the solution of nonlinear integro-differential equations by firstly discretizing the ADM, followed by coupling some numerical integration schemes or quadrature rules [17].

We can solve integro-differential equations with some basis functions by the Chebyshev collocation method [4], Galerkin methods [8], Runge-Kutta methods [9], Taylor collection method [28], rationalized Haar functions method [31], Galerkin methods with hybrid functions [32], and ADM [7, 17, 34]. In addition to these numerical methods, Khuri [29] used Laplace transform numerical scheme. Moreover, properties of the integro-differential equations have been studied by several authors [1, 10, 11, 14, 17, 18, 20, 24–27, 33].

In this paper, we shall be concerned with the nonlinear Volterra-Fredholm integro-differential equations of the second kind of the form

$$y''(x) = f(x) + \int_a^x \Psi_1(x,t)(\Delta_1(y(t)) + \Theta_1(y(t)))dt + \int_a^b \Psi_2(x,t)(\Delta_2(y(t)) + \Theta_2(y(t)))dt, \tag{1.1}$$

with the initial conditions

$$y(0) = \alpha, \;\; y'(0) = \beta, \;\; a \leqslant x \leqslant b. \tag{1.2}$$

The main objective of the present paper is to study the behavior of the solution that can be formally determined by analytical approximated method as the LDADM.

2. Description of the Method

Some powerful method has been focusing on the development of more advanced and efficient methods for nonlinear Volterra-Fredholm integro-differential equations such as the LDADM [7, 17, 34]. We will describe this method in this section.

2.1. *Laplace Discrete Adomian Decomposition Method (LDADM)*

To solve the nonlinear Volterra-Fredholm integro-differential Eqs. (1.1)-(1.2) we use the Laplace transform method, we recall the Laplace transform of the second derivative of $y(x)$, that is

$$\mathcal{L}\{y''(x)\} = s^2\mathcal{L}\{y(x)\} - sy(0) - y'(0). \tag{2.1}$$

Thus on applying the Laplace transform to both sides of Eq. (1.1) we obtain

$$\mathcal{L}\{y''(x)\} = \mathcal{L}\{f(x) + \int_a^x \Psi_1(x,t)(\Delta_1(y(t)) + \Theta_1(y(t)))dt + \int_a^b \Psi_2(x,t)(\Delta_2(y(t)) + \Theta_2(y(t)))dt\}. \tag{2.2}$$

From (2.1)

$$\begin{aligned} s^2\mathcal{L}\{y(x)\} - sy(0) - y'(0) =& \mathcal{L}\{f(x)\} + \mathcal{L}\{\int_a^x \Psi_1(x,t)(\Delta_1(y(t)) + \Theta_1(y(t)))dt \\ &+ \int_a^b \Psi_2(x,t)(\Delta_2(y(t)) + \Theta_2(y(t)))dt\}, \end{aligned} \tag{2.3}$$

by using (1.2)

$$\begin{aligned} s^2\mathcal{L}\{y(x)\} - s\alpha - \beta =& \mathcal{L}\{f(x)\} + \mathcal{L}\{\int_a^x \Psi_1(x,t)(\Delta_1(y(t)) + \Theta_1(y(t)))dt \\ &+ \int_a^b \Psi_2(x,t)(\Delta_2(y(t)) + \Theta_2(y(t)))dt\}, \end{aligned}$$

$$\mathcal{L}\{y(x)\} = \frac{\alpha}{s} + \frac{\beta}{s^2} + \frac{1}{s^2}\mathcal{L}\{f(x)\} + \frac{1}{s^2}\mathcal{L}\{\int_a^x \Psi_1(x,t)(\Delta_1(y(t)) + \Theta_1(y(t)))dt$$
$$+ \int_a^b \Psi_2(x,t)(\Delta_2(y(t)) + \Theta_2(y(t)))dt\}. \tag{2.4}$$

The decomposition method represents the solution $y(x)$ as a series of the form

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \tag{2.5}$$

and the nonlinear term $\Theta_1(y(t)), \Theta_2(y(t))$ are decomposed into an infinite series of the form

$$\Theta_1(y(t)) = \sum_{n=0}^{\infty} A_n, \quad \Theta_2(y(t)) = \sum_{n=0}^{\infty} B_n, \tag{2.6}$$

where A_n, B_n are the Adomian polynomials of $y_1, y_2, \cdots, y_n$, given by the formula

$$A_n = \frac{1}{n!}\frac{d^n}{d\gamma^n}[\Theta_1(\sum_{i=0}^{n} \gamma^i y_i)]_{\gamma=0}, \quad B_n = \frac{1}{n!}\frac{d^n}{d\gamma^n}[\Theta_2(\sum_{i=0}^{n} \gamma^i y_i)]_{\gamma=0}. \tag{2.7}$$

On substituting Eqs. (2.5)-(2.6) and (2.7) in Eq. (2.4) and making comparison between the right and left hand sides, we thus obtain:

$$\begin{aligned}\mathcal{L}\{y_0(x)\} &= \frac{\alpha}{s} + \frac{\beta}{s^2} + \frac{1}{s^2}\mathcal{L}\{f(x)\},\\ \mathcal{L}\{y_{k+1}(x)\} &= \frac{1}{s^2}\mathcal{L}\{\int_a^x \Psi_1(x,t)(\Delta_1(y_k(t)) + A_k)dt + \int_a^b \Psi_2(x,t)(\Delta_2(y_k(t)) + B_k)dt\}.\end{aligned} \tag{2.8}$$

Finally, on applying the inverse Laplace transform to the first part of Eq. (2.8) $y_0(x)$ is given and consequently A_0 will be defined. Also, using A_0 enables us to evaluate $y_1(x)$. The determination of $y_0(x)$ and $y_1(x)$ leads to the determination of A_1 that will allow us to determine $y_2(x)$, and so on. The recursive relation is given by

$$\begin{aligned}y_0(x) &= \mathcal{L}^{-1}\{\frac{\alpha}{s} + \frac{\beta}{s^2} + \frac{1}{s^2}\mathcal{L}\{f(x)\}\},\\ y_{k+1}(x) &= \mathcal{L}^{-1}\{\frac{1}{s^2}\mathcal{L}\{\int_a^x \Psi_1(x,t)(\Delta_1(y_k(t)) + A_k)dt + \int_a^b \Psi_2(x,t)(\Delta_2(y_k(t)) + B_k)dt\}\}.\end{aligned}$$

3. Numerical Example

In this section, we proposed a numerical solution for nonlinear integro-differential equations by using the LDADM see numerical results in Table 1 and Figs. 1-2.

Example 3.1. Consider the following nonlinear integro-differential equation:

$$y''(x) = \frac{1}{2}e^x + \frac{1}{2}\int_0^x e^{x-2t} y^2(t)dt, \tag{3.1}$$

with the following initial conditions

$$y(0) = y'(0) = 1. \tag{3.2}$$

Therefore the exact solution is $y(x) = e^x$.

Solution: Taking the Laplace transform of both sides of the Eq. (3.1)

$$\mathcal{L}\{y''(x)\} = \mathcal{L}\{\frac{1}{2}e^x\} + \mathcal{L}\{\frac{1}{2}\int_0^x e^{x-2t}y^2(t)dt\}. \tag{3.3}$$

From (2.1)

$$s^2\mathcal{L}\{y(x)\} - sy(0) - y'(0) = \mathcal{L}\{\frac{1}{2}e^x\} + \mathcal{L}\{\frac{1}{2}\int_0^x e^{x-2t}y^2(t)dt\},$$

by (3.2)

$$\mathcal{L}\{y(x)\} = \frac{1}{s} + \frac{1}{s^2} + \frac{1}{2s^2(s-1)} + \frac{1}{2s^2}\mathcal{L}\{\int_0^x e^{x-2t}y^2(t)dt\},$$

the recursive relation is given by

$$y_0(x) = \mathcal{L}^{-1}\{\frac{1}{s} + \frac{1}{s^2} + \frac{1}{2s^2(s-1)}\}$$
$$y_{k+1}(x) = \mathcal{L}^{-1}\{\frac{1}{2s^2}\mathcal{L}\{\int_0^x e^{x-2t}A_k dt\}\}, k \geqslant 0.$$

Table 1: Numerical Results of the Example 3.1.

x	Exact	ADM	LDADM	Error_{ADM}	Error_{LDADM}
0.0	1.00000000	1.00000000	1.00000000	0.000000000	0.000000000
0.2	1.22140276	1.22010210	1.22140206	1.30066×10^{-3}	0.700×10^{-6}
0.4	1.49182470	1.45262489	1.49182168	3.919981×10^{-2}	0.302×10^{-5}
0.6	1.82211880	1.81302140	1.82211150	9.0974×10^{-3}	0.730×10^{-5}
0.8	2.22554093	2.22032852	2.22552695	5.21241×10^{-3}	1.398×10^{-5}
1.0	2.71828183	2.71223606	2.71825824	6.04577×10^{-3}	2.359×10^{-5}

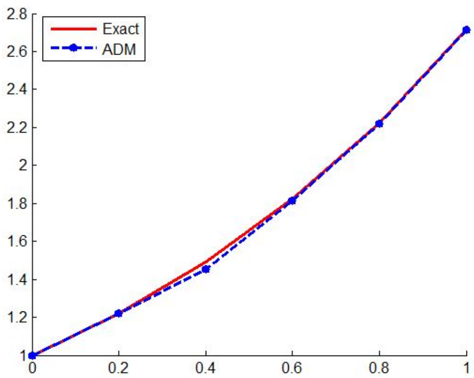

Figure 1: Numerical Results of the Example 3.1.

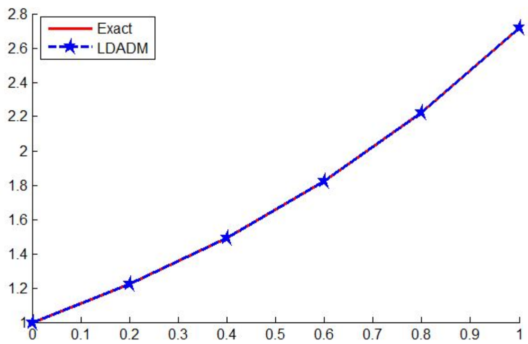

Figure 2: Numerical Results of the Example 3.1.

4. Conclusions

In this paper, new modification of the Adomian decomposition method inspired by the property of discretization is proposed. We developed a new Laplace Discrete Adomian decomposition method in which has been successfully applied to finding efficient numerical solutions of nonlinear Volterra-Fredholm integro-differential equations. The LDADM gives approximate solutions iteratively with less number of computational steps comparison with ADM (see Table 1, Fig. 1 and Fig. 2). The results reveal that the proposed method is simple to execute and effective.

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