The Journal of Mathematics and Computer Science

Available online at
http://www.TJMCS.com

Journal of Mathematics and Computer Science Vol .1 No.2 (2010) 76-79

Basic Unary Transformations and Functions operating in Fuzzy Plane

A. Taleshian[1] and S.Rezvani
Department of Mathematics, Faculty of Sciences,
Mazandaran University.P.O.Box 47416-1467,Babolsar, Iran.
taleshian@umz.ac.ir
salim_rezvani@umz.ac.ir

Received: Sep 2009, Revised: Nov 2009
Online Publication: Apr 2010

Abstract
In this paper first a series of basic transformation such integral, Rising and Falling has been defined. then the integrals have been proved. So falling and rising planes have been studied and a theorem about it has been proved. At the end, operations fuzzy time planes is shown and related proposition to it is proved.

Keywords: fuzzy plane, Y-function, operations fuzzy time planes, Extend, Shift, Exp, Integrate.

Introduction

[1] Basic transformation about fuzzy interval time has been studied. First, basic concepts in fuzzy plane time have been studied and we argue a series of operations on fuzzy time planes by using [1], [20], [4, 7 and 18].we define summary of formula of basic unary transformation such as integral, Rising and Falling. Then we continue to argue about integrals and we prove some theorems. Time planes usually don't appear from nowhere, but they are constructed from other time planes. Plane operators are more general construction functions. They take one or more fuzzy time planes and construct a new one out of them.

We distinguish two ways of constructing new fuzzy time planes, first by means of Y-functions and then by means of plane operators. Y-functions map fuzzy values to fuzzy values. They can therefore be used to construct a new plane from a given one by applying the y-function point by point to the membership function values. Plane operators are more general construction functions.

In fact, our gold for presenting of this paper is that there are fuzzy planes which can be defined 2-dimension basic transformation for them, be defined some theorems for them.

[1]Corresponding author Tel/Fax: +98-112-5342460

Basic Unary Transformations

Definition (Basic Unary Transformations**)**

Let $p \in F_{R^k}$ be a fuzzy plane. We define the following (parameterized) plane operators**:**

$$\hat{S} = \sup\big(p(x,y)\big)\ [1]$$

$$f_m = \text{first maximom}\ [1]$$

$$l_m = \text{last maximom}\ [1]$$

$$\text{identity}(p) = p$$

$$\mathrm{integrate}^{+}(\mathrm{p})(\mathrm{f}(\mathrm{x})) \stackrel{\mathrm{def}}{=} \lim_{a\to\infty}\lim_{b\to\infty}\frac{\int_{-a}^{x}\int_{-b}^{x}\mathrm{p}(\mathrm{f}(\mathrm{y}_1,\mathrm{y}_2))\ \mathrm{dy}_1\,\mathrm{dy}_2}{\int_{-a}^{+a}\int_{-b}^{+b}\mathrm{p}(\mathrm{f}(\mathrm{y}_1,\mathrm{y}_2))\ \mathrm{dy}_1\,\mathrm{dy}_2}$$

$$\mathrm{integrate}^{-}(\mathrm{p})(\mathrm{f}(\mathrm{x})) \stackrel{\mathrm{def}}{=} \lim_{a\to\infty}\lim_{b\to\infty}\frac{\int_{x}^{+a}\int_{x}^{+b}\mathrm{p}(\mathrm{f}(\mathrm{y}_1,\mathrm{y}_2))\ \mathrm{dy}_1\,\mathrm{dy}_2}{\int_{-a}^{+a}\int_{-b}^{+b}\mathrm{p}(\mathrm{f}(\mathrm{y}_1,\mathrm{y}_2))\ \mathrm{dy}_1\,\mathrm{dy}_2}$$

Integrate

This operator integrates over the membership function and normalizes the integral to values ≤ 1. The two integration operators $\mathrm{integrate}^{+}$ and $\mathrm{integrate}^{-}$ can be simplified for finite fuzzy time planes.

Proposition (Integration for Finite planes)

If the fuzzy plane p is finite then

$\mathrm{integrate}^{+}(\mathrm{p})(\mathrm{f}(\mathrm{x})) \stackrel{\mathrm{def}}{=} \frac{\int_{-a}^{x}\int_{-b}^{x}\mathrm{p}(\mathrm{f}(\mathrm{y}_1,\mathrm{y}_2))\ \mathrm{dy}_1\,\mathrm{dy}_2}{|\mathrm{p}|}$ And

$\mathrm{integrate}^{-}(\mathrm{p})(\mathrm{f}(\mathrm{x})) \stackrel{\mathrm{def}}{=} \frac{\int_{x}^{+a}\int_{x}^{+b}\mathrm{p}(\mathrm{f}(\mathrm{y}_1,\mathrm{y}_2))\ \mathrm{dy}_1\,\mathrm{dy}_2}{|\mathrm{p}|}$

The proofs are straightforward [1].

Proposition (Integration for planes with Finite Kernel)

If the infinite fuzzy plane p has a finite kernel with $\mathrm{p}_1 \stackrel{\mathrm{def}}{=} \mathrm{p}(-\infty,-\infty)$ and $\mathrm{p}_2 \stackrel{\mathrm{def}}{=} \mathrm{p}(+\infty,+\infty)$ then $\mathrm{integrate}^{+}(\mathrm{p})(\mathrm{f}(\mathrm{x})) = \frac{\mathrm{p}_1}{\mathrm{p}_1+\mathrm{p}_2}$ and $\mathrm{integrate}^{-}(\mathrm{p})(\mathrm{f}(\mathrm{x})) = \frac{\mathrm{p}_2}{\mathrm{p}_1+\mathrm{p}_2}$.

Proof: by using [2]

$$integrate^{+}(p)(f(x)) = \lim_{a\to\infty}\lim_{b\to\infty}\frac{\int_{-a}^{x}\int_{-b}^{x}p(f(y_1,y_2))\ dy_1\,dy_2}{\int_{-a}^{+a}\int_{-b}^{+b}p(f(y_1,y_2))\ dy_1\,dy_2}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{|p|_{-a}^{i^{fk}} + |p|_{i^{fk}}^{x} + |p|_{-b}^{i^{fk}} + |p|_{i^{fk}}^{x}}{|p|_{-a}^{i^{fk}} + |p|_{i^{fk}}^{i^{lk}} + |p|_{i^{lk}}^{a} + |p|_{-b}^{i^{fk}} + |p|_{i^{fk}}^{i^{lk}} + |p|_{i^{lk}}^{b}}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{|p|_{-a}^{i^{fk}} + |p|_{-b}^{i^{fk}}}{|p|_{-a}^{i^{fk}} + |p|_{i^{lk}}^{a} + |p|_{-b}^{i^{fk}} + |p|_{i^{lk}}^{b}}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{(i^{fk}+a).i_1 + (i^{fk}+b).i_1}{(i^{fk}+a).i_1 + (a-i^{lk}).i_2 + (i^{fk}+b).i_1 + (b-i^{lk}).i_2}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{a.i_1 + b.i_1}{a.i_1 + a.i_2 + b.i_1 + b.i_2}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{(a+b)i_1}{a(i_1+i_2) + b(i_1+i_2)} = \lim_{a\to\infty}\lim_{b\to\infty}\frac{(a+b)i_1}{(a+b)(i_1+i_2)} = \frac{i_1}{(i_1+i_2)}$$

$$integrate^{-}(p)(f(x)) \stackrel{\mathrm{def}}{=} \lim_{a\to\infty}\lim_{b\to\infty}\frac{\int_{x}^{+a}\int_{x}^{+b}p(f(y_1,y_2))\ dy_1\,dy_2}{\int_{-a}^{+a}\int_{-b}^{+b}p(f(y_1,y_2))\ dy_1\,dy_2}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{|p|_{x}^{i^{lk}} + |p|_{i^{lk}}^{a} + |p|_{x}^{i^{lk}} + |p|_{i^{lk}}^{b}}{|p|_{-a}^{i^{fk}} + |p|_{i^{fk}}^{i^{lk}} + |p|_{i^{lk}}^{a} + |p|_{-b}^{i^{fk}} + |p|_{i^{fk}}^{i^{lk}} + |p|_{i^{lk}}^{b}}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{|p|_{i^{lk}}^{a} + |p|_{i^{lk}}^{b}}{|p|_{-a}^{i^{fk}} + |p|_{i^{lk}}^{a} + |p|_{-b}^{i^{fk}} + |p|_{i^{lk}}^{b}}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{(a-i^{lk}).i_2 + (b-i^{lk}).i_2}{(i^{fk}+a).i_1 + (a-i^{lk}).i_2 + (i^{fk}+b).i_1 + (b-i^{lk}).i_2}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{a.i_2 + b.i_2}{a.i_1 + a.i_2 + b.i_1 + b.i_2}$$

$$= \lim_{a\to\infty}\lim_{b\to\infty}\frac{(a+b)i_2}{a(i_1+i_2) + b(i_1+i_2)} = \lim_{a\to\infty}\lim_{b\to\infty}\frac{(a+b)i_2}{(a+b)(i_1+i_2)} = \frac{i_2}{(i_1+i_2)}$$

.

Rising and Falling Fuzzy planes

Definition (Rising and Falling Fuzzy planes and plane Operators**)**

A fuzzy set p is rising $\boldsymbol{iff}$ for its membership function $p(x,y) = (1,1)$ for all $(x,y) > p^{fm}$. P is falling $\boldsymbol{iff}$ for its membership function $p(x,y) = (1,1)$ for all $(x,y) < p^{lm}$.

Proposition

The basic unary transformations $extend^{+}$and $int^{+}$are rising plane operators and the unary transformations $extend^{-}$ and int^{-} are falling plane operators.

Proof: Any composition $f_1 \circ \ldots \circ f_n \circ f$ where f is a rising (falling) plane operator is again a Rising (falling) plane operator. The proofs are straightforward [1].

Linear Y-Functions

A small, but important class of y-functions are linear y-functions. They are important firstly because very natural operators, like standard complement, intersection and union of fuzzy time planes can be described with linear y-

functions. Secondly they are important because they allow us to transform planes represented by polygons in a very efficient way: only the vertices of the polygons need to be transformed.
The main characterization of linear y-functions is therefore that they map non intersecting straight plane segments to straight plane segments.

Definition (Y-Functions)

$Y - FCT^n \stackrel{\text{def}}{=} \{f: [(0,0),(1,1)]^n \longrightarrow [(0,0),(1,1)]\}$ Is the set of n-place y-functions.
They map fuzzy values to fuzzy values.
$Y - FCT \stackrel{\text{def}}{=} \bigcup_{n\geq 1} Y - FCT^n.$

Definition (plane Operators)

$S - OP\mu^n = \{g: F_{R^k}^n \longrightarrow F_{R^k}\}$Is the set of n-place plane operators.
They map fuzzy planes to fuzzy planes.
$S - OP\mu \stackrel{\text{def}}{=} \bigcup_{n\geq 0} S - OP\mu^n$.
Every y-function can be used to construct a new fuzzy time plane from given ones by applying the y-function to the fuzzy values.

Definition (Associated plane Operators)

If $f \in Y - FCT^n$is a y-function then $g_f \in S - OP\mu^n$ defined $g_f(s_1, s_2, \dots, s_n)(x,y) = f\big(s_1(x,y), \dots, s_n(x,y)\big)$is the associated plane operator.

Definition (Linear Y-Function)

A y-function $f \in Y - FCT^n$ is linear if the mapping
$f'\Big(\big(z,(x_1,y_1)\big), \dots, \big(z,(x_n,y_n)\big)\Big) \stackrel{\text{def}}{=} \Big(z, f\big((x_1,y_1), \dots, (x_n,y_n)\big)\Big)$
Maps non-intersecting plane segments
$\big(z_1,(x_{11},y_{11})\big) - \big(z_2,(x_{12},y_{12})\big), \dots, \big(z_1,(x_{n1},y_{n1})\big) - \big(z_2,(x_{n2},y_{n2})\big)$
To a line segment
$\Big(z_1, f\big((x_{11},y_{11}), \dots, (x_{n1},y_{n1})\big)\Big) - \Big(z_2, f\big((x_{12},y_{12}), \dots, (x_{n2},y_{n2})\big)\Big).$

One-place linear y-functions can be characterized in the following way

Proposition (Characterization of One-Place Linear y-Functions)

A one-place y-function f is linear if and only if $f(x,y) = f(0,0) + \big(f(1,1) - f(0,0)\big).(x,y)$ holds.
Proof: Suppose f is linear. We take the straight plane segment between $\big((0,0),(0,0)\big)$ and $\big((1,1),(1,1)\big)$. The mapping $\mathrm{f}'\big(\mathrm{z},(\mathrm{x},\mathrm{y})\big) = \big(\mathrm{z}, \mathrm{f}(\mathrm{x},\mathrm{y})\big)$ maps this plane segment to a plane segment between $\big((0,0), f(0,0)\big)$ and $\big((1,1), f(1,1)\big)$.
Therefore

$$f(x,y) = f(0,0) + \frac{f(1,1) - f(0,0)}{(1,1)-(0,0)}.\big((x,y) - (0,0)\big) \quad \textbf{(Line equation)}$$
$$= f(0,0) + \big(f(1,1) - f(0,0)\big).(x,y)$$

Other direction: clearly.
An example for a one-place linear y-function is the standard negation
$n(x,y) = 1 - (x,y).$

The characterization of two-place linear y-functions

Proposition (Characterization of Two-Place Linear y-Functions)

A two-place y-function f is linear if and only if the following condition holds:
$f\big((x_1,y_1),(x_2,y_2)\big)$=

$$= \begin{cases} f\big((0,0),(0,0)\big) + \left(f\left(\dfrac{(x_1,y_1)}{(x_2,y_2)},(1,1)\right) - f\big((0,0),(0,0)\big)\right).(x_2,y_2) & if\ (x_1,y_1) \leq (x_2,y_2) \\ f\left((0,0), \dfrac{(x_1,y_1)-(x_2,y_2)}{(1,1)-(x_2,y_2)}\right) + \left(f(1,1) - f\left((0,0), \dfrac{(x_1,y_1)-(x_2,y_2)}{(1,1)-(x_2,y_2)}\right)\right).(x_2,y_2) & \text{otherwise} \end{cases}$$

Proof: Suppose f is linear. We consider the case $(x_1,y_1) \leq (x_2,x_2)$ first. To this end we take the straight plane segment between $\big((0,0),(0,0)\big)$ and$\big((1,1),(1,1)\big)$. The line equation for this curve is just$\|y\| = \|x\|$. Now take an arbitrary $(\mathrm{x}_2,\mathrm{y}_2) \in [(0,0),(1,1)]$ and an arbitrary $(\mathrm{x}_1,\mathrm{y}_1) \leq (\mathrm{x}_2,\mathrm{y}_2)$. The line equation for the plane segment starting At $\big((0,0),(0,0)\big)$ and crossing $\big((\mathrm{x}_2,\mathrm{y}_2),(\mathrm{x}_1,\mathrm{y}_1)\big)$ is $(\mathrm{x},\mathrm{y}) = \frac{\left((\mathrm{x}_1,\mathrm{y}_1)-(0,0)\right)}{\left((\mathrm{x}_2,\mathrm{y}_2)-(0,0)\right)}.(\mathrm{w}_1,\mathrm{w}_2)$. For $(\mathrm{w}_1,\mathrm{w}_2) = (1,1)$ We get $(z_1,z_2) = \frac{(\mathrm{x}_1,\mathrm{y}_1)}{(\mathrm{x}_2,\mathrm{y}_2)}$.
Since f is linear we have

$$f\big((x_1,y_1),(x_2,y_2)\big) = f\big((0,0),(0,0)\big) + \frac{f\big((z_1,z_2),(1,1)\big) - f\big((0,0),(0,0)\big)}{(0,0)-(1,1)}.(x_2,y_2)$$
$$= f\big((0,0),(0,0)\big) + \left(f\left(\frac{(x_1,y_1)}{(x_2,y_2)},(1,1)\right) - f\big((0,0),(0,0)\big)\right).(x_2,y_2)$$

Now consider the case$(x_1,y_1) \geq (x_2,x_2)$.

The plane starting at $((1,1),(1,1))$ and crossing $((x_2,y_2),(x_1,y_1))$ crosses the y-axis
At $(z_1,z_2)=\frac{((x_1,y_1)-(x_2,y_2))}{((1,1)-(x_2,y_2))}$.

Since f is linear we have

$$f((x_1,y_1),(x_2,y_2)) = f((0,0),(z_1,z_2)) + \frac{f((1,1),(1,1)) - f((0,0),(z_1,z_2))}{(1,1)-(0,0)}.(x_2,y_2)$$

$$= f\left((0,0),\frac{(x_1,y_1)-(x_2,y_2)}{(1,1)-(x_2,y_2)}\right) + \left(f((1,1),(1,1)) - f\left((0,0),\frac{(x_1,y_1)-(x_2,y_2)}{(1,1)-(x_2,y_2)}\right)\right).(x_2,y_2)$$

The other direction, showing that the two conditions imply linearity, is again straightforward.

Simple examples for linear two-place y-functions are the minimum and maximum function. The minimum function is used to realize standard intersection of two fuzzy time planes, and the maximum function is used to realize standard union of two fuzzy time planes.

References

[1] H.J. ohlbach - University of Munich - 31 Augast 2004.
[2] Hans Jurgen Ohlbach. Calendrical calculations with time partitionings and fuzzy time intervals. In H. J. Ohlbach and S. Schaffert, editors, Proc. of PPSWR04, number 3208 in LNCS. Springer Verlag, 2004.
[3] Hans Jurgen Ohlbach. Fuzzy time intervals and relations-the FuTIRe library. Technical report, Inst. f. Informatik, LMU Munchen, 2004. See http://www.pms.informatik.unimuenchen.de/mitarbeiter/ohlbach/systems/FuTIRe.
[4] Hans Jurgen Ohlbach. Relations between fuzzy time intervals. In C. Combi and G. Ligozat, editors, Proc. of the 11th International Symposium on Temporal Representation and Reasoning, pages 44-51, Los Alamitos, California, 2004. IEEE.
[5] Hans Jurgen Ohlbach. The role of labelled partitionings for modeling periodic temporal notions. In C. Combi and G. Ligozat, editors, Proc. of the 11th International Symposium on Temporal Representation and Reasoning, pages 60-63, Los Alamitos, California, 2004.IEEE.
[6] Franois Bry, Bernhard Lorenz, Hans Jurgen Ohlbach, and Stephanie Spranger. On reasoning on time and location on the web. In N. Henze F. Bry and J. Malusynski, editors, Principles and Practice of Semantic Web Reasoning, volume 2901 of LNCS, pages 69-83.Springer Verlag, 2003.
[7] James. F. Allen. Maintaining knowledge about temporal intervals. Communication of the Acm, 832 - 843, 1983.
[8] Fronz Baader, Diego Calvanese, Deborah Me Guinness, Daniele Nardi, and peter patel Schneider, editors. The Description logic Han dbook. Theary, Implementation and Applicanso Cambridge University press, 2003.
[9] T. Bernerz - Lee, M. Fishchetti andM.Dertouzos.Weaving the web: The original Design and Ultimate Desting of the word wid web. Harper, son Froncisco, September 1999.
[10] Diana R. Cukierman. A Formalization of structured temporal objects and Repetition. PhD thesis, siman Franser University, Vancouver, Canada, 2003.
[11] Didier Dubois and Henri prade, editors. Fundamentals of fuzzy sets. kluwer Academi publisher, 2000.
[12] Joseph o Rouke. Computational Geometry in C. Cambridge University press. 1998.
[13] Gabor Nagypal and Boris Motik. A fuzzy model for representing uncertain, subjective and vague temporal knowledge in ontologies. In Proceedings of the International Conference onOntologies, Databases and Applications of Semantics, (ODBASE), volume 2888 of LNCS.Springer-Verlag, 2003.
[14] Klaus U. Schulz and Felix Weigel. Systematic and architecture for a resource representing knowledge about named entities. In Jan Maluszynski Francois Bry, Nicola Henze, editor, Principles and Practice of SemanticWeb Reasoning, pages 189-208, Berlin, 2003. Springer-Verlag.
[15] The ACM Compating Classification System, 2001 Http: // www.acm . Ogr/class/1998/home page.hutml
[16] Nachum Dershowitz and Edward M. Reingold. Calendrical Calculations. Cambridge University Press, 1997.
[17] Hans Jurgen Ohlbach. About real time, calendar systems and temporal notions. In H. Barringer and D. Gabbay, editors, Advances in Temporal Logic, pages 319-338. Kluwer Academic Publishers, 2000.
[18] Hans Jurgen Ohlbach. Calendar logic. In I. Hodkinson D.M. Gabbay and M. Reynolds, editors, Temporal Logic: Mathematical Foundations and Computational Aspects, pages489 586. Oxford University Press, 2000.
[19] Hans Jurgen Ohlbach and Dov Gabbay. Calendar logic. Journal of Applied Non-Classical Logics, 8(4), 1998.
[20] L. A. Zadeh. Fuzzy sets. Information and Control, 8:338-353, 1965.
[21] Jacob E. Goodman and Joseph O'Rourke, editors. Handbook of Discrete and Computational Geometry. CRC Press, 1997.

(34)