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On the Korobov and Changhee mixed-type polynomials and numbers

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Abstract

By using the Bosonic p-adic integral, Kim et al. [D. S. Kim, T. Kim, H.-I. Kwon, J.-J. Seo, Adv. Stud. Theor. Phys., **8** (2014), 745–754] studied some identities of the Korobov and Daehee mixed-type polynomials. In this paper, by using the fermionic p-adic integral, we define the Korobov and Changhee mixed-type polynomials and give some interesting identities of those polynomials.

Keywords: Korobov polynomials, Changhee polynomials, Korobov and Changhee mixed-type polynomials.
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1. Introduction

Let p be an odd prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$, and $\mathbb{C}_p$ denote the ring of p-adic integers, the filed of p-adic rational numbers, and the completion of algebraic closure of $\mathbb{Q}_p$, respectively. The p-adic norm $|\cdot|_p$ is normalized as $|p|_p = \frac{1}{p}$. Let $C(\mathbb{Z}_p)$ be the space of continuous functions on $\mathbb{Z}_p$.

For $f \in C(\mathbb{Z}_p)$, the fermionic p-adic integrals on $\mathbb{Z}_p$ is defined by Kim to be

$$I_{-1}(f) = \int_{\mathbb{Z}_p} f(x)d\mu_{-1}(x) = \lim_{N\to\infty} \sum_{x=0}^{p^N-1} f(x)(-1)^x, \quad \text{(see [1–21])}. \tag{1.1}$$

From (1.1), it is well-known that

$$I_{-1}(f_1) + I_{-1}(f) = 2f(0), \tag{1.2}$$

where $f_1(x) = f(x+1)$. By using (1.2), we get

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$$\int_{\mathbb{Z}_p} (1-t)^{qy+x} d\mu_{-1}(y) = \frac{2}{(1-t)^q+1}(1-t)^x. \tag{1.3}$$

Recall that the q-Chaghee polynomials are defined by the generating function

$$\frac{2}{(1+t)^q+1}(1+t)^x = \sum_{x=0}^{\infty} Ch_{n,q}(x)\frac{t^n}{n!}, \quad (\text{see [20]}). \tag{1.4}$$

From (1.3) and (1.4), we have

$$\frac{2}{(1-t)^q+1}(1-t)^x = \sum_{n=0}^{\infty} Ch_{n,q}(x)(-1)^n\frac{t^n}{n!}. \tag{1.5}$$

By replacing t by $1-e^t$ in (1.5), we have

$$\begin{aligned} \text{LHS of (1.5)} &= \frac{2}{1+e^{qt}}e^{xt} \\ &= \frac{2}{1+e^{qt}}e^{\frac{x}{q}qt} \\ &= \sum_{n=0}^{\infty} E_n\left(\frac{x}{q}\right)q^n\frac{t^n}{n!}, \end{aligned} \tag{1.6}$$

and

$$\begin{aligned} \text{RHS of (1.5)} &= \sum_{n=0}^{\infty} Ch_{n,q}(x)(-1)^n\frac{(1-e^t)^n}{n!} \\ &= \sum_{n=0}^{\infty} Ch_{n,q}(x)\frac{1}{n!}n!\sum_{m=n}^{\infty} S_2(m,n)\frac{t^m}{m!} \\ &= \sum_{m=0}^{\infty}\left(\sum_{n=0}^{m} Ch_{n,q}(x)S_2(m,n)\right)\frac{t^m}{m!}. \end{aligned} \tag{1.7}$$

From (1.6) and (1.7), we obtain the following theorem.

Theorem 1.1. *For* $m \in \mathbb{N}\cup\{0\}$, *we have*

$$E_m\left(\frac{x}{q}\right)q^m = \sum_{n=0}^{m} Ch_{n,q}(x)S_2(m,n).$$

We observe that

$$\begin{aligned} \int_{\mathbb{Z}_p} (1-t)^{qy+x} d\mu_{-1}(y) &= \sum_{m=0}^{\infty}\int_{\mathbb{Z}_p}\binom{qy+x}{m}(-t)^m d\mu_{-1}(y) \\ &= \sum_{m=0}^{\infty}\int_{\mathbb{Z}_p}(qy+x)_m d\mu_{-1}(y)(-1)^m\frac{t^m}{m!}. \end{aligned} \tag{1.8}$$

From (1.3), (1.5), and (1.8), we obtain the following theorem.

Theorem 1.2. *For* $m \in \mathbb{N}\cup\{0\}$, *we have*

$$\int_{\mathbb{Z}_p}(qy+x)_m d\mu_{-1}(y) = Ch_{m,q}(x). \tag{1.9}$$

The Stirling numbers of the first kind are defined by

$$(x)_n = \sum_{l=0}^{n} S_1(n,l)x^l, \qquad (n \geqslant 0).$$

The Stirling numbers of the second kind are defined by

$$x^n = \sum_{l=0}^{n} S_2(n,l)(x)_l, \qquad (n \geqslant 0).$$

By using the Bosonic p-adic integral, Kim et al. ([11, 12, 14]) studied some identities of the Korobov and Daehee mixed-type polynomials. In this paper, we observe the Korobov and Daehee mixed-type polynomials in a slightly different way and use the Fermionic p-adic integral in stead of the Bosonic p-adic integral. From the Fermionic p-adic integral, we define the Korobov and Changhee mixed-type polynomials and give some interesting identities of those polynomials.

2. The Korobov and Changhee mixed-type polynomials

Let us define Korobov and Changhee mixed-type polynomials $KCh_{n,q}(x)$ of the first kind as follows:

$$KCh_{n,q}(x) = (-1)^n \int_{\mathbb{Z}_p} (qy+x)_n d\mu_{-1}(y), \qquad (n \geqslant 0). \tag{2.1}$$

Then, by (1.9) and (2.1), we have

$$KCh_{n,q}(x) = Ch_{l,q}(x)(-1)^n.$$

By (2.1), we derive the generating function of $KCh_{n,q}(x)$ as follows:

$$\begin{aligned}
\sum_{n=0}^{\infty} KCh_{n,q}(x)\frac{t^n}{n!} &= \int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} (qy+x)_n \frac{(-t)^n}{n!} d\mu_{-1}(y) \\
&= \int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} \binom{qy+x}{n} (-t)^n d\mu_{-1}(y) \\
&= \int_{\mathbb{Z}_p} (1-t)^{qy+x} d\mu_{-1}(y) \\
&= \frac{2}{(1-t)^q+1}(1-t)^x.
\end{aligned}$$

Note that the generating function of the Stirling number is given by

$$(e^t-1)^n = n! \sum_{l=n}^{\infty} S_2(l,n)\frac{t^l}{l!}, \quad \text{(see [4, 5, 8, 10, 13, 14, 16, 20, 21])}.$$

Recall that the Euler polynomials was defined by the generating function as follows:

$$\int_{\mathbb{Z}_p} e^{(y+x)t} d\mu_{-1}(y) = \frac{2}{e^t+1}e^{xt} = \sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!}.$$

Replacing t by $1-e^t$, we have

$$\begin{aligned}
\sum_{m=0}^{\infty} KCh_{m,q}(x)(-1)^m \frac{(e^t-1)^m}{m!} &= \frac{2}{1+e^{qt}} e^{\frac{x}{q}qt} \\
&= \sum_{n=0}^{\infty} E_n\left(\frac{x}{q}\right) q^n \frac{t^n}{n!},
\end{aligned} \tag{2.2}$$

and

$$\begin{aligned}\sum_{m=0}^{\infty} KCh_{m,q}(x)(-1)^m\frac{(e^t-1)^m}{m!} &= \sum_{m=0}^{\infty} KCh_{m,q}(x)(-1)^m\sum_{n=m}^{\infty} S_2(n,m)\frac{t^n}{n!}\\ &= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}(-1)^m KCh_{m,q}(x)S_2(n,m)\right)\frac{t^n}{n!}.\end{aligned} \tag{2.3}$$

Thus by (2.2) and (2.3), we obtain the following theorem.

Theorem 2.1. *For $n \geqslant 0$, we have*

$$E_n\left(\frac{x}{q}\right)q^n = \sum_{l=0}^{n}(-1)^l KCh_{l,q}(x)S_2(l,n).$$

In view of (2.1) , we define the Korobov and Changhee mixed-type polynomials of the second kind as following:

$$\widehat{KCh}_{n,q}(x) = (-1)^n\int_{\mathbb{Z}_p}(-qy+x)_n d\mu_{-1}(y), \qquad (n \geqslant 0). \tag{2.4}$$

From (2.4), we get

$$\begin{aligned}\sum_{n=0}^{\infty}\widehat{KCh}_{n,q}\frac{t^n}{n!} &= \sum_{n=0}^{\infty}(-1)^n\int_{\mathbb{Z}_p}(-qy+x)_n d\mu_{-1}(y)\frac{t^n}{n!}\\ &= \int_{\mathbb{Z}_p}(1-t)^{-qy+x}d\mu_{-1}(y).\end{aligned} \tag{2.5}$$

From (1.2), we have

$$\int_{\mathbb{Z}_p}(1-t)^{-qy+x}d\mu_{-1}(y) = \frac{2}{(1-t)^{-q}+1}(1-t)^x. \tag{2.6}$$

By (2.5) and (2.6), we derive the generating function of $\widehat{KCh}_{n,q}(x)$ as follows:

$$\sum_{n=0}^{\infty}\widehat{KCh}_{n,q}(x)\frac{t^n}{n!} = \frac{2}{(1-t)^{-q}+1}(1-t)^x. \tag{2.7}$$

From (2.4), we have

$$\begin{aligned}\widehat{KCh}_{n,q}(x) &= (-1)^n\int_{\mathbb{Z}_p}(-qy+x)_n d\mu_{-1}(y)\\ &= \sum_{l=0}^{n}S_1(n,l)\int_{\mathbb{Z}_p}(-qy+x)^l d\mu_{-1}(y)(-1)^n\\ &= \sum_{l=0}^{n}S_1(n,l)(-1)^{l+n}q^l\int_{\mathbb{Z}_p}\left(y-\frac{x}{q}\right)^l d\mu_{-1}(y)\\ &= \sum_{l=0}^{n}S_1(n,l)(-1)^{n+l}q^l E_l\left(-\frac{x}{q}\right).\end{aligned} \tag{2.8}$$

Thus, by (2.8), we obtain the following theorem.

Theorem 2.2. *For $n \geqslant 0$, we have*

$$\widehat{KCh}_{n,q}(x) = \sum_{l=0}^{n}S_1(n,l)(-1)^{n+l}q^l E_l\left(-\frac{x}{q}\right). \tag{2.9}$$

We observe that

$$\begin{aligned}\sum_{n=0}^{\infty} E_n(-x)\frac{t^n}{n!} &= \frac{2}{e^t+1}e^{-xt}\\ &= \frac{2}{1+e^{-t}}e^{-(x+1)t}\\ &= \sum_{n=0}^{\infty} E_n(1+x)(-1)^n\frac{t^n}{n!}.\end{aligned} \tag{2.10}$$

By (2.10), we get

$$E_{n,q}(-x) = E_n(1+x)(-1)^n. \tag{2.11}$$

By (2.8) and (2.11), we obtain the following theorem.

Theorem 2.3. *For* $n \geqslant 0$,

$$\widehat{KCh}_{n,q}(x) = \sum_{l=0}^{n} S_1(n,l)(-1)^n q^l E_l\left(1+\frac{x}{q}\right).$$

By replacing t by $1-e^t$ in (2.7)

$$\begin{aligned}\sum_{n=0}^{\infty} \widehat{KCh}_{n,q}(x)\frac{(1-e^t)}{n!} &= \frac{2}{1+e^{qt}}e^{qt}e^{tx}\\ &= \frac{2}{1+e^{qt}}e^{(q+x)t}\\ &= \frac{2}{1+e^{qt}}e^{(1+\frac{x}{q})qt}\\ &= \sum_{n=0}^{\infty} E_n\left(1+\frac{x}{q}\right)q^n\frac{t^n}{n!},\end{aligned} \tag{2.12}$$

and

$$\begin{aligned}\sum_{m=0}^{\infty} \widehat{KCh}_{m,q}(x)\frac{1}{m!}(1-e^t)^m &= \sum_{m=0}^{\infty} \widehat{KCh}_{m,q}(x)\frac{1}{m!}m!(-1)^m\sum_{n=m}^{\infty} S_2(n,m)\frac{t^n}{n!}\\ &= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n} \widehat{KCh}_{m,q}(x)(-1)^m S_2(n,m)\right)\frac{t^n}{n!}.\end{aligned} \tag{2.13}$$

Thus, by (2.12) and (2.13), we obtain the following theorem.

Theorem 2.4. *For* $n \geqslant 0$, *we have*

$$E_n\left(1+\frac{x}{q}\right)q^n = \sum_{m=0}^{n} \widehat{KCh}_{m,q}(x)(-1)^m S_2(n,m).$$

We observe that

$$\begin{aligned}\frac{\widehat{KCh}_{n,q}}{n!} &= \frac{(-1)^n}{n!}\int_{\mathbb{Z}_p}(-qy)_n\,d\mu_{-1}(y)\\ &= \int_{\mathbb{Z}_p}\binom{qy+n-1}{n}d\mu_{-1}(y)\\ &= \sum_{l=0}^{n}\binom{n-1}{n-l}\int_{\mathbb{Z}_p}\binom{qy}{l}d\mu_{-1}(y)\\ &= \sum_{l=0}^{n}\binom{n-1}{n-l}\frac{1}{l!}\int_{\mathbb{Z}_p}(qy)_l\,d\mu_{-1}(y)\\ &= \sum_{l=0}^{n}\binom{n-1}{l-1}\frac{KCh_{l,q}}{l!}.\end{aligned} \tag{2.14}$$

Thus, by (2.14), we obtain the following theorem.

Theorem 2.5. *For $n \geqslant 1$, we have*

$$\frac{\widehat{KCh}_{n,q}}{n!} = \sum_{l=0}^{n} \binom{n-1}{l-1} \frac{KCh_{l,q}}{l!}.$$

3. The Korobov and Changhee mixed-type polynomials of order r

For $r \in \mathbb{N}$, let us consider Korobov and Changhee mixed-type polynomials of order r as follows:

$$KCh_{n,q}^{(r)}(x) = (-1)^n \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (qx_1 + \cdots + qx_r + x)_n d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_r). \tag{3.1}$$

Then we have

$$\begin{aligned} KCh_{n,q}^{(r)}(x) &= (-1)^n \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (qx_1 + \cdots + qx_r + x)_n d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_r) \\ &= \sum_{l=0}^{n} S_1(n,l)(-1)^n \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (qx_1 + \cdots + qx_r + x)^r d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_r) \\ &= \sum_{l=0}^{n} S_1(n,l) q^r (-1)^n \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (x_1 + \cdots + x_r + \frac{x}{q})^r d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_r). \end{aligned} \tag{3.2}$$

Recall that the Euler polynomials of order r was defined by the generating function as follows:

$$\int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} e^{(x_1 + \cdots + x_r + x)t} d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_r) = \left(\frac{2}{e^t + 1}\right)^r e^{xt} = \sum_{n=0}^{\infty} E_n^{(r)}(x) \frac{t^n}{n!}. \tag{3.3}$$

From (3.2) and (3.3), we obtain the following theorem.

Theorem 3.1. *For $n \geqslant 0$, we have*

$$KCh_{n,q}^{(r)}(x) = \sum_{l=0}^{n} S_1(n,l) q^r (-1)^n E_n^{(r)} \left(\frac{x}{q}\right).$$

From (3.1), we can derive the generating function of $KCh_{n,q}(x)$ as follows:

$$\begin{aligned} \sum_{n=0}^{\infty} KCh_{n,q}^{(r)}(x) \frac{t^n}{n!} &= \sum_{n=0}^{\infty} (-1)^n \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (qx_1 + \cdots + qx_r + x)_n d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_r) \frac{t^n}{n!} \\ &= \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} \binom{qx_1 + \cdots + qx_r + x}{n} (-t)^n d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_r) \\ &= \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (1-t)^{qx_1 + \cdots + qx_r + x} d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_r) \\ &= \left(\frac{2}{(1-t)^q + 1}\right)^r (1-t)^x. \end{aligned} \tag{3.4}$$

As is known, the q-Changhee polynomials of order r are defined by the generating function to be

$$\left(\frac{2}{(1+t)^q + 1}\right)^r (1+t)^x = \sum_{n=0}^{\infty} Ch_{n,q}^{(r)}(x) \frac{t^n}{n!}.$$

By replacing t by $1-e^t$ in (3.4)

$$\begin{aligned}\sum_{n=0}^{\infty} KCh_{n,q}^{(r)}(x)\frac{(1-e^t)^n}{n!} &= \sum_{n=0}^{\infty} KCh_{n,q}^{(r)}(x)\frac{1}{n!}n!\sum_{l=n}^{\infty} S_2(l,n)(-1)^n\frac{t^l}{l!} \\ &= \sum_{l=0}^{\infty}\left(\sum_{n=0}^{l} KCh_{n,q}^{(r)}(x)(-1)^n S_2(l,n)\right)\frac{t^l}{l!},\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}\sum_{n=0}^{\infty} KCh_{n,q}^{(r)}(x)\frac{(1-e^t)^n}{n!} &= \left(\frac{2}{e^{qt}+1}\right)^r e^{tx} \\ &= \left(\frac{2}{e^{qt}+1}\right)^r e^{\frac{x}{q}qt} \\ &= \sum_{l=0}^{\infty} E_l^{(r)} q^l\left(\frac{x}{q}\right)\frac{t^l}{l!}.\end{aligned} \tag{3.6}$$

From (3.5) and (3.6), we obtain the following theorem.

Theorem 3.2. *For $n \geqslant 0$,*

$$q^n E_n^{(r)}(x) = \sum_{l=0}^{n} KCh_{l,q}^{(r)}(x)(-1)^l S_2(n,l).$$

Let us consider Korobov and Changhee mixed-type polynomials of second kind with order r as follows:

$$\widehat{KCh}_{n,q}^{(r)}(x) = (-1)^n\int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}(-qx_1-qx_2-\cdots-qx_r+x)_n d\mu_{-1}(x_1)\cdots d\mu_{-1}(x_r), \tag{3.7}$$

where $n \geqslant 0$.

Thus, by (3.7), we get

$$\begin{aligned}\widehat{KCh}_{n,q}^{(r)}(x) &= \sum_{l=0}^{n} S_1(n,l)(-1)^n\int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}(-qx_1-qx_2-\cdots-qx_r+x)^l d\mu_{-1}(x_1)\cdots d\mu_{-1}(x_r) \\ &= \sum_{l=0}^{n} S_1(n,l)(-1)^{n+l}q^l\int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}(x_1+\cdots+x_r-\frac{x}{q})^l d\mu_{-1}(x_1)\cdots d\mu_{-1}(x_r) \\ &= \sum_{l=0}^{n} S_1(n,l)(-1)^{n+l}q^l E_l^{(r)}(-\frac{x}{q}).\end{aligned} \tag{3.8}$$

By (3.8), we obtain the following theorem.

Theorem 3.3. *For $n \geqslant 0$,*

$$\widehat{KCh}_{n,q}^{(r)}(x) = \sum_{l=0}^{n} S_1(n,l)q^l(-1)^{n+l}E_l^{(r)}(-\frac{x}{q}). \tag{3.9}$$

We observe that

$$\begin{aligned}\sum_{n=0}^{\infty} E_n^{(r)}(-x)\frac{t^n}{n!} &= \left(\frac{2}{e^t+1}\right)^r e^{-xt} \\ &= \left(\frac{2}{1+e^{-t}}\right)^r e^{-rt}e^{-xt} \\ &= \left(\frac{2}{1+e^{-t}}\right)^r e^{-(x+r)t} \\ &= \sum_{n=0}^{\infty} E_n^{(r)}(x+r)(-1)^n\frac{t^n}{n!}.\end{aligned} \tag{3.10}$$

By (3.10), we have

$$E_n^{(r)}(-x) = E_n^{(r)}(x+r)(-1)^n. \tag{3.11}$$

By (3.9) and (3.11), we obtain the following theorem.

Theorem 3.4. *For* $n \geqslant 0$,

$$\widehat{KCh}_{n,q}^{(r)}(x) = \sum_{l=0}^{n} S_1(n,l)q^l(-1)^n E_l^{(r)}\left(\frac{x}{q}+r\right).$$

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