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Application of reduced differential transformation method for solving Fourth-Order Parabolic Partial Differential Equations

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Abstract

The purpose of this paper is to obtain the approximate solution of fourth-order parabolic partial differential equations by the reduced differential transform method (RDTM).This method provides the solution in the form of a convergent series with easily calculable terms. Comparing RDTM with some other methods in the literature shows present approach is very simple, effective, powerful and can be easily applied to other linear or nonlinear PDEs in science and engineering.

Keywords: Reduced differential transform method (RDTM), differential transform method (DTM) fourth-order parabolic partial differential equations, initial value problems.

1. Introduction

There are many problems arising in science and engineering are modeled using linear or nonlinear partial differential equations (PDEs). Boundary and initial value problems in PDEs occur in fluid mechanics, mathematical physics, astrophysics, biology, materials science, electromagnetism, image processing, computer graphics, etc. PDEs are categorized into different types, including elliptic, parabolic, and hyperbolic PDEs. In this article we concentrate our discussion on fourth-order parabolic PDEs. These PDEs describe various physical phenomenon including deformation of beams, viscoelastic and inelastic flows, transverse vibrations of a homogeneous beam, plate deflection theory, engineering and applied sciences [1-12]. In recent years, various methods have been proposed for solving the fourth-order parabolic PDEs, such that adomian decomposition method (ADM) [13,14], variational iteration method (VIM) [15,16], B-spline methods [17-19], homotopy perturbation method (HPM) [20] and homotopy analysis method (HAM) [21].

In this paper, we applied the RDTM, which is the modified version of differential transform method (DTM), to fourth-order parabolic PDEs. RDTM doesn't require any discretization or linearization and it reduces significantly the computational work. Also, it provides an analytical approximation, in many cases an exact solution, in a rapidly convergent sequence with elegantly computed terms. computational work.

2. Reduced differential transform method (RDTM)

In this section, some basic definitions and properties for RDTM, which could be found in [22-26], have been reviewed.

Definition 2.1. Consider a function of n+1 variables $u(\tilde{x},t)=u(x_1,x_2,...x_n,t)$ where $\tilde{x}\in R^n$, $\tilde{x}=(x_1,x_2,...,x_n)$. The reduced differential transform of $u(\tilde{x},t)$ with respect to t is defined by

$$U_k(\tilde{x})=\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(\tilde{x},t)\right]_{t=0} \qquad , k\in N \tag{1}$$

In Eq.(1) $U_k(\tilde{x})$ is the transformed function and $u(\tilde{x},t)$ is the original function.

Definition 2.2. The reduced differential inverse transform of $U_k(\tilde{x})$ is defined as follows:

$$u(\tilde{x},t)=\sum_{k=0}^{\infty}U_k(\tilde{x}).t^k \tag{2}$$

From Eq.(1) and Eq.(2) , we get

$$u(\tilde{x},t)=\sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{\partial^k}{\partial t^k}u(\tilde{x},t)\right]_{t=0} \tag{3}$$

Notice that the RDTM is close to the one dimensional DTM because the RDTM is considered as the standard DTM of $u(\tilde{x},t)$ with respect to the variable t. However, the corresponding recursive algebraic equation is the function of the variable $\tilde{x}=(x_1,x_2,...,x_n)$.

The following theorems that can be deduced from Eqs.(1)-(3) are given below:

Theorem 1. If $f(\tilde{x},t)=\alpha g(\tilde{x},t)\pm\beta h(\tilde{x},t)$, then $F_k(\tilde{x})=\alpha G_k(\tilde{x})\pm\beta H_k(\tilde{x})$,where α and β are constant.

Theorem 2. If $f(\tilde{x},t)=x_1^{m_1}x_2^{m_2}\cdots x_n^{m_n}t^n$,then $F_k(\tilde{x})=x_1^{m_1}x_2^{m_2}\cdots x_n^{m_n}\delta(k-n)$ where $\delta(k)=\begin{cases}1,k=0\\0,k\neq 0\end{cases}$.

Theorem 3. If $f(\tilde{x},t)=g(\tilde{x},t)\cdot h(\tilde{x},t)$, then $F_k(\tilde{x})=\sum_{r=0}^{k}G_r(\tilde{x})\cdot H_{k-r}(\tilde{x})$

Theorem 4. If $f(\tilde{x},t)=x_1^{m_1}\cdot x_2^{m_2}\cdots x_n^{m_n}\cdot t^n\cdot g(\tilde{x},t)$, then $F_k(\tilde{x})=x_1^{m_1}\cdot x_2^{m_2}\cdots x_n^{m_n}\cdot G_{k-n}(\tilde{x})$

Theorem 5. If $f(\tilde{x},t)=\frac{\partial^n}{\partial t^n}g(\tilde{x},t)$, then $F_k(\tilde{x})=(k+1)(k+2)\cdots(k+n)G_{k+n}(\tilde{x})$

Theorem 6. If $f(\tilde{x},t)=\frac{\partial^n}{\partial x_i^n}g(\tilde{x},t)$, then $F_k(\tilde{x})=\frac{\partial^n}{\partial x_i^n}G_k(\tilde{x}),\ i=1,2,...,n$

Theorem 7. If $f(\tilde{x},t)=\sin\left(\sum_{i=1}^{n}\alpha_i x_i+\beta t\right)$, then $F_k(\tilde{x})=\left(\frac{\beta^k}{k!}\right)\cdot\sin\left(\frac{k\pi}{2}+\sum_{i=1}^{n}\alpha_i x_i\right)$

Theorem 8. If $f(\tilde{x},t)=\cos\left(\sum_{i=1}^{n}\alpha_i x_i+\beta t\right)$, then $F_k(\tilde{x})=\left(\frac{\beta^k}{k!}\right)\cdot\cos\left(\frac{k\pi}{2}+\sum_{i=1}^{n}\alpha_i x_i\right)$

3. Applications of RDTM to fourth-order parabolic PDEs

In this section, in order to show the applicability and efficiency of the RDTM for solving the fourth-order parabolic PDEs, some illustrative examples are given.

Example 3.1 Firstly, consider the following one dimensional non-homogeneous fourth-order parabolic PDE [13].

$$\frac{\partial^2 u}{\partial t^2}+(1+x)\frac{\partial^4 u}{\partial x^4}=\left(x^4+x^3-\frac{6}{7!}x^7\right)\cos t,\ 0<x<1, t>0 \tag{4}$$

subject to the initial condition

$$u(x,0)=\frac{6}{7!}x^7,\ \ \frac{\partial u}{\partial t}(x,0)=0 \tag{5}$$

whose exact solution is

$$u(x,t)=\frac{6}{7!}x^7\cos t \tag{6}$$

By applying the RDTM on Eq.(4), the following recursive equation is obtained:

$$U_{k+2}(x)=\frac{-1}{(k+1)(k+2)}\left\{(1+x)\frac{\partial^4}{\partial x^4}U_k(x)+F_k(x)\right\} \tag{7}$$

where $F_k(x)$ is the transformation of the function $f(x,t)=\left(x^4+x^3-\frac{6}{7!}x^7\right)\cos t$. From theorem 8, the reduced differential transform $F_k(x)$ is,

$$F_k(x)=\left(x^4+x^3-\frac{6}{7!}x^7\right)\frac{1}{k!}\cos\left(\frac{k\pi}{2}\right) \tag{8}$$

From Eq.(1), the initial conditions given in Eq. (5) can be transformed at $t=0$ as

$$U_0(x)=\frac{6}{7!}x^7,\ U_1(x)=0 \tag{9}$$

Substituting Eqs.(8)-(9) into Eq.(7) and by straightforward iterative steps, the following $U_k(x)$ (for k=0,1,2,...,n) values are obtained.

$$\begin{gathered}U_0(x)=\frac{6}{7!}x^7,\ U_1(x)=0,\ U_2(x)=-\frac{6}{7!}\cdot\frac{1}{2}x^7,\ U_3(x)=0,\ U_4(x)=\frac{6}{7!}\cdot\frac{1}{24}x^7\\ U_5(x)=0,\ U_6(x)=-\frac{6}{7!}\cdot\frac{1}{6!}x^7,\ U_7(x)=0,\ U_8(x)=\frac{6}{7!}\cdot\frac{1}{8!}x^7,\cdots\end{gathered} \tag{10}$$

Then, using the inverse transformation Eq.(2), the 8*th* order approximate solution as,

$$\begin{aligned}\tilde{u}(x,t)=\sum_{k=0}^{8}U_k(x)t^k&=\frac{6}{7!}x^7-\frac{6}{7!}\cdot\frac{1}{2!}x^7t^2+\frac{6}{7!}\cdot\frac{1}{4!}x^7t^4-\frac{6}{7!}\cdot\frac{1}{6!}x^7t^6+\frac{6}{7!}\cdot\frac{1}{8!}x^7t^8+\cdots\\&=\frac{6}{7!}x^7\left(1-\frac{1}{2!}t^2+\frac{1}{4!}t^4-\frac{1}{6!}t^6+\frac{1}{8!}t^8+\cdots\right)\end{aligned}$$

which is the first eight terms of the poisson series of the exact solution Eq.(6).

Example 3.2 Now, Consider the following singular fourth-order parabolic PDE in two space variables [14]

$$\frac{\partial^2 u}{\partial t^2}+2\left(\frac{1}{x^2}+\frac{x^4}{6!}\right)\frac{\partial^4 u}{\partial x^4}+2\left(\frac{1}{y^2}+\frac{y^4}{6!}\right)\frac{\partial^4 u}{\partial y^4}=0,\ \frac{1}{2}<x,y<1,\ t>0 \tag{11}$$

subject to the initial condition

$$u(x,y,0)=0,\ \frac{\partial u}{\partial t}(x,y,0)=2+\frac{x^6}{6!}+\frac{y^6}{6!} \tag{12}$$

whose exact solution is

$$u(x,y,t)=\left(2+\frac{x^6}{6!}+\frac{y^6}{6!}\right)\sin t \tag{13}$$

Using above theorems, the transformed form of Eq. (11) is find as

$$U_{k+2}(x,y)=\frac{-1}{(k+1)(k+2)}\left\{2\left(\frac{1}{x^2}+\frac{x^4}{6!}\right)\frac{\partial^4 U_k(x,y)}{\partial x^4}+2\left(\frac{1}{y^2}+\frac{y^4}{6!}\right)\frac{\partial^4 U_k(x,y)}{\partial y^4}\right\} \tag{14}$$

From Eq.(1), the initial conditions given in Eq. (12) can be transformed at $t=0$ as

$$U_0(x,y)=0,\ U_1(x,y)=2+\frac{x^6}{6!}+\frac{y^6}{6!} \tag{15}$$

Substituting Eq.(15) into Eq.(14) and by straightforward iterative steps, the following $U_k(x)$ (for k=0,1,2,...,n) values are obtained.

$$\begin{aligned}&U_0(x)=0,\ U_1(x)=2+\frac{x^6}{6!}+\frac{y^6}{6!},\ U_2(x)=0,\ U_3(x)=-\frac{1}{3!}\left(2+\frac{x^6}{6!}+\frac{y^6}{6!}\right),\ U_4(x)=0\\&U_5(x)=\frac{1}{5!}\left(2+\frac{x^6}{6!}+\frac{y^6}{6!}\right),\ U_6(x)=0,\ U_7(x)=-\frac{1}{7!}\left(2+\frac{x^6}{6!}+\frac{y^6}{6!}\right),\ U_8(x)=0,\cdots\end{aligned} \tag{16}$$

Then, using the inverse transformation Eq.(2), we get the 8*th* order approximate solution as,

$$\begin{aligned}\tilde{u}(x,t)=\sum_{k=0}^{n}U_k(x)t^k&=\left(2+\frac{x^6}{6!}+\frac{y^6}{6!}\right)t-\frac{1}{3!}\left(2+\frac{x^6}{6!}+\frac{y^6}{6!}\right)t^3+\frac{1}{5!}\left(2+\frac{x^6}{6!}+\frac{y^6}{6!}\right)t^5\\&-\frac{1}{7!}\left(2+\frac{x^6}{6!}+\frac{y^6}{6!}\right)t^7\\&=\left(2+\frac{x^6}{6!}+\frac{y^6}{6!}\right)\left(t-\frac{1}{3!}t^3+\frac{1}{5!}t^5-\frac{1}{7!}t^7\right)\end{aligned}$$

which is the first eight terms of the poisson series of the exact solution Eq.(13).

Example 3.3 Consider the following PDE in three space variables [14]

$$\frac{\partial^2 u}{\partial t^2}+\left(\frac{y+z}{2\cos x}-1\right)\frac{\partial^4 u}{\partial x^4}+\left(\frac{z+x}{2\cos y}-1\right)\frac{\partial^4 u}{\partial y^4}+\left(\frac{x+y}{2\cos z}-1\right)\frac{\partial^4 u}{\partial z^4}=0,\ \ 0<x,y,z<\pi/3, t>0 \quad (17)$$

subject to the initial condition

$$u(x,y,z,0)=-\frac{\partial u}{\partial t}(x,y,z,0)=\left(x+y+z\right)-\left(\cos x+\cos y+\cos z\right) \quad (18)$$

whose exact solution is

$$u(x,y,z,t)=\left(x+y+z-\cos x-\cos y-\cos z\right)e^{-t} \quad (19)$$

Using the RDTM, , the transformed form of Eq. (17) can be viewed as the following recursive formula

$$\begin{aligned}U_{k+2}(x,y,z)=\frac{-1}{(k+1)(k+2)}&\left\{\left(\frac{y+z}{2\cos x}-1\right)\frac{\partial^4 U_k(x,y,z)}{\partial x^4}+\left(\frac{z+x}{2\cos y}-1\right)\frac{\partial^4 U_k(x,y,z)}{\partial y^4}\right.\\ &\left.+\left(\frac{x+y}{2\cos z}-1\right)\frac{\partial^4 U_k(x,y,z)}{\partial z^4}\right\}\end{aligned} \quad (20)$$

From Eq.(1), the initial conditions given in Eq.(18) can be transformed at $t=0$ as

$$U_0(x,y,z)=-U_1(x,y,z)=\left(x+y+z\right)-\left(\cos x+\cos y+\cos z\right) \quad (21)$$

Substituting Eq.(21) into Eq.(20) and by straightforward iterative steps, we get the following $U_k(x)$ (for k=0,1,2,...,n) values.

$$\begin{aligned}&U_0(x)=\left(x+y+z-\cos x-\cos y-\cos z\right),\quad U_1(x)=-\left(x+y+z-\cos x-\cos y-\cos z\right),\\ &U_2(x)=\frac{1}{2}\left(x+y+z-\cos x-\cos y-\cos z\right),\ U_3(x)=-\frac{1}{6}\left(x+y+z-\cos x-\cos y-\cos z\right),\\ &U_4(x)=\frac{1}{24}\left(x+y+z-\cos x-\cos y-\cos z\right), U_5(x)=-\frac{1}{120}\left(x+y+z-\cos x-\cos y-\cos z\right),\\ &U_6(x)=\frac{1}{720}\left(x+y+z-\cos x-\cos y-\cos z\right),\cdots\end{aligned} \quad (22)$$

Then, using the inverse transformation Eq.(2), we get the *nth* order approximate solution as,

$$\tilde{u}(x,t)=\sum_{k=0}^{n}U_k(x)t^k=\left(x+y+z-\cos x-\cos y-\cos z\right)\left(1-t+\frac{1}{2!}t^2-\frac{1}{3!}t^3+\frac{1}{4!}t^4-\frac{1}{5!}t^5+\frac{1}{6!}t^6+\cdots\right)$$

which is the first eight terms of the poisson series of the exact solution Eq.(19).

Example 3.4 As the last example, consider the following three dimensional non-homogeneous fourth-order PDE [14]

$$\frac{\partial^2 u}{\partial t^2}+\frac{1}{4!z}\frac{\partial^4 u}{\partial x^4}+\frac{1}{4!x}\frac{\partial^4 u}{\partial y^4}+\frac{1}{4!y}\frac{\partial^4 u}{\partial z^4}=\left\{-\frac{x}{y}-\frac{y}{z}-\frac{z}{x}+\frac{1}{x^5}+\frac{1}{y^5}+\frac{1}{z^5}\right\}\cos t,\ \ \frac{1}{2}<x,y,z<1,\ t>0 \quad (23)$$

subject to the initial condition

$$u(x,y,z,0)=\frac{x}{y}+\frac{y}{z}+\frac{z}{x},\quad \frac{\partial u}{\partial t}(x,y,z,0)=0 \tag{24}$$

whose exact solution is

$$u(x,y,t)=\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)\cos t \tag{25}$$

Applying RDTM to Eq.(23), the following recursive formula is obtained:

$$\begin{aligned}U_{k+2}(x,y,z)=\frac{-1}{(k+1)(k+2)}\Bigg\{&\frac{1}{4!z}\frac{\partial^4 U_k(x,y,z)}{\partial x^4}+\frac{1}{4!x}\frac{\partial^4 U_k(x,y,z)}{\partial y^4}\\&+\frac{1}{4!y}\frac{\partial^4 U_k(x,y,z)}{\partial z^4}-F_k(x,y,z)\Bigg\}\end{aligned} \tag{26}$$

where $F_k(x)$ is the transformation of the function $f(x,y,z,t)=\left\{-\frac{x}{y}-\frac{y}{z}-\frac{z}{x}+\frac{1}{x^5}+\frac{1}{y^5}+\frac{1}{z^5}\right\}\cos t$.

From theorem 8, the redeuced differential transform $F_k(x)$ is,

$$F_k(x,y,z)=\left\{-\frac{x}{y}-\frac{y}{z}-\frac{z}{x}+\frac{1}{x^5}+\frac{1}{y^5}+\frac{1}{z^5}\right\}\frac{1}{k!}\cos\left(\frac{k\pi}{2}\right) \tag{27}$$

From Eq.(1), the initial conditions given in Eq.(24) can be transformed at $t=0$ as

$$U_0(x,y,z)=\frac{x}{y}+\frac{y}{z}+\frac{z}{x},\quad U_1(x,y,z)=0 \tag{28}$$

Substituting Eq.(27) and Eq.(28) into Eq.(26) and by straightforward iterative steps, the following $U_k(x)$ (for k=0,1,2,...,n) values are obtained.

$$\begin{aligned}&U_0(x)=\frac{x}{y}+\frac{y}{z}+\frac{z}{x},\ U_1(x)=0,\ U_2(x)=-\frac{1}{2!}\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right),\ U_3(x)=0,\\&U_4(x)=-\frac{1}{4!}\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right),U_5(x)=0,\ U_6(x)=\frac{1}{6!}\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right),\ U_7(x)=0,\cdots\end{aligned} \tag{29}$$

Then, using the inverse transformation Eq.(2), we get the *nth* order approximate solution as,

$$\begin{aligned}\tilde{u}(x,t)=\sum_{k=0}^{n}U_k(x)t^k&=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}-\frac{1}{2!}\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)t^2+\frac{1}{4!}\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)t^4-\frac{1}{6!}\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)t^6+\cdots\\&=\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)\left(1-\frac{1}{2!}t^2+\frac{1}{4!}t^4-\frac{1}{6!}t^6+\cdots\right)\end{aligned}$$

which is the first eight terms of the poisson series of the exact solution Eq.(25).

4. Conclusion

In this paper, we introduced the modified version of the DTM, namely the reduced differential transform method (RDTM) for solving fourth-order parabolic PDEs. The main advantage of the RDTM is to provide the user an analytical approximation to the solution, in many cases, an exact solution, in a rapidly convergent sequence with elegantly computed terms. The solution procedure of the RDTM is simpler and effective than other analytic methods such as the Adomian Decomposition Method (ADM), the Variational Iteration Method (VIM), the Homotopy Perturbation Method (HPM) and Spline methods. The results show that the RDTM is a powerful and effectiveness method for solving linear and nonlinear PDEs.

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