



## A fully fuzzy approach to data envelopment analysis

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### Abstract

Data envelopment analysis (DEA) is a method to evaluate the efficiency of some decision making units which by using one or more inputs will make one or more outputs. In real world, most of the problems don't have a certain mode. Fuzzy theory is one of the ways of considering uncertainty in the mathematical programming problems. In this study by using this idea, the DEA model on a fully fuzzy mode is proposed. The feature of this proposed model is that it considers 3 situations for problem and solving them simultaneously. The first situation occurs on a desired condition with the highest output and lowest input. The second is made of centric point of inputs and outputs and is analogous with the first condition. The third or undesired situation is when there are upper bound of input and lower bound of output. Results showed that the highest efficiency of some units is 1, so these units are efficient. To collate the efficient units on the proposed method, we can use the obtained centric points for efficiency of units.

**Keywords:** Data envelopment analysis, fully fuzzy model, Decision making unit, Triangular fuzzy number, efficiency.

## 1. Introduction

Measuring the efficiency of decision making units by a linear programming model was first introduced by Charnes et al. and their proposed model was named "Data Envelopment Analysis"[1]. This name was due to constructing an efficient frontier from efficient units by the model that this frontier will cover (envelop) the inefficient units. This model is used where there are some inputs and outputs and the goal is measuring the efficiency of some symmetric decision making units. There are lots of researches about DEA model in last decades. One of the aspects that attract the attention of researchers is consideration of uncertainty in this model. This is due to inability of researchers to exactly measure and define model's parameters. One of the proposed ideas to consider uncertainty in problems is the fuzzy theory. This idea was first stated by Professor zadeh[2]. This idea had a not zero-oneview on the issues. On the zero-one opinion, time is either day or night, but on the fuzzy opinion time can be 70% day and 30% night.

Since in the real world all the variables can't be measured exactly, so they were considered as fuzzy forms on the last years. One of the fields that is covered by fuzzy theory is DEA. The researches that were done on this field can be divided in 5 categories. The first are researches that use  $\alpha$ -cut method on their models, so they can change their fuzzy problem to a parametric problem that is dependent on  $\alpha$  parameter [3][4][5]. Some of the researches on this field had reached to an interval that has high and low bounds according to  $\alpha$  [6][7]. Some other studies proposed two other methods for DEA fuzzy models that one of them resulted in interval numbers for efficiency of decision making units according to  $\alpha$ -cut method [8]. The second are studies that use fuzzy ranking method. These studies design their models as they could reach to fuzzy numbers for efficiency of decision making units and in the next step they will use different methods of ranking fuzzy numbers [9][10]. The third class of studies used defuzzification approach. These studies first change fuzzy inputs and outputs to crisp numbers and then solved DEA model [11][12]. Fourth class of studies used possibility theory [13][14]. According to this opinion, every fuzzy variable is defined by a probability distribution. The fifth class are studies that make other developments on this field. For example, Saati et al. introduced a DEA model by considering discretionary and non-discretionary factors [15]. Studies with fully fuzzy method are in this class, like works of HatamiMarbini and his colleagues [16]. They used the proposed method of Allahviiranlo et al. [17] to expand fuzzy problems.

In this study we focus on the main model of DEA and it is written in a fully fuzzy mode. Since solving the fully fuzzy problems is difficult, we use the proposed method of Kumar et.al to solve the mentioned problems [18]. The main difference of this study with last studies is that this study can solve fully fuzzy model simultaneously in 3 situations. The first situation is on a desired mode that the outputs are at maximum and the inputs are at minimum. In this proposed model, maximum efficiency has been considered and this efficiency has been compared with efficiency of centric points of inputs and outputs and of situations that outputs are at minimum and inputs are at maximum.

This study is organized as follows: on the second section the DEA model is presented and parts of model are defined. On the third section we have the definitions of fuzzy theory that is needed for modeling. This model is proposed on the forth section and solved on the fifth section. On the sixth section a numerical example is solved by the proposed model and on the seventh section we will have conclusion.

## 2. Basic Model of DEA

Model of Charnes et.al (1978) was the basic model of DEA (model 1). This is a linear programming model that can be solved by the common softwares. By considering the quotient of total weight of outputs on total weight of inputs, the efficiency on this basic model can be calculated.

$$\begin{aligned}
 \text{Max} \quad & E_p = \sum_{r=1}^s u_r y_{rp} \\
 \text{S.T.} \quad & \sum_{i=1}^m v_i x_{ip} = 1, \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad \forall j \\
 & u_r, v_i \geq 0, \quad \forall r, i.
 \end{aligned} \tag{1}$$

In model 1,  $E_p$  is the efficiency of DMUp. Consider there are  $n$  DMUs in which  $x_{ij}$  ( $i=1, 2, \dots, m$ ) are inputs and  $y_{rj}$  ( $r=1, 2, \dots, s$ ) are outputs of DMUj ( $j=1, 2, \dots, n$ ).

### 3. Fuzzy theory

Zadeh (1965) was the first to recommend the idea of fuzzy sets. This view is against the certainty view and has been used in many fields. In this part we have some definitions of this theory:

a) If the universal set is defined as  $X$  then a fuzzy set  $\tilde{A}$  of  $X$  can be defined by its membership function  $\mu_{\tilde{A}} : X \rightarrow [0, 1]$

Where for each  $x \in X$ ,  $\mu_{\tilde{A}}(x)$  is a real number in interval  $[0, 1]$ .  $\mu_{\tilde{A}}(x)$  shows the grade of membership function of  $x$  in  $\tilde{A}$ . The fuzzy subset of  $\tilde{A}$  can be written as a set of pairs of element  $x$  and  $\mu_{\tilde{A}}(x)$

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$$

b) The  $\alpha$  – cut set of a fuzzy set like  $\tilde{A}$  is defined as a set like  $[\tilde{A}]_{\alpha}$  in which the degree of its membership function is more than the value of  $\alpha$

$$[\tilde{A}]_{\alpha} = \{x | \mu_{\tilde{A}}(x) \geq \alpha, \alpha \in [0, 1]\}$$

c) A fuzzy set  $\tilde{A}$  in  $X = R^n$  is a convex fuzzy set if and only if all its  $\alpha$  – cut sets are convex.

d) A fuzzy set  $\tilde{A}$  in  $X$  is a normal fuzzy set if there is a  $x \in X$  in which  $\mu_{\tilde{A}}(x) = 1$ .

e) A fuzzy number is a continuous, convex and normal fuzzy set of the real line of  $R^1$ .

f) A fuzzy number  $L$  is said to be positive (negative) if  $\mu_L(x) = 0, \forall x < 0$  ( $\forall x > 0$ ). For example (1, 3, 7) is a positive number, (-7, -5, -1) is a negative number and (-3, 1, 5) is neither positive nor negative [19].

g) A fuzzy number  $\tilde{A} = (f, g, h)$  is called a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-f}{g-f}, & f \leq x \leq g \\ \frac{x-h}{g-h}, & g \leq x \leq h \\ 0, & \text{otherwise.} \end{cases}$$

h) Two triangular fuzzy numbers  $\tilde{A} = (f, g, h)$  and  $\tilde{B} = (p, q, r)$  are said to be equal iff  $f=p, g=q$  and  $h=r$ .

i) A function  $\Re : F(R) \rightarrow R$  is a ranking function where  $F(R)$  includes a set of fuzzy numbers that is defined on the set of real numbers. This function maps each fuzzy number into the real line. Let

$\tilde{A} = (f, g, h)$  be a triangular fuzzy number then  $\Re(\tilde{A}) = \frac{f+2g+h}{4}$  [20].

j) Let  $\tilde{A} = (f, g, h)$  and  $\tilde{B} = (p, q, r)$  be two triangular fuzzy numbers then we have:

I)  $\tilde{A} \oplus \tilde{B} = (f, g, h) \oplus (p, q, r) = (f+p, g+q, h+r)$

II)  $-\tilde{B} = -(p, q, r) = (-r, -q, -p)$

III)  $\tilde{A} \ominus \tilde{B} = (f, g, h) \ominus (p, q, r) = (f-r, g-q, h-p)$

IV) Let  $\tilde{A} = (f, g, h)$  be a triangular fuzzy number and  $\tilde{B} = (x, y, z)$  be a positive triangular fuzzy number then we have:

$$\tilde{A} \otimes \tilde{B} \simeq \begin{cases} (fx, gy, hz), & f \geq 0 \\ (fz, gy, hz), & f < 0, \quad h \geq 0 \\ (fz, gy, hx), & h < 0. \end{cases}$$

#### 4. Fully fuzzy DEA model

This model is formed when all the inputs, outputs and decision variables on the DEA are defined as the fuzzy numbers (model 2).

$$\begin{aligned}
 \text{Max} \quad & \tilde{E}_p = \sum_{r=1}^s \tilde{u}_r \otimes \tilde{y}_{rp} \\
 \text{S.T.} \quad & \sum_{i=1}^m \tilde{v}_i \otimes \tilde{x}_{ip} = \tilde{1}, \quad (2) \\
 & \sum_{r=1}^s \tilde{u}_r \otimes \tilde{y}_{rj} - \sum_{i=1}^m \tilde{v}_i \otimes \tilde{x}_{ij} \leq \tilde{0}, \quad \forall j \\
 & \tilde{u}_r, \tilde{v}_i \geq 0, \quad \forall r, i.
 \end{aligned}$$

As you can see from model 2, all the decision variables and model parameters are fuzzy and their relations are fuzzy too. The components of model 2 are defined according to equation 3.

$$\begin{cases}
 \tilde{u}_r = (u_r, s_r, t_r) \\
 \tilde{y}_{rp} = (y_{rp}, b_{rp}, z_{rp}) \\
 \tilde{v}_i = (v_i, l_i, d_i) \\
 \tilde{x}_{ip} = (x_{ip}, a_{ip}, c_{ip})
 \end{cases} \quad (3)$$

In the equation 3 all the numbers are defined as triangular fuzzy, that by putting these components on model 3 we will reach to model 4.

$$\begin{aligned}
 \text{Max} \quad & \tilde{E}_p = \sum_{r=1}^s (u_r, s_r, t_r) \otimes (y_{rp}, b_{rp}, z_{rp}) \\
 \text{S.T.} \quad & \sum_{i=1}^m (v_i, l_i, d_i) \otimes (x_{ip}, a_{ip}, c_{ip}) \leq (1,1,1), \quad (4) \\
 & \sum_{r=1}^s (u_r, s_r, t_r) \otimes (y_{ri}, b_{ri}, z_{ri}) - \sum_{i=1}^m (v_i, l_i, d_i) \otimes (x_{ij}, a_{ij}, c_{ij}) \leq (0,0,0), \quad \forall j \\
 & (u_r, s_r, t_r), (v_i, l_i, d_i) \geq 0, \quad \forall r, i.
 \end{aligned}$$

In model 4 the first constraint is less or equal due to increase of solution's space and avoiding non-feasible answers. This model is a fully fuzzy model of DEA that we will solve it in the next section.

#### 5. Solving the model

It's proved that models with interval coefficients are NP-hard and to solve them we should use heuristic methods [21]. So solving fully fuzzy models that are more complex than interval models will be so difficult. In this paper, to solve model 4 we follow the approach of Kumar et.al. To solve this fully fuzzy model, these authors first did multiply on all decision variables and parameters as it was described on section 3. After doing fuzzy multiply, model 4 is rewrite as follow (model 5).

$$\begin{aligned}
 \text{Max} \quad & \tilde{E}_p = \sum_{r=1}^s (u_r y_{rp}, s_r b_{rp}, t_r z_{rp}) \\
 \text{S.T.} \quad & \sum_{i=1}^m (v_i x_{ip}, l_i a_{ip}, d_i c_{ip}) \leq (1,1,1), \quad (5) \\
 & \sum_{r=1}^s (u_r y_{ri}, s_r b_{ri}, t_r z_{ri}) - \sum_{i=1}^m (v_i x_{ij}, l_i a_{ij}, d_i c_{ij}) \leq (0,0,0), \quad \forall j
 \end{aligned}$$

$$u_r \geq s_r, s_r \geq t_r, v_i \geq l_i, l_i \geq d_i, t_r \geq 0, d_i \geq 0 \quad \forall r, i.$$

At the end line of model 5, the decision variables of problem are defined as they will remain their triangular fuzzy mode after being solved. Then Kumar et al. wrote a ranking function for objective function and changed every constraint into 3 constraints, as one constraint is for upper bound, one for centric point and one for lower bound. After doing this, we will have model 6.

$$\text{Max} \quad \tilde{E}_p = \sum_{r=1}^s \Re(u_r y_{rp}, s_r b_{rp}, t_r z_{rp})$$

$$\text{S.T.} \quad \sum_{i=1}^m v_i x_{ip} \leq 1,$$

$$\sum_{i=1}^m l_i a_{ip} \leq 1, \quad (6)$$

$$\sum_{i=1}^m d_i c_{ip} \leq 1,$$

$$\sum_{r=1}^s u_r y_{ri} - \sum_{i=1}^m d_i c_{ij} \leq 0, \quad \forall j$$

$$\sum_{r=1}^s s_r b_{ri} - \sum_{i=1}^m l_i a_{ij} \leq 0, \quad \forall j$$

$$\sum_{r=1}^s t_r z_{ri} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad \forall j$$

$$u_r \geq s_r, s_r \geq t_r, v_i \geq l_i, l_i \geq d_i, t_r \geq 0, d_i \geq 0 \quad \forall r, i.$$

Model 6 is the proposed model of this study. This model is used when the data of a problem (inputs and outputs) are triangular fuzzy. One of the features of this proposed model is that it can be solved by all the common linear programming softwares and it can give fully fuzzy answers.

## 6. Numerical example

In this section we have a numerical example from Guo & Tanaka study that its data are on table 1 [22].

**Table 1- example of Guo and Tanaka**

DMU	Input 1	Input 2	Output 1	Output 2
1	(3.5, 4, 4.5)	(1.9, 2.1, 2.3)	(2.4, 2.6, 2.8)	(3.8, 4.1, 4.4)
2	(2.9, 2.9, 2.9)	(1.4, 1.5, 1.6)	(2.2, 2.2, 2.2)	(3.3, 3.5, 3.7)
3	(4.4, 4.9, 5.4)	(2.2, 2.6, 3)	(2.7, 3.2, 3.7)	(4.3, 5.1, 5.9)
4	(3.4, 4.1, 4.8)	(2.2, 2.3, 2.4)	(2.5, 2.9, 3.3)	(5.5, 5.7, 5.9)
5	(5.9, 6.5, 7.1)	(3.6, 4.1, 4.6)	(4.4, 5.1, 5.8)	(6.5, 7.4, 8.3)

By putting the data of table 1 on the proposed model, we will have model 7. This model is written to calculate the first decision making unit's efficiency and 4 other models have to be written to calculate the efficiency of other units.

$$\text{Max} \quad \tilde{E}_1 = 1/4(2.4u_1 + 3.8u_2 + 5.2s_1 + \dots)$$

$$\begin{aligned}
S.T. \quad & 3.5v_1 + 1.9v_2 \leq 1, \\
& 4l_1 + 2.1l_2 \leq 1, \\
& 4.5d_1 + 2.3d_2 \leq 1, \quad (7) \\
& 2.4u_1 + 3.8u_2 - 4.5d_1 - 2.3d_2 \leq 0, \\
& \cdot \\
& \cdot \\
& 2.6s_1 + 4.1s_2 - 4l_1 - 2.1l_2 \leq 0, \\
& \cdot \\
& \cdot \\
& 2.8t_1 + 4.4t_2 - 3.5v_1 - 1.9v_2 \leq 0, \\
& \cdot \\
& \cdot
\end{aligned}$$

$$u_1 \geq s_1, s_1 \geq t_1, v_1 \geq l_1, l_1 \geq d_1, t_1 \geq 0, d_1 \geq 0$$

$$u_2 \geq s_2, s_2 \geq t_2, v_2 \geq l_2, l_2 \geq d_2, t_2 \geq 0, d_2 \geq 0$$

In model 7, there are 18 constraints totally that we wrote some of them. In proposed model, there is  $3n+3$  constraints and  $n$  is the number of DMUs. By solving model 7 on Lingo 11.0 software, the efficiency of DMUs is calculated and its results are on table 2.

**Table 2- comparison of results of different methods**

DMU	Efficiencies of our study	[12]	[4]	[23]	[16]
1	(0.624, 0.747, 0.899)	0.855	0.915	0.9074	(0.789, 0.855, 0.921)
2	(0.834, 0.882, 0.935)	1	1	1	(0.981, 0.999, 1.017)
3	(0.555, 0.747, 1)	0.861	0.948	0.9074	(0.726, 0.861, 0.996)
4	(0.854, 0.924, 1)	1	1	1	(0.964, 0.999, 1.034)
5	(0.631, 0.799, 1)	1	0.991	1	(0.867, 0.999, 1.132)

As you can see on table 2, the efficiency of every decision making unit is a triangular fuzzy number that its upper bound is 1 or a number near 1. On DEA models, the efficient units are shown by 1 and the efficiency of other units are compared with these efficient units. So obtaining numbers more than 1 is uncommon on DEA models. Therefore the proposed model of this study is designed as a desired situation that can be obtain by using lower bound of inputs and upper bound of outputs and also maximum efficiency of every unit can be determine that this number is the highest efficiency on table 2. This is like constructing a virtual unit to ranking efficient units. In the proposed model of this study, the efficiency of DMUs is measured by centric numbers of inputs and outputs with desired efficiency. By considering lower bound of output and upper bound of input, we can achieve the lower bound of efficiency. This is the worst situation for efficiency of a unit and shows that the efficiency of this unit won't get lower than this number. To ranking the units it can be say that units which their upper bound of efficiency is 1 are efficient and other units are inefficient (of course here the concept of efficiency means the desired efficiency). On the proposed numerical example, units three, four and five are efficient and others are inefficient. It is also recommended that to rank efficient units, it's better to use efficiency centric number. As a result, units four, five, three, two and one are more efficient units in this numerical example respectively.

Table 2 shows both the results of this study and the results of other studies. In all these studies, units two, four and five have the highest efficiency and after them units three and one have the highest efficiency. These results conform to the results of this study about centric numbers. Also in this study the efficiency of units one and three are equal that is the same as Wen et.al study. Among the mentioned studies, only HatamiMarbini had studied fully fuzzy model of DEA and since the model designing view of his study is different from view of proposed model of this study, so their results can't be compared correctly.

## 7. Conclusions

The Data Envelopment Analysis (DEA) models had developed in last years. To maximize the efficiency of every decision making unit, these models are looking for proper weight sets for every input and output. Sometimes in real world it is impossible to show parameters with crisp numbers. Hence most of the authors are considering uncertainty on designing and solving problems. One of the views that was under consideration was fuzzy theory. This view was used in the DEA problem and hundreds of studies were done for it. One of the fuzzy models of DEA was fully fuzzy model. In this study we use a special way to solve fully fuzzy model of DEA and this problem can be solved by existing softwares. The advantage of this model is that it can choose a desired situation for decision making units and compare its situation with desired situation. Another advantage of it is its ability to ranking efficient units without defuzzification the answers. For future works after it was considered that fully fuzzy models are NP-hard, heuristic and meta-heuristic methods can be used to solve problems and results can be compared. It is also recommended that the method used in this study can be used on input-oriented and BCCmodels and its results can be compared.

## References

- [1] Charnes, A., Cooper, W.W., Rhodes, E.L., "Measuring the efficiency of decision making units", *European Journal of Operational Research*, No. 2,, 1978, pp. 429-444,
- [2] Zadeh, L.A., "Fuzzy Sets". *Information and Control*, No. 8, 1965, pp. 338-353.
- [3] Azadeh, A., Alem, S.M., "A flexible deterministic, stochastic and fuzzy data envelopment analysis approach for supply chain risk and vendor selection problem: simulation analysis", *Expert Systems with Applications*, No. 37 (12), 2010, pp. 7438–7448.
- [4] ZerafatAngiz, L.M., Emrouznejad, A., Mustafa, A., "Fuzzy assessment of performance of a decision making units using DEA: a non-radial approach", *Expert Systems with Applications*, No. 37 (7), 2010, pp. 5153–5157.
- [5] ZerafatAngiz, L.M., Emrouznejad, A., Mustafa, A., "Fuzzy data envelopment analysis: a discrete approach", *Expert Systems with Applications*, No. 39, 2012, pp. 2263–2269.
- [6] Kao, C., Liu, S.T., "Fuzzy efficiency measures in data envelopment analysis", *Fuzzy Sets and Systems*, No. 113 (3), 2000a, pp. 427–437.
- [7] Kao, C., Liu, S.T., "Data envelopment analysis with missing data: an application to University libraries in Taiwan", *Journal of Operational Research Society*, No. 51 (8), 2000b, pp. 897–905.
- [8] HosseinzadehLotfi, F., AdabitarbarFirozja, M., Erfani, V., "Efficiency measures in data envelopment analysis with fuzzy and ordinal data", *International Mathematical Forum*, No. 4 (20), 2009, pp. 995–1006.
- [9] Jahanshahloo, G.R., HosseinzadehLotfi, F., Shahverdi, R., Adabitarbar, M., Rostamy-Malkhalifeh, M., Sohraiee, S., "Ranking DMUs by l1 \_ normwith fuzzy data in DEA" *Chaos, Solitons and Fractals*, No. 39, 2009, pp. 2294–2302.

- [10] Soleimani-damaneh, M., "Establishing the existence of a distance-based upper bound for a fuzzy DEA model using duality", *Chaos, Solitons and Fractals*, No. 41, 2009, pp. 485–490.
- [11] Juan, Y.K., "A hybrid approach using data envelopment analysis and case-based reasoning for housing refurbishment contractors selection and performance Improvement", *Expert Systems with Applications*, No. 36 (3), 2009, pp. 5702–5710.
- [12] Lertworasirikul, S., "Fuzzy Data Envelopment Analysis (DEA)", Ph.D. Dissertation, Dept. of Industrial Engineering, North Carolina State University, 2002.
- [13] Lertworasirikul, S., Fang, S.C., Joines, J.A., Nuttle, H.L.W., Fuzzy data envelopment analysis (DEA): a possibility approach. *Fuzzy Sets and Systems* 139 (2), 2003, pp. 379–394.
- [14] Wang, Y.M., Luo, Y., Liang, L., "Fuzzy data envelopment analysis based upon fuzzy arithmetic with an application to performance assessment of manufacturing enterprises". *Expert Systems with Applications* 36, 2009, pp. 5205–5211.
- [15] Saati, S., Hatami-Marbini, A. and Tavana, M. "A data envelopment analysis model with discretionary and non-discretionary factors in fuzzy environments", *Int. J. Productivity and Quality Management*, 8(1), 2011, pp.45–63.
- [16] Hatami-Marbini, A., Tavana, M., Ebrahimi, A., "A fully fuzzified data envelopment analysis model", *International Journal of Information and Decision Sciences*, No. 3(3), 2011, pp. 252-264.
- [17] Allahviranloo, T., HosseinzadehLotfi, F., Kiasary, M.Kh, Kiani, N.A., Alizadeh, L., "Solving fully fuzzy linear programming problem by the ranking function", *Applied Mathematical Sciences*, Vol. 2, No. 1, 2008, pp.19–32.
- [18] Kumar, A., Kaur, J., Singh, P., "Fuzzy optimal solution of fully fuzzy linear programming problems with inequality constraints", *International Journal of Mathematical and Computer Sciences*, No. 6, 2010, pp. 37-41.
- [19] Dehghan, M., Hashemi, B., Ghatee, M., "Computational methods for solving fully fuzzy linear systems", *Applied Mathematics and Computations*, No. 179, 2006, pp. 328–343.
- [20] Liou, T.S., Wang, M.J., "Ranking fuzzy numbers with integral value", *Fuzzy Sets and Systems*, No. 50, 1992, pp. 247–255.
- [21] Kreinovich, V., Lakeyev, A.V., Rohn, J., Kahl, P.T., "Computational Complexity and Feasibility of Data Processing and Interval Computations", *Applied Optimization*, vol. 10, 1998.
- [22] Guo, P., Tanaka, H., "Fuzzy DEA: a perceptual evaluation method", *Fuzzy Sets and Systems*, Vol. 119, No. 1, 2001, pp.149–160.
- [23] Wen, M., You, C., Kang, R., "A new ranking method to fuzzy data envelopment analysis" . *Computers and Mathematics with Applications*, 59, 2010, 3398-3404.