Ranking decision making units by compromise programming

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Abstract
Data envelopment analysis (DEA) is a very useful management and decision tool. The ranking of decision making units (DMU) has become an important component in the decision process. In this paper we developed a new method for measuring the efficiency score of Decision-Making Units (DMUs) by using compromise programming. The proposed method calculates distance to the ideal for each DMU. The DMU with shorter distance to the ideal has better efficiency. A numerical example is provided to illustrate the application of the proposed DEA model.

Keywords: Data envelopment analysis, Ideal decision making (IDMU), Compromise programming

1. Introduction

In recent years, Data Envelopment Analysis (DEA) has become a central technique in productivity and efficiency analysis applied in different aspects of economics and management sciences.DEA is a nonparametric method of measuring the efficiency of a DMU such as a firm or a public sector agency, first introduced by Charnes, Cooper and Rhodes (CCR) in 1978 [2]. The efficiency score is measured as a ratio between a weighted sum of outputs and a weighted sum of inputs, even if the production function is unknown. The weights are chosen so as to find the best advantage for each unit to maximize its relative efficiency, under the restriction that this score is bound by unit efficiency. If a unit with its optimal weights receives the score efficiency of 1, it is efficient, and for a score smaller than 1 it is inefficient. These optimal weights differ from unit to unit. There are DEA researchers that emphasize the difficulty to rank all the units on one scale, claiming that DEA provides only a dichotomy classification into two groups: efficient and inefficient. If the number of units is small relative to the number of inputs and outputs, most of the units will be efficient.
Within DEA is a sub-group of papers in which many researchers have sought to improve the differential capabilities of DEA and to fully rank both efficient, as well as inefficient, decision-making units. The ranking methods have been divided in this paper into six, somewhat overlapping, areas. The first area involves the evaluation of a cross-efficiency matrix, in which the units are self and peer evaluated. The second idea, generally knows as the super-efficiency method, ranks through the exclusion of the unit being scored from the dual linear program and an analysis of the

change in the Pareto Frontier. The third grouping is based on benchmarking, in which a unit is highly ranked if it is chosen as a useful target for many other units. The fourth group utilizes multivariate statistical techniques, which are generally applied after the DEA dichotomic classification. The fifth research area ranks inefficient units through proportional measures of inefficiency. The last approach requires the collection of additional, preferential information from relevant decision-makers and combines multiple-criteria decision methodologies with the DEA approach. In this paper, we present a new approach for ranking DMUs in DEA. First, we calculate distance to IDMU for each DMU. Then, we order DMUs based on their distance to IDMU, smaller distance corresponding to upper ranking. The described idea in this paper is inspired of Kao [4].

The current paper is organized as follows. The next section, we give a ranking method by ideal DMU. A numerical example is provided in section 3. And the main idea of the paper is concluded in section 4.

2. Proposed method

Assume that there are n DMUs to be evaluated, each DMU with m inputs and s outputs. We denote by xij (i = 1,. . .,m) and yrj (r = 1,. . ., s) the values of inputs and outputs of DMUj (j = 1,. . .,n), which are all known and positive. An IDMU can be defined as follows:

Definition 1. [16] An IDMU is a virtual DMU, which can use the least inputs to generate the most outputs.

According to the above definition, we denote by x_i^{min} (i = 1,. . .,m) and y_r^{max} (r = 1,. . ., s) the inputs and outputs of the IDMU. They are determined by the following formulae:

$$x_i^{min} = \min_j\{x_{ij}\}, \quad i = 1, \dots, m; \qquad y_r^{max} = \max_j\{y_{rj}\}, \quad r = 1, \dots, s$$

The efficiency of any DMU is worse than that of the IDMU, no matter what weights are applied to individual input or output. Thus, we have $E_j/E^* < 1$, where the efficiency of the DMUj is E_j and the efficiency of the IDMU is E^*.

According to the implication of efficiency, the efficiency of the IDMU can be defined as

$$E^* = \frac{\sum_{r=1}^s u_r\, y_r^{max}}{\sum_{i=1}^m v_i x_i^{min}} \qquad (1)$$

In order to standardize the weights u_r and v_i, the efficiency of IDMU must be equal to 1. The difference between E_j and 1, by s_j, is the relative distance between the DMUj and the IDMU. Thus efficiency of DMUj is 1-s_j. The problem is then transformed to finding the set of weights u_r and v_i, which produce the smallest total squared difference between the efficiency of DMU and that of the IDMU. The associated model is:

$$\min \ \sum_{j=1}^n s_j^2$$

$$\text{s.t.} \ \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} + s_j = 1 \quad j = 1, \dots, n \qquad (2)$$

$$\frac{\sum_{r=1}^s u_r\, y_r^{max}}{\sum_{i=1}^m v_i x_i^{min}} = 1$$

$$v_i \geq \varepsilon, u_r \geq \varepsilon, s_j \geq 0, i = 1, \dots, m; r = 1, \dots, s$$

Using Charnes and Cooper transformation, the above fractional programming model can be solved through the following nonlinear programming model:

$$\min \ \sum_{j=1}^n s_j^2$$

$$\text{s.t.} \sum_{r=1}^{s} u_r y_{rj} + \left(s_j - 1\right) \sum_{i=1}^{m} v_i x_{ij} = 0, \quad j = 1, \dots, n \qquad (3)$$

$$\sum_{r=1}^{s} u_r \, y_r^{max} = 1$$

$$\sum_{i=1}^{m} v_i x_i^{min} = 1$$

$$v_i \geq \varepsilon, u_r \geq \varepsilon, s_j \geq 0, i = 1, \dots, m; r = 1, \dots, s$$

Note that the distance variable s_j is always positive because every DMUj is dominated by the IDMU.

The efficiency of DMUj is $1\text{-}s_j$.

In model (2), we consider a virtual DMU as IDMU and assigned the one score to it as a benchmark to this virtual DMU such that for other DMUs is got

$$\frac{\sum_{r=1}^{s} u_r \, y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \quad j = 1, \dots, n \qquad (4)$$

As a geometrical interpretation, IDMU ($\sum_{i=1}^{m} v_i x_i^{min}$, $\sum_{r=1}^{s} u_r \, y_r^{max}$) is point in R^2 and the benchmark is a straight line that passes through the origin with gradient one. Then v_i, u_r ($i = 1, \dots, m, r = 1, \dots, s$) must be determined such that the distance between these points and the benchmark line be as small as possible. In order to don't solve a fractional programming, (2) is transformed to (3). Therefore (2) and (3) are equivalent.

In model (3) if set $Y_j = \sum_{r=1}^{s} u_r \, y_{rj}$ and $X_j = \sum_{i=1}^{m} v_i x_{ij}$ is got

$$\sum_{r=1}^{s} u_r y_{rj} + \left(s_j - 1\right) \sum_{i=1}^{m} v_i x_{ij} = 0 \Rightarrow Y_j - \lambda_j X_j = 0 \quad \left(0 \leq \lambda_j \leq 1\right) j = 1, \dots, n$$

$$\Rightarrow \lambda_j = \frac{Y_j}{X_j} \quad j = 1, \dots, n \qquad (5)$$

Note that the efficiency of jth DMU is λ_j. Since λ_j is the complement of s_j, as it is clear from the first set of constraints in models (2) and (3), the DMU with the smallest distance to the ideal, s_j, obviously has the largest efficiency, λ_j. In model (3), constraints two and three are applied in order to standardize weights.

Geometrically, in model (3), v_i, u_r ($i = 1, \dots, m, r = 1, \dots, s$) must be determined such that slop of lines $Y_j - \lambda_j X_j = 0$ ($j = 1, \dots, n$) be nearest value to 1.

3. Numerical example

In this section, we use set of DMUs shown in Table 1 [6] to make an experiment to compare the calculation results of the proposed ranking model with AP method [1], MAJ model [5], Revised MAJ model [12], SA-DEA model [13], L1 norm model [7], Tchebycheff norm model [8], CSW model [3], Gradient line model [9], Advantage model [10], SBM model [14,15] and Mont carlo model [11]. The results are shown in Table 2. The non-Archimedean infinitesimal was set as $\varepsilon = 10^{-9}$.

Table 1. Data [6]

Units	I1	I2	I3	I4	I5	O1	O2
DMU1	583	8	2.75	16.731	17129300	285	0.8848
DMU2	741	8	2	18.999	8903705	95	0.8597
DMU3	600	7	2.75	19.437	15864760	307	0.9226
DMU4	593	8	2.75	19.326	14802089	260	0.8928
DMU5	746	7	2	20.125	8398300	154	0.812
DMU6	992	9	2.75	21.821	19330020	254	0.8624
DMU7	775	8	2.75	13.333	17182320	292	0.9109
DMU8	1852	14	3.25	21.696	30126900	473	0.8632
DMU9	625	5	2	16.285	7638220	106	0.8898
DMU10	673	6	2	16.789	8659940	148	0.8668
DMU11	423	6	2	13.304	10799980	151	0.9435
DMU12	1292	18	3.25	18.333	47102720	782	0.9571
DMU13	1300	8	2.75	17.730	17451040	288	0.8996
DMU14	582	8	2.75	19.178	15850628	260	0.9054
DMU15	620	8	2	16.056	7938560	124	0.8744
DMU16	1256	10	2.75	21.516	23034560	378	0.8465
DMU17	765	10	2.75	19.145	15692740	303	0.8945
DMU18	842	7	2.75	16.972	8029240	153	0.9074
DMU19	1011	4	2.75	17.692	7702609	57	0.8764
DMU20	1128	9	2.75	21.927	22143650	357	0.9028
DMU21	3456	18	3.5	20.217	24892550	393	0.9195

| DMU22 | 1008 | 3 | 2.25 | 10.213 | 7405200 | 36 | 0.8611 |
| DMU23 | 910 | 4 | 2.25 | 12.941 | 8839280 | 72 | 0.7735 |

Table 2. Ranking of efficient units and compare proposed method with other methods.

Model	D3	D4	D7	D9	D10	D11	D12	D15	D18	D22
AP	4	8	7	5	9	2	1	10	6	3
MAJ	4	6	5	7	9	3	1	10	8	2
Revised MAJ	4	8	6	5	9	3	1	10	7	2
SA-DEA	4	8	7	5	9	2	1	10	6	3
L1 norm	2	4	3	8	7	10	1	6	5	9
Tchebycheff norm	4	7	5	9	8	2	1	10	6	3
CSW	3	5	8	11	6	2	1	9	4	22
Gradient line	4	8	7	5	9	3	1	10	6	2
Advantage	13	15	4	5	8	3	2	7	6	1
SBM	4	8	6	5	9	3	1	10	7	2
Mont carlo	5	3	1	6	10	2	4	9	8	7
Proposed method	8	9	7	3	5	1	2	4	10	6

Note: the data with gray background color, the unit is infeasible in that model.

4. Conclusions

This paper present a new model for ranking DMUs based on compromise programming. In this model ideal DMU is defined. Then, DMUs are ranked based on distance to ideal DMU, smaller distance corresponding upper rank. The example given in this paper illustrates the advantages, potential and application of the proposed DEA models. Comparing with the exiting approaches, it is reasonable and efficient. Additionally, the proposed approach can provide decision makers with a new alternative to rank DMUs.

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