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Novel insights on distributed delayed flexible impulsive control for mixed delay complex-valued neural networks

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Abstract

This research investigates novel synchronization criteria for uncertain mixed-delay complex-valued neural networks (CVNNs) using distributed delayed flexible impulsive control (DDFIC). To cope with the mixed delays, we offer a novel distributed delayed flexible impulsive differential inequality that incorporates the average impulsive distributed delay and average impulsive interval. In addition, new synchronization requirements for linear matrix inequalities (LMIs) are developed for the proposed CVNNs using the Lyapunov function and Jensen's inequality. The DDFIC gains are also determined by solving the LMIs. Lastly, we include simulation examples to exemplify the proposed criteria, and to illustrate the effectiveness of the DDFIC through diagrammatic representations.

Keywords: Mixed delay, distributed delayed flexible impulsive control, complex-valued neural networks, exponential synchronization.

2020 MSC: 34K41, 93C27, 93D23.

1. Introduction

Neural networks (NNs) have gained extensive acknowledgment from the global scientific community due to its substantial applications in diverse domains including image processing, computing parallelism, pattern recognition, signal processing, and associative memory. CVNNs possess more sophisticated attributes due to its complex-valued states, weights of connections, output values, and activation functions. Furthermore, CVNNs could solve various problems that real-valued NNs cannot, including the symmetry detection problem, so highlighting their improved computational capacity and performance [4, 5, 9, 13].

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1.1. Background

The chaos in networks is marked by their unpredictable and highly sensitive behavior, where small changes in initial conditions can produce dramatically different outcomes. Certain parameters and time delays in CVNNs can create chaos, causing the system to behave in a complicated and unpredictable way, making synchronization a challenging task [7, 8, 30, 40, 46]. Synchronization is essential for neural networks, as numerous CVNNs depend on synchronous behavior to function efficiently. As a consequence, the scientific community has given considerable attention to research on the synchronization of delayed CVNNs [27, 56].

To establish an accurate and obvious synchronization criteria for CVNNs, proper control mechanisms should be chosen. In general, various popular control strategies are employed to synchronize two related or dissimilar chaotic systems, including adaptive control [1, 21, 50], pinning control [45, 51], coupling control [22], feedback control [2, 34], sample-data control [19], impulsive control [23, 38], event-triggered control [3, 12, 57], and so on. Because of its desirable properties, like simplicity, effectiveness, and flexibility, the impulsive control technique has been thoroughly and widely analyzed by various experts from various domains. For instance, in [11], the authors conducted an investigation into the fixed-time synchronization results for delayed CVNNs under impulsive control, as well as its application in image encryption. In [37], the authors analyzed stability criteria for discrete-time distributed delayed CVNNs based on impulsive effects. In [16], the authors analyzed synchronization criteria for delayed CVNNs based on hybrid impulses effects. The synchronization results for delayed CVNNs systems with impulsive control were discussed in [6, 25, 33], while exponential synchronization (ES) of delayed memristor-based CVNNs under impulsive control was investigated in [20].

On the other hand, time delays in impulses are unavoidable, as evidenced by the NNs' dependency on both the past and present states, as well as the restricted sampling speed and communication of impulsive information. Basically, delayed impulses are of two kinds: synchronizing impulses and de-synchronizing impulses [28, 48, 59]. The majority of current efforts on the control problem of impulsive dynamical NNs solely consider how impulses rely on the current positions of the NNs. Nonetheless, there are numerous real-world uses for delayed impulses, such as in financial systems, communication security, and population dynamics. Consequently, when investigating impulsive systems, delayed impulses should be considered. For example, the authors of [15, 35, 47, 55] analyzed the exponential stability (ES) of delayed chaotic NNs and CVNNs based on delayed impulsive control. In [17, 36, 39, 52, 58], the authors analyzed ES exponential stability for mixed delay CVNNs based on delayed impulses. However, the impulsive delays considered in the previous studies are assumed to occur at fixed times. In [24, 32, 41–44], the authors examined the ES results of delayed NNs at random time points.

1.2. Motivation and the key contributions

The delayed impulsive control mentioned above involves specific past states, but it can also happen as distributed ones. Distributed delayed impulsive control is yet another kind of delayed impulsive control, could be used to synchronize the NNs effectively. Few studies have used distributed delayed impulsive control, such as [14, 29, 49], which examined ES and lag synchronization of delayed NNs using this method. In the preceding discussions, the distributed time delays in impulses are regarded as fixed and have restricted upper bounds. However, these distributed time delays may also be flexible. In [18, 31], the authors examined synchronization results for delayed real-valued and complex-valued NNs using flexible impulsive control and delay-dependent flexible impulsive control, respectively, without accounting for distributed delayed impulses. In this research, we concentrate on the distributed delayed impulsive control in which the distributed delay is taken as flexible. To the best of the authors' knowledge, the distributed delayed flexible impulsive control (DDFIC) has not been examined for the proposed networks. Consequently, this study addresses these research gaps and summarizes its major achievements as follows.

(1) Considering mixed-delay CVNNs in which mixed delay includes bounded transmission delay, discrete distributed delay, and flexible impulsive delay.

(2) Developing a new delay differential inequality based on DDFIC with average impulsive distributed delay (AIDD) and average impulsive interval (AII) concepts.
(3) Deriving new LMI-based criteria for robust exponential synchronization (RES) and ES for the proposed unstable chaotic CVNNs, utilizing a Lyapunov function approach combined with Jensen's inequality, and obtaining the corresponding controller gains by solving the resulting LMIs.
(4) Examining a distributed delay in impulsive control, which is not necessarily dependent upon the transmission delays inherent in the network. Further, rendering the impulsive distributed delay flexible with a substantial upper bound.

Taking the aforementioned primary distinctions into account, it is crucial to note that this work differs from prior research [10, 14, 18, 26, 29, 31, 39, 49, 53, 54] and presents novel findings by using distinct perspectives.

1.3. Structure of the paper

The following is an overview of this paper. The preliminary knowledge, which encompasses the model outline, lemmas, and definitions, is outlined in Section 2. The primary findings are delineated in Section 3. The criteria that were obtained are illustrated in Section 4 through visual representations and numerical depictions. Finally, the conclusion is presented in Section 5.

1.4. Notations

Let $\mathbb{C}$, $\mathbb{C}^n$, $\mathbb{C}^{n\times n}$, $\mathbb{R}^{n\times n}$ indicate sets of complex numbers, complex vectors in n-dimension, $n\times n$ complex-valued, and real matrices, respectively, $\mathbb{Z}^+$ is a set of positive integers, and $\mathbb{R}^+ = [0, +\infty)$. For any vector $w \in \mathbb{C}^n$, we define the norm $\|w\| = \sqrt{\sum_{i=1}^{n}|w_i|^2}$. For any $A \in \mathbb{C}^{n\times n}$, we have $\|A\| = \max_{1\leqslant j\leqslant n}\sum_{i=1}^{n}|a_{ij}|$ and for $\mathbf{q_1}$, $\mathbf{q_2} \in \mathbb{C}$, $\mathbf{q_1} \leqslant \mathbf{q_2}$ if and only if $\mathrm{Re}(\mathbf{q_1}) \leqslant \mathrm{Re}(\mathbf{q_2})$ and $\mathrm{Im}(\mathbf{q_1}) \leqslant \mathrm{Im}(\mathbf{q_2})$, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ are maximum and minimum eigen values of a hermitian matrix P, respectively, $C([-\rho,0],\mathbb{C}^n)$ shows set of all continuous complex-valued functions, $\omega(s) : [-\rho, 0] \to \mathbb{C}^n$, then define the norm $\|\omega(s)\| = \sup_{s\in[-\rho,0]}\sqrt{\sum_{j=1}^{n}|\omega_j(s)|^2}$.

2. Model description and preliminary concepts

Consider the following uncertain mixed delayed CVNNs as the master system

$$\begin{cases} \dot{\mathbf{p}}(t) = -(A+\Delta A)\mathbf{p}(t) + (B+\Delta B)f(\mathbf{p}(t)) + (C+\Delta C)f(\mathbf{p}(t-\tau(t))) \\ \qquad\qquad + (D+\Delta D)\displaystyle\int_{t-\mu(t)}^{t} f(\mathbf{p}(s))ds + J_e, \quad t>0, \\ \mathbf{p}(t) = \boldsymbol{\phi}(t), \quad \forall t \in [-\rho_1, 0], \end{cases} \tag{2.1}$$

where, neuron state vector $\mathbf{p}(t) = [\mathbf{p_1}(t), \mathbf{p_2}(t), \ldots, \mathbf{p_n}(t)]^T \in \mathbb{C}^n$, $A = \mathrm{diag}\{a_1, a_2, \ldots, a_n\}; a_i > 0 \in \mathbb{R}^{n\times n}$, $B, C, D \in \mathbb{C}^{n\times n}$, activation function f; $f(\mathbf{p}(\cdot)) = [f_1(\mathbf{p}(\cdot)), f_2(\mathbf{p}(\cdot)), \ldots, f_n(\mathbf{p}(\cdot))]^T \in \mathbb{C}^n$, $\tau(t)$, $\mu(t)$ are transmission delays, the initial condition $\boldsymbol{\phi}(t) = [\boldsymbol{\phi_1}(t), \boldsymbol{\phi_2}(t), \ldots, \boldsymbol{\phi_n}(t)]^T \in C([-\rho_1, 0], \mathbb{C}^n)$, $\rho_1 = \max\{\tau, \mu\}$, and external input $J_e = [J_1, J_2, \ldots, J_n] \in \mathbb{C}^n$. The corresponding slave system takes the form of

$$\begin{cases} \dot{\mathbf{u}}(t) = -(A+\Delta A)\mathbf{u}(t) + (B+\Delta B)f(\mathbf{u}(t)) + (C+\Delta C)f(\mathbf{u}(t-\tau(t))) \\ \qquad\qquad + (D+\Delta D)\displaystyle\int_{t-\mu(t)}^{t} f(\mathbf{u}(s))\,ds + U(t) + J_e, \quad t>0, \\ \mathbf{u}(t) = \boldsymbol{\chi}(t), \quad \forall t \in [-\rho, 0], \end{cases} \tag{2.2}$$

where, $\mathbf{u}(t) = [\mathbf{u_1}(t), \mathbf{u_2}(t), \ldots, \mathbf{u_n}(t)]^T \in \mathbb{C}^n$, $f(\mathbf{u}(\cdot)) = [f_1(\mathbf{u}(\cdot)), f_2(\mathbf{u}(\cdot)), \ldots, f_n(\mathbf{u}(\cdot))]^T \in \mathbb{C}^n$, $U(t)$ is the impulsive control, and initial condition $\boldsymbol{\chi}(t) = [\boldsymbol{\chi_1}(t), \boldsymbol{\chi_2}(t), \ldots, \boldsymbol{\chi_n}(t)]^T \in C([-\rho, 0], \mathbb{C}^n)$; $\rho = \max\{\rho_1, \varepsilon\}$.

Define the error function $\boldsymbol{w}(t) = \mathbf{u}(t) - \mathbf{p}(t)$, then error CVNNs between slave CVNNs (2.2) and master CVNNs (2.1) is expressed as

$$\begin{cases} \dot{\boldsymbol{w}}(t) = -(A+\Delta A)\boldsymbol{w}(t) + (B+\Delta B)h(\boldsymbol{w}(t)) + (C+\Delta C)h(\boldsymbol{w}(t-\tau(t))) \\ \qquad\qquad + (D+\Delta D)\displaystyle\int_{t-\mu(t)}^{t} h(\boldsymbol{w}(s))\,ds + U(t), \quad t>0, \\ \boldsymbol{w}(t) = \boldsymbol{\psi}(t), \quad \forall t \in [-\rho, 0], \end{cases} \tag{2.3}$$

with $h(\boldsymbol{w}(\cdot)) = (h_1(\boldsymbol{w}(\cdot)), h_2(\boldsymbol{w}(\cdot)), \dots, h_n(\boldsymbol{w}(\cdot)))^T \in \mathbb{C}^n$, $h(\boldsymbol{w}(\cdot)) = f(\boldsymbol{w}(\cdot) + \mathbf{p}(\cdot)) - f(\mathbf{p}(\cdot))$, and $\boldsymbol{\psi}(t) = [\boldsymbol{\chi}(t) - \boldsymbol{\phi}(t)] \in C([-\rho, 0], \mathbb{C}^n)$. The proposed DDFIC is structured as

$$U(t) = \sum_{k=1}^{\infty} (C_k \int_{t-\tau_k}^{t} \boldsymbol{w}(s)ds - \boldsymbol{w}(t_k^-))\ \delta(t-t_k),$$

with the impulsive strength $C_k \in \mathbb{C}^{n\times n}$ at impulsive instant t_k, impulsive instant sequence $\{t_k\}_{k\in\mathbb{Z}^+}$; $0 = t_0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < \cdots$, with $\lim_{k\to+\infty} t_k = +\infty$, impulsive distributed delay τ_k; $0 < \tau_k \leqslant \varepsilon$, ε is any positive constant, Dirac function $\delta(t)$, and let $\boldsymbol{w}(t_k^+) = \boldsymbol{w}(t_k)$. Further, the solutions to CVNNs (2.3) are assumed to be piecewise right continuous at $t = t_k$ and exhibit first-kind discontinuities at $t = t_k$. For $t \neq t_k$, $U(t) = 0$. As $t = t_k$, we obtain, $U(t) = C_k \int_{t_k-\tau_k}^{t_k} \boldsymbol{w}(s)ds$. Thus, the error CVNNs (2.3) becomes

$$\begin{cases} \dot{\boldsymbol{w}}(t) = -(A+\Delta A)\boldsymbol{w}(t) + (B+\Delta B)h(\boldsymbol{w}(t)) + (C+\Delta C)h(\boldsymbol{w}(t-\tau(t))) \\ \qquad\qquad + (D+\Delta D)\displaystyle\int_{t-\mu(t)}^{t} h(\boldsymbol{w}(s))\,ds, \quad t \neq t_k,\ t>0, \\ \boldsymbol{w}(t_k) = C_k \displaystyle\int_{t_k-\tau_k}^{t_k} \boldsymbol{w}(s)\,ds, \quad t = t_k,\ k \in \mathbb{Z}^+, \\ \boldsymbol{w}(t) = \boldsymbol{\psi}(t), \quad \forall t \in [-\rho, 0]. \end{cases} \tag{2.4}$$

Assume that solution $\boldsymbol{w}(t)$ of CVNNs (2.4) exists and is unique.

Throughout this work, we make use of the following hypotheses

(H1) For uncertain matrices $\Delta A, \Delta B, \Delta C, \Delta D \in \mathbb{C}^{n\times n}$, there exists constants $a, b, c, d > 0$, that satisfies

$$\|\Delta A\| \leqslant a, \quad \|\Delta B\| \leqslant b, \quad \|\Delta C\| \leqslant c, \quad \|\Delta D\| \leqslant d.$$

(H2) For any $\boldsymbol{w}_1(\cdot), \boldsymbol{w}_2(\cdot) \in \mathbb{C}$, activation functions $h(\boldsymbol{w}_j(\cdot)); j = 1, 2$, satisfy

$$|h(\boldsymbol{w}_1(\cdot)) - h(\boldsymbol{w}_2(\cdot))| \leqslant L\,|\boldsymbol{w}_1(\cdot) - \boldsymbol{w}_2(\cdot)|,$$

where $L > 0$ is a diagonal matrix.

(H3) The transmission delays $\tau(t)$, $\mu(t)$ and impulsive delay τ_k satisfy

$$0 \leqslant \tau(t) \leqslant \tau,\ 0 \leqslant \tau < \infty,\ \ 0 \leqslant \mu(t) \leqslant \mu,\ 0 \leqslant \mu < \infty,\ \ \tau_0 = 0,\ \tau_k \leqslant t_k - t_{k-1}.$$

Lemma 2.1 ([31]). *For any matrices $G, \Delta H \in \mathbb{C}^{n\times n}$, $\|\Delta H\| \leqslant \nu$; $\nu > 0$, $\mathbf{k_1}$, $\mathbf{k_2} \in \mathbb{C}^n$, and for any $\epsilon > 0$, there exists hermitian matrix $W > 0$ that satisfies*

1. $\mathbf{k}_1^*\mathbf{k}_2 + \mathbf{k}_2^*\mathbf{k}_1 \leqslant \mathbf{k}_1^* W \mathbf{k}_1 + \mathbf{k}_2^* W^{-1} \mathbf{k}_2$;
2. $\pm 2\mathbf{k}_1^* G^*(\Delta H)\mathbf{k}_2 \leqslant \epsilon \mathbf{k}_1^* G^* G \mathbf{k}_1 + \epsilon^{-1}\nu^2 \mathbf{k}_2^* \mathbf{k}_2$.

Lemma 2.2 ([39]). *For all vectors $\mathbf{g} : [m_1, m_2] \to \mathbb{C}^n$ with scalars $m_1 < m_2$, positive hermitian matrix $P \in \mathbb{C}^{n\times n}$, such that*

$$\left(\int_{m_1}^{m_2} \mathbf{g}(s)\,ds\right)^* P \left(\int_{m_1}^{m_2} \mathbf{g}(s)\,ds\right) \leqslant (m_2 - m_1) \int_{m_1}^{m_2} \mathbf{g}^*(s)\,P\,\mathbf{g}(s)\,ds.$$

Definition 2.3 ([15])**.** The AIDD $\bar{\tau}$ and AII T_a of impulsive distributed delay sequence $\{\tau_k\}$ and impulsive instant sequence $\{t_k\}$, respectively, are described by:

$$\bar{\tau} = \lim_{t\to+\infty} \inf \frac{\tau_1 + \tau_2 + \cdots + \tau_{N(t,t_0)}}{N(t,t_0)}, \qquad T_a = \lim_{t\to+\infty} \sup \frac{t-t_0}{N(t,t_0)},$$

in which, $N(t,t_0)$ is the number of impulsive instants in $\{t_k\}_{k\in\mathbb{Z}^+}$ on $(t_0,t]$.

Lemma 2.4. *Suppose* $r(\cdot) \in PC([t_0-\rho,+\infty)\times\mathbb{C},\mathbb{R}^+)$ *and if there exist* $\alpha\in\mathbb{R}$, $\eta\in\mathbb{R}^+$, $\gamma\in(0,1)$ *that satisfy following inequality:*

$$\dot{r}(t) \leqslant \alpha r(t) + \eta \sup_{s\in[t-\rho_1,t]} r(s),\ t\neq t_k,\ t>t_0,$$
$$r(t_k) \leqslant \gamma\int_{t_k-\tau_k}^{t_k} r(s)ds,\ t=t_k, \quad r(t_0)=r(s),\ s\in[t_0-\rho,t_0].$$

Moreover, we define

$$Q(t) = \begin{cases} r(t)e^{-\lambda(t-t_k)}, & t\in[t_k,t_{k+1}),\ k\in\mathbb{Z}^+,\\ r(t), & t\in[t_0-\rho,t_0],\end{cases} \tag{2.5}$$

such that $\lambda > \alpha+\eta > 0$, *and* $\alpha + \frac{\eta}{\gamma}e^{-\lambda\rho} - \lambda < 0$. *Then the condition*

$$r(t) \leqslant \bar{r}(t_0)\gamma^k e^{-\lambda\sum_{j=1}^{k}\tau_j} e^{\lambda(t-t_0)},\ t\in[t_k,t_{k+1}),\ k\in\mathbb{Z}^+, \tag{2.6}$$

is satisfied with $\bar{r}(t_0) = \sup_{s\in[t_0-\rho,t_0]} r(s)$.

Proof. First, we prove $D^+Q(t) < 0$, for any $t^*\in[t_k,t_{k+1})$, which satisfies

$$Q(s) \leqslant Q(t^*),\ t_k\leqslant s\leqslant t^*, \tag{2.7}$$
$$Q(s) \leqslant \frac{Q(t^*)}{\gamma},\ t_{k-1}\leqslant s\leqslant t_k. \tag{2.8}$$

To prove the above results, construct the function, for any $\xi>0$,

$$Q_\xi(t) = \begin{cases} r(t)e^{-(\lambda+\xi)(t-t_k)}, & t\in[t_k,t_{k+1}),\\ r(t), & t\in[t_0-\rho,t_0].\end{cases}$$

Let $\delta\in[0,\rho]$, and if $t^*-\delta\geqslant t_k$, utilizing (2.7),

$$\begin{aligned} e^{\xi(t^*-t_k)}D^+Q_\xi(t^*) &= D^+\left(r(t^*)e^{-(\lambda+\xi)(t^*-t_k)}\right)\\ &\leqslant (\alpha-\lambda-\xi)r(t^*)e^{-\lambda(t^*-t_k)} + \eta r(t^*-\delta)e^{-\lambda(t^*-t_k)}\\ &\leqslant (\alpha-\lambda-\xi)Q(t^*) + \eta Q(t^*-\rho)e^{\lambda(t^*-\rho-t_k)}e^{-\lambda(t^*-t_k)}\\ &\leqslant (\alpha-\lambda-\xi)Q(t^*) + \eta Q(t^*-\rho)e^{-\lambda\rho} \leqslant (\alpha-\lambda-\xi+\eta e^{-\lambda\rho})Q(t^*) < -\xi Q(t^*).\end{aligned}$$

Besides, if $t^*-\delta<t_k$, by using (2.8),

$$\begin{aligned} e^{\xi(t^*-t_{k-1})}D^+Q_\xi(t^*) &\leqslant (\alpha-\lambda-\xi)Q(t^*) + \eta Q(t^*-\rho)e^{\lambda(t^*-\rho-t_{k-1})}e^{-\lambda(t^*-t_{k-1})}\\ &= (\alpha-\lambda-\xi+\frac{\eta}{\gamma}e^{-\lambda\rho})Q(t^*) < -\xi Q(t^*).\end{aligned}$$

Hence, based on the above conclusions, we can notice

$$D^+Q(t) = e^{\xi(t^*-t_k)}D^+Q_\xi(t^*) + \xi e^{\xi(t^*-t_k)}Q_\xi(t^*) < -\xi Q(t^*) + \xi e^{\xi(t^*-t_k)}Q_\xi(t^*) = 0.$$

Following this, we will show that

$$r(t) \leqslant \bar{r}(t_0)\gamma^k e^{-\lambda\sum_{j=1}^k \tau_j} e^{\lambda(t-t_0)},\ t \in [t_k, t_{k+1}). \tag{2.9}$$

Using (2.5), proof of (2.9) is transformed into the proof of

$$Q(t) \leqslant \bar{r}(t_0)\gamma^k e^{-\lambda\sum_{j=1}^k \tau_j} e^{\lambda(t_k-t_0)},\ t \in [t_k, t_{k+1}). \tag{2.10}$$

Firstly, we prove (2.10) valid for $k = 0$, that is, $Q(t) \leqslant \bar{r}(t_0),\ t \in [t_0, t_1)$. Note that $Q(t_0) = r(t_0) \leqslant \bar{r}(t_0)$. Assume that this is not valid, thus is a $t^* \in [t_0, t_1)$ with

$$Q(t^*) = \bar{r}(t_0),\ \ Q(t) \leqslant \bar{r}(t_0),\ t \in [t_0, t^*),\ \ D^+Q(t^*) < 0,$$

which leads to contradiction that $D^+Q(t^*) \geqslant 0$. Thus, for $k = 0$, (2.10) holds. Next suppose (2.10) valid for $k \leqslant n$, and using mathematical induction method,

$$Q(t) \leqslant \bar{r}(t_0)\gamma^k e^{-\lambda\sum_{j=1}^k \tau_j} e^{\lambda(t_k-t_0)},\ \forall t \in [t_k, t_{k+1}),$$

which gives

$$Q(t) \leqslant \bar{r}(t_0)\gamma^n e^{-\lambda\sum_{j=1}^n \tau_j} e^{\lambda(t_n-t_0)},\ t \in [t_n, t_{n+1}).$$

Next, we will prove

$$Q(t) \leqslant \bar{r}(t_0)\gamma^{n+1} e^{-\lambda\sum_{j=1}^{n+1} \tau_j} e^{\lambda(t_{n+1}-t_0)},\ \forall t \in [t_{n+1}, t_{n+2}). \tag{2.11}$$

We obtain,

$$\begin{aligned} Q(t_{n+1}) &\leqslant \gamma \int_{t_{n+1}-\tau_{n+1}}^{t_{n+1}} \bar{r}(t_0)\gamma^n e^{-\lambda\sum_{j=1}^n \tau_j} e^{\lambda(s-t_0)} ds \\ &\leqslant \gamma^{n+1}\bar{r}(t_0) e^{-\lambda\sum_{j=1}^n \tau_j} \int_{t_{n+1}-\tau_{n+1}}^{t_{n+1}} e^{\lambda(s-t_0)} ds \leqslant \gamma^{n+1}\bar{r}(t_0) e^{-\lambda\sum_{j=1}^{n+1} \tau_j} e^{\lambda(t_{n+1}-t_0)}. \end{aligned}$$

So, for $t = t_{n+1}$, (2.11) holds. Now assume there is a $t^* \in [t_{n+1}, t_{n+2})$ with

$$Q(t^*) = \bar{r}(t_0)\gamma^{n+1} e^{-\lambda\sum_{j=1}^{n+1} \tau_j} e^{\lambda(t_{n+1}-t_0)},\ Q(t) \leqslant Q(t^*),\ t \in [t_{n+1}, t^*),$$

and $D^+Q(t^*) \geqslant 0$. And while $s \in [t_n, t_{n+1})$, and $\tau_{n+1} \leqslant (t_{n+1} - t_n)$, we have

$$\begin{aligned} Q(s) &\leqslant \gamma^n \bar{r}(t_0) e^{-\lambda\sum_{j=1}^n \tau_j} e^{\lambda(t_n-t_0)} \\ &= \frac{\gamma^{n+1}}{\gamma}\bar{r}(t_0) e^{-\lambda\sum_{j=1}^n \tau_j} e^{\lambda(t_{n+1}-t_0)} e^{-\lambda(t_{n+1}-t_n)} \\ &= \frac{\gamma^{n+1}}{\gamma}\bar{r}(t_0) e^{-\lambda\sum_{j=1}^{n+1} \tau_j} e^{\lambda\tau_{n+1}} e^{\lambda(t_{n+1}-t_0)} e^{-\lambda(t_{n+1}-t_n)} \\ &\leqslant \frac{Q(t^*)}{\gamma} e^{-\lambda(t_{n+1}-\tau_{n+1}-t_n)} \leqslant \frac{Q(t^*)}{\gamma},\ s \in [t_n, t_{n+1}), \end{aligned}$$

which gives $D^+Q(t^*) < 0$ that contradicts $D^+Q(t^*) \geqslant 0$. Hence, we have deduced

$$Q(t) \leqslant \gamma^k \bar{r}(t_0) e^{-\lambda\sum_{j=1}^k \tau_j} e^{\lambda(t_k-t_0)},\ \ t \in [t_k, t_{k+1}),$$

showing that (2.9) is valid. □

Remark 2.5. In Lemma 2.4, the requirement (2.6) indicates that the distributed delay τ_k is crucial for enabling the synchronization of the proposed system. Current literature [10, 14, 26, 29, 39, 49, 53, 54] imposes the constraint that $t_k - t_{k-1} \leqslant \sigma$; $\sigma > 0$, and the impulsive distributed delay must be contingent upon the time delay present within the networks. However, in our findings, we have mitigated these constraints by employing distributed delay τ_k in conjunction with AII and AIDD, hence enhancing flexibility.

Definition 2.6. The solution $\boldsymbol{w}(t)$ of CVNNs (2.4) is said to achieve RES, if there exists $M > 0$, $\vartheta > 0$ with

$$\|\boldsymbol{w}(t)\| \leqslant M \sup_{s\in[t_0-\rho,t_0]} \|\boldsymbol{w}(s)\| e^{-\vartheta(t-t_0)}.$$

Specifically, when $\Delta A = \Delta B = \Delta C = \Delta D = 0$, the CVNNs (2.4) becomes ES.

3. Synchronization criteria for mixed delay CVNNs

Within this section, some novel sufficient synchronization criteria for uncertain mixed delayed CVNNs (2.4) is presented associated with DDFIC, formulating by LMIs.

Theorem 3.1. *Suppose* (H2) *holds. Let* $\alpha \in \mathbb{R}$, $q, \gamma \in (0, 1)$, $\eta \in \mathbb{R}^+$, *and positive constants* ϵ_1, ϵ_2, ϵ_3, ϵ_4, a, b, c, d, μ, $\bar{\tau}$, T_a. *If there exists positive diagonal matrices* $W_1, W_2, W_3 \in \mathbb{R}^{n\times n}$, *positive matrix* $Q \in \mathbb{C}^{n\times n}$, *positive hermitian matrix* P, *with* $\lambda^* = \max\left\{\alpha + \frac{\eta}{\gamma}e^{-\lambda\rho}, 0\right\}$, *and the following inequalities*

$$\begin{bmatrix} -AP-PA+a^2\epsilon_1 I+b^2\epsilon_2L^2+LW_1L-\alpha P & P & P & P & P & PB & PC & PD \\ \star & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -\epsilon_2 I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\epsilon_3 I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\epsilon_4 I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -W_1 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -W_2 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -W_3 \end{bmatrix} < 0, \tag{3.1}$$

$$\begin{bmatrix} c^2\epsilon_3L^2 - q\eta P & LW_2 \\ \star & -W_2 \end{bmatrix} < 0, \tag{3.2}$$

$$\begin{bmatrix} d^2\epsilon_4L^2 - (1-q)\eta P & \mu LW_3 \\ \star & -W_3 \end{bmatrix} < 0, \tag{3.3}$$

$$\begin{bmatrix} -\gamma P & Q \\ \star & -P \end{bmatrix} < 0, \tag{3.4}$$

$$\frac{\ln\gamma + \lambda^*(T_a - \bar{\tau})}{T_a} < 0, \tag{3.5}$$

are satisfied, then the uncertain CVNNs (2.4) *is said to achieve RES with DDFIC gain* $C_k = P^{-1}Q^*$.

Proof. Define the Lyapunov function be $r(t, \boldsymbol{w}(t)) = r(t)$, and $r(t) = \boldsymbol{w}^*(t)P\boldsymbol{w}(t)$. Calculating derivative along solution of CVNNs (2.4), we get

$$\begin{aligned} \dot{r}(t) = 2\boldsymbol{w}^*(t)P\dot{\boldsymbol{w}}(t) = 2\boldsymbol{w}^*(t)P\Big[& -(A+\Delta A)\boldsymbol{w}(t) + (B+\Delta B)h(\boldsymbol{w}(t)) \\ & + (C+\Delta C)h(\boldsymbol{w}(t-\tau(t))) + (D+\Delta D)\int_{t-\mu(t)}^{t} h(\boldsymbol{w}(s))ds\Big]. \end{aligned} \tag{3.6}$$

Utilizing hypothesis (H2) and Lemmas 2.1 and 2.2, then (3.6) becomes

$$\dot{r}(t) \leqslant \boldsymbol{w}^*(t)(-AP-PA)\boldsymbol{w}(t) + \epsilon_1^{-1}\boldsymbol{w}^*(t)P^2\boldsymbol{w}(t) + a^2\epsilon_1\boldsymbol{w}^*(t)\boldsymbol{w}(t) + \boldsymbol{w}^*(t)LW_1L\boldsymbol{w}(t)$$

$$
\begin{aligned}
&+ \boldsymbol{w}^*(t)PBW_1^{-1}B^*P\boldsymbol{w}(t) + \epsilon_2^{-1}\boldsymbol{w}^*(t)P^2\boldsymbol{w}(t) + b^2\epsilon_2\boldsymbol{w}^*(t)L^2\boldsymbol{w}(t)\\
&+ \boldsymbol{w}^*(t-\tau(t))LW_2L\boldsymbol{w}(t-\tau(t)) + \boldsymbol{w}^*(t)PCW_2^{-1}C^*P\boldsymbol{w}(t) + \epsilon_3^{-1}\boldsymbol{w}^*(t)P^2\boldsymbol{w}(t)\\
&+ c^2\epsilon_3\boldsymbol{w}^*(t-\tau(t))L^2\boldsymbol{w}(t-\tau(t)) + \mu(t)\int_{t-\mu(t)}^{t}\boldsymbol{w}^*(s)LW_3L\boldsymbol{w}(s)ds + \boldsymbol{w}^*(t)PDW_3^{-1}D^*P\boldsymbol{w}(t)\\
&+ \epsilon_4^{-1}\boldsymbol{w}^*(t)P^2\boldsymbol{w}(t) + d^2\epsilon_4\mu(t)\int_{t-\mu(t)}^{t}\boldsymbol{w}^*(s)L^2\boldsymbol{w}(s)ds\\
\leqslant\, &\boldsymbol{w}^*(t)\Big(-AP - PA + (\epsilon_1^{-1}+\epsilon_2^{-1}+\epsilon_3^{-1}+\epsilon_4^{-1})P^2 + a^2\epsilon_1 + LW_1L + PBW_1^{-1}B^*P\\
&+ b^2\epsilon_2L^2 + PCW_2^{-1}C^*P + PDW_3^{-1}D^*P\Big)\boldsymbol{w}(t) + \boldsymbol{w}^*(t-\tau(t))(LW_2L + c^2\epsilon_3L^2)\boldsymbol{w}(t-\tau(t))\\
&+ \mu(t)\int_{t-\mu(t)}^{t}\boldsymbol{w}^*(s)(LW_3L + d^2\epsilon_4L^2)\boldsymbol{w}(s)ds.
\end{aligned}
$$

By using (3.1), we get

$$
\begin{aligned}
\dot{r}(t) &\leqslant \boldsymbol{w}^*(t)(\alpha P)\boldsymbol{w}(t) + \boldsymbol{w}^*(t-\tau(t))(LW_2L + c^2\epsilon_3L^2)\boldsymbol{w}(t-\tau(t))\\
&\quad + \mu(t)\max_{t-\mu\leqslant s\leqslant t}\boldsymbol{w}^*(s)(LW_3L + d^2\epsilon_4L^2)\boldsymbol{w}(s)\int_{t-\mu(t)}^{t}ds\\
&= \alpha\boldsymbol{w}^*(t)P\boldsymbol{w}(t) + \boldsymbol{w}^*(t-\tau(t))(LW_2L + c^2\epsilon_3L^2)\boldsymbol{w}(t-\tau(t))\\
&\quad + \mu^2(t)\max_{t-\mu\leqslant s\leqslant t}\boldsymbol{w}^*(s)(LW_3L + d^2\epsilon_4L^2)\boldsymbol{w}(s)\\
&\leqslant \alpha\boldsymbol{w}^*(t)P\boldsymbol{w}(t) + \max_{t-\tau\leqslant s\leqslant t}\boldsymbol{w}^*(s)(LW_2L + c^2\epsilon_3L^2)\boldsymbol{w}(s) + \mu^2\max_{t-\mu\leqslant s\leqslant t}\boldsymbol{w}^*(s)(LW_3L + d^2\epsilon_4L^2)\boldsymbol{w}(s)\\
&\leqslant \alpha\boldsymbol{w}^*(t)P\boldsymbol{w}(t) + \max_{t-\rho_1\leqslant s\leqslant t}\boldsymbol{w}^*(s)(LW_2L + c^2\epsilon_3L^2)\boldsymbol{w}(s) + \mu^2\max_{t-\rho_1\leqslant s\leqslant t}\boldsymbol{w}^*(s)(LW_3L + d^2\epsilon_4L^2)\boldsymbol{w}(s).
\end{aligned}
$$

Using (3.2) and (3.3), then

$$
\begin{aligned}
\dot{r}(t) &\leqslant \alpha\boldsymbol{w}^*(t)P\boldsymbol{w}(t) + q\eta\max_{t-\rho_1\leqslant s\leqslant t}\boldsymbol{w}^*(s)P\boldsymbol{w}(s) + (1-q)\eta\max_{t-\rho_1\leqslant s\leqslant t}\boldsymbol{w}^*(s)P\boldsymbol{w}(s)\\
&\leqslant \alpha\boldsymbol{w}^*(t)P\boldsymbol{w}(t) + \eta\max_{t-\rho_1\leqslant s\leqslant t}\boldsymbol{w}^*(s)P\boldsymbol{w}(s) \leqslant \alpha r(t) + \eta\max_{s\in[t-\rho_1,t]}r(s).
\end{aligned}
\tag{3.7}
$$

If $t = t_k$, we get

$$
\begin{aligned}
r(t_k) = \boldsymbol{w}^*(t_k)P\boldsymbol{w}(t_k) &\leqslant [C_k\int_{t_k-\tau_k}^{t_k}\boldsymbol{w}(s)ds]^*P[C_k\int_{t_k-\tau_k}^{t_k}\boldsymbol{w}(s)ds]\\
&= \int_{t_k-\tau_k}^{t_k}\boldsymbol{w}^*(s)ds(C_k^*PC_k)\int_{t_k-\tau_k}^{t_k}\boldsymbol{w}(s)ds.
\end{aligned}
$$

Employing (3.4), we obtain

$$
r(t_k) \leqslant \int_{t_k-\tau_k}^{t_k}\boldsymbol{w}^*(s)ds(\gamma P)\int_{t_k-\tau_k}^{t_k}\boldsymbol{w}(s)ds = \gamma\int_{t_k-\tau_k}^{t_k}r(s)ds. \tag{3.8}
$$

Combining (3.7) and (3.8), and employing Lemma 2.4, we obtain

$$
r(t) \leqslant \bar{r}(t_0)\gamma^k e^{-\lambda\sum_{j=1}^{k}\tau_j}e^{\lambda(t-t_0)},
$$

with $\bar{r}(t_0) = \sup_{s\in[t_0-\rho,t_0]}r(s)$. Moreover $\{t_k\}, \{\tau_k\} \in N(t,t_0)$, and if $N(t,t_0)$ impulses be affected on $(t_0,t]$, thus above inequality transforms to

$$
r(t) \leqslant \bar{r}(t_0)\gamma^{N(t,t_0)}e^{-\lambda\sum_{j=1}^{N(t,t_0)}\tau_j}e^{\lambda(t-t_0)} = \bar{r}(t_0)e^{[(\frac{\frac{N(t,t_0)\ln\gamma}{N(t,t_0)} - \frac{\lambda\sum_{j=1}^{N(t,t_0)}\tau_j}{N(t,t_0)}}{\frac{t-t_0}{N(t,t_0)}}) + \lambda](t-t_0)}.
$$

Using Definition 2.3, and supposing $t \to +\infty$, we get

$$r(t) \leqslant \bar{r}(t_0)e^{[\frac{\ln\gamma-\lambda\bar{\tau}}{T_a}+\lambda](t-t_0)} = \bar{r}(t_0)e^{[\frac{\ln\gamma+\lambda(T_a-\bar{\tau})}{T_a}](t-t_0)}.$$

As $w(t)$ holds $\lambda_{\min}(P)\|w(t)\| \leqslant r(t) \leqslant \lambda_{\max}(P)\|w(t)\|$, it follows that $\lambda_{\min}(P)\|w(t)\| \leqslant r(t)$, and

$$\bar{r}(t_0) \leqslant \lambda_{\max}(P) \sup_{s\in[t_0-\rho,t_0]} \|w(s)\|.$$

Then,

$$\begin{aligned}\lambda_{\min}(P)\|w(t)\| &\leqslant \lambda_{\max}(P) \sup_{s\in[t_0-\rho,t_0]} \|w(s)\|e^{(\frac{\ln\gamma+\lambda^*(T_a-\bar{\tau})}{T_a})(t-t_0)},\\ \|w(t)\| &\leqslant \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \sup_{s\in[t_0-\rho,t_0]} \|w(s)\|e^{\frac{\ln\gamma+\lambda^*(T_a-\bar{\tau})}{T_a}(t-t_0)} \leqslant M \sup_{s\in[t_0-\rho,t_0]} \|w(s)\|e^{\kappa(t-t_0)},\end{aligned}$$

where $M = \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} > 0$, and $\kappa = \frac{\ln\gamma+\lambda^*(T_a-\bar{\tau})}{T_a} < 0$; $\lambda^* = \max\{\alpha + \frac{\eta}{\gamma}e^{-\lambda\rho}, 0\}$. Then, $\|w(t)\|$ tends to 0, as $t \to +\infty$. Thus, the uncertain mixed delay CVNNs (2.4) is RES. □

Corollary 3.2. *Under* (H2), *let* $\alpha \in \mathbb{R}$, $\eta \in \mathbb{R}^+$, μ, T_a, $\bar{\tau} > 0$, *and* $q, \gamma \in (0, 1)$. *If there exists positive diagonal matrices* $W_1, W_2, W_3 \in \mathbb{R}^{n\times n}$, *positive matrix* $Q \in \mathbb{C}^{n\times n}$, *positive hermitian matrix* P, *with* $\lambda^* = \max\left\{\alpha + \frac{\eta}{\gamma}e^{-\lambda\rho}, 0\right\}$, *and the following inequalities:*

$$\begin{bmatrix} -AP-PA+LW_1L-\alpha P & PB & PC & PD\\ \star & -W_1 & 0 & 0\\ \star & \star & -W_2 & 0\\ \star & \star & \star & -W_3\end{bmatrix} < 0, \tag{3.9}$$

$$\begin{bmatrix} -q\eta P & LW_2\\ \star & -W_2\end{bmatrix} < 0, \tag{3.10}$$

$$\begin{bmatrix} -(1-q)\eta P & \mu LW_3\\ \star & -W_3\end{bmatrix} < 0, \tag{3.11}$$

$$\begin{bmatrix} -\gamma P & Q\\ \star & -P\end{bmatrix} < 0, \tag{3.12}$$

$$\frac{\ln\gamma+\lambda^*(T_a-\bar{\tau})}{T_a} < 0, \tag{3.13}$$

are satisfied, then the CVNNs (2.4) *achieves ES with DDFIC gain* $C_k = P^{-1}Q^*$.

Proof. Same as proof of Theorem 3.1, under the condition $\Delta A = \Delta B = \Delta C = \Delta D = 0$. □

Corollary 3.3. *Assume* (H2) *holds and let* $\alpha \in \mathbb{R}$, $\gamma \in (0, 1)$, *and* $\eta \in \mathbb{R}^+$, $\bar{\tau}$, $T_a > 0$. *If there exist positive matrix* $Q \in \mathbb{C}^{n\times n}$, W_1, $W_2 \in \mathbb{R}^{n\times n}$ *are positive diagonal matrices, positive hermitian matrix* P, *with* $\lambda^* = \max\left\{\alpha + \frac{\eta}{\gamma}e^{-\lambda\rho}, 0\right\}$, *and the following inequalities:*

$$\begin{bmatrix} -AP-PA+LW_1L-\alpha P & PB & PC\\ \star & -W_1 & 0\\ \star & \star & -W_2\end{bmatrix} < 0, \qquad \begin{bmatrix} -\eta P & LW_2\\ \star & -W_2\end{bmatrix} < 0,$$

$$\begin{bmatrix} -\gamma P & Q\\ \star & -P\end{bmatrix} < 0, \qquad \frac{\ln\gamma+\lambda^*(T_a-\bar{\tau})}{T_a} < 0,$$

are fulfilled, then the CVNNs (2.4) *achieves ES with DDFIC gain* $C_k = P^{-1}Q^*$.

Proof. Same as proof of Corollary 3.2, when $\int_{t-\mu(t)}^{t} h(w(s))ds = 0$. □

Remark 3.4. In [17, 35, 36, 39, 52, 55], the criteria for stability and synchronization were discussed of CVNNs including both transmission delays and delayed impulsive effects, but these impulsive delays were considered as fixed times. In [14, 49], synchronization results were investigated for delayed NNs under distributed delayed impulsive control, but impulsive delays were taken as fixed along with the restriction of smaller upper bounds. However, in our studies, we relaxed those restrictions that made impulsive distributed delay as flexible and has large upper bounds. Further, we assert that we considered the comprehensive model of CVNNs with uncertainties, transmission delay, discrete distributed delay, and DDFIC. Thus, our proposed results are different, new and general from these existing literature [14, 29, 49].

4. Numerical examples

The following section offers a discussion of numerical examples and its graphical depictions, aiming at demonstrating the efficacy of DDFIC in achieving proposed criteria.

Example 4.1. Consider the 2-neuron uncertain master CVNNs (2.1) and the slave CVNNs (2.2) with following parameters

$$A = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 2+2i & 0.1-0.1i \\ 4-4i & 3+3i \end{bmatrix}, \quad C = \begin{bmatrix} 1.5-3i & -0.1-0.1i \\ -0.1-0.1i & -10.5-i \end{bmatrix},$$
$$D = \begin{bmatrix} -1.5-1.5i & 5+5i \\ 0.1+0.1i & -1.5-1.5i \end{bmatrix}, \quad \Delta A = \begin{bmatrix} 0.1+0.1i & 0 \\ 0 & 0.1+0.1i \end{bmatrix}, \quad \Delta B = \begin{bmatrix} 0.08+0.1i & 0 \\ 0 & 0.08+0.1i \end{bmatrix},$$
$$\Delta C = \begin{bmatrix} 0.2+0.03i & 0 \\ 0 & 0.2+0.03i \end{bmatrix}, \quad \Delta D = \begin{bmatrix} 0.25+0.05i & 0 \\ 0 & 0.25+0.05i \end{bmatrix}, \quad h(\cdot) = \tanh(\cdot),$$
$$\tau(t) = \frac{e^t}{e^t+1}, \quad \tau_k = 1.99 + \frac{1}{100k}, \quad \mu(t) = 0.5|\sin(t)|,$$
$$L = \mathrm{diag}\{0.1, 0.1\}, \quad J_e = (0,0)^T.$$

Then $\tau = 0.25$, $\mu = 0.5$, and so $\rho = 2$. Let $a = 0.15$, $b = 0.13$, $c = 0.3$, $d = 0.35$, $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0.1$, and with initial conditions for master CVNNs (2.1) are $\mathbf{p_1}(t) = -1+3i$, $\mathbf{p_2}(t) = 2-4i$, $\forall t \in [-0.5, 0]$, and for slave CVNNs (2.2) is taken as $\mathbf{u_1}(t) = 2+i$, $\mathbf{u_2}(t) = 3-3.5i$, $\forall t \in [-2, 0]$. The behavior of real and imaginary parts of trajectories for CVNNs (2.4) absence of control is given by Figure 1 (A) and (B). To synchronize the proposed CVNNs, let $\alpha = 0.16$, $\eta = 0.26$, $\gamma = 0.01$, $q = 0.9$, $\lambda = 3$, $\bar{\tau} = 2$, $T_a = 3$, and $\lambda^* = 0.224$.

Employing LMI toolbox, we got the feasible solution matrices for the LMIs (3.1), (3.2), (3.3), (3.4), and (3.5) are

$$P = \begin{bmatrix} 0.1864 & 0.0215-0.0029i \\ 0.0215+0.0029i & 0.2153 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.0057+0.0056i & 0.0061+0.0059i \\ 0.0060+0.0061i & 0.0064+0.0064i \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 19.5322 & 0 \\ 0 & 12.5002 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 2.7001 & 0 \\ 0 & 4.4428 \end{bmatrix}, \quad W_3 = \begin{bmatrix} 1.0579 & 0 \\ 0 & 1.6263 \end{bmatrix},$$

and the DDFIC gain is

$$C_k = \begin{bmatrix} 0.0279-0.0270i & 0.0293-0.0294i \\ 0.0253-0.0252i & 0.0265-0.0274i \end{bmatrix}.$$

Under the given initial conditions, real part and imaginary part of trajectories of CVNNs (2.4) based on DDFIC is depicted by Figure 1 (C) and (D). Hence, the CVNNs (2.4) achieves RES, by Theorem 3.1. Next, we validate the results to synchronize the mixed delay CVNNs (2.1) and CVNNs (2.2) with $\Delta A = \Delta B =$

$\Delta C = \Delta D = 0$. Consider the other parametric values same as of Example 4.1. The behavior of real part and imaginary part of paths for mixed delayed CVNNs (2.4) in the absence of the effect of control are given by Figure 2 (A) and (B).

Employing LMI toolbox, the feasible solutions obtained for LMIs (3.9), (3.10), (3.11), (3.12), and (3.13) of Corollary 3.2 and there exist the matrices

$$P = \begin{bmatrix} 0.8380 & 0.1436 - 0.0722i \\ 0.1436 + 0.0722i & 0.7443 \end{bmatrix}, \qquad Q = \begin{bmatrix} 0.0316 + 0.0313i & 0.0321 + 0.0277i \\ 0.0267 + 0.0312i & 0.0272 + 0.0275i \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 8.6623 & 0 \\ 0 & 6.5479 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 4.0693 & 0 \\ 0 & 9.0208 \end{bmatrix}, \quad W_3 = \begin{bmatrix} 2.8757 & 0 \\ 0 & 4.5438 \end{bmatrix},$$

and the DDFIC gain is

$$C_k = \begin{bmatrix} 0.0350 - 0.0284i & 0.0300 - 0.0289i \\ 0.0336 - 0.0352i & 0.0280 - 0.0343i \end{bmatrix}.$$

According to the provided initial conditions, real part and imaginary part of trajectories of CVNNs (2.4) based on DDFIC are given by Figure 2 (C) and (D). Hence, CVNNs (2.4) achieves ES, by Corollary 3.2.

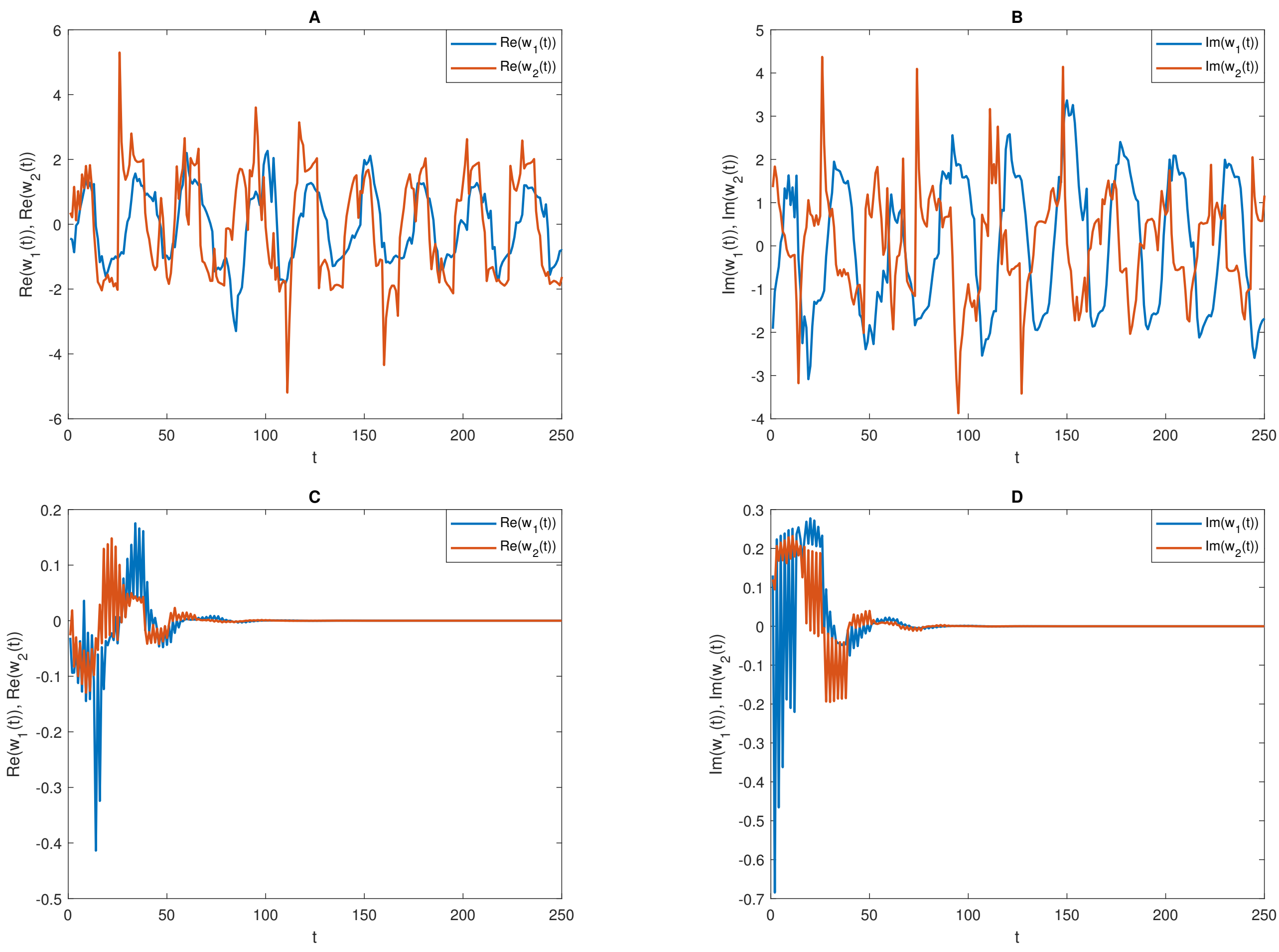

Figure 1: Real and imaginary parts of paths to CVNNs (2.4) with uncertainties, transmission delay and discrete distributed delay; (A) and (B) show without impulses; (C) and (D) prove the efficiency of DDFIC.

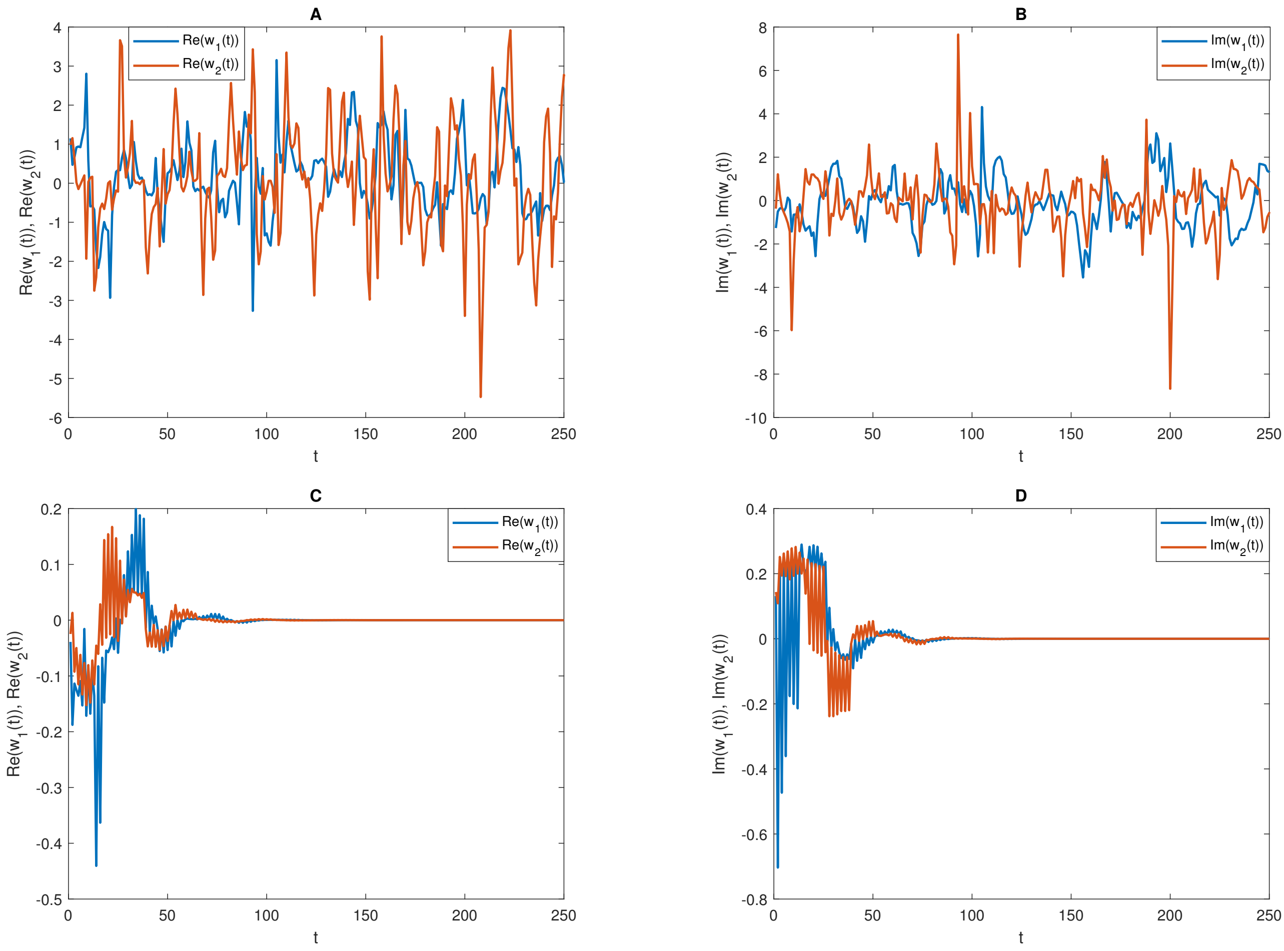

Figure 2: Real and imaginary parts of paths to CVNNs (2.4) in absence of uncertainties; (A) and (B) show without impulsive effect; (C) and (D) prove the efficiency of DDFIC.

5. Conclusion

This paper addresses the RES and ES criteria for uncertain mixed delayed CVNNs with DDFIC. Before introducing the main findings, we presented the novel distributed delayed flexible impulsive differential inequality employing the ideas of AII and AIDD. Subsequently, we developed innovative RES and ES criteria utilizing LMIs by the use of Lyapunov functions and Jensen's inequality for the suggested uncertain mixed delayed CVNNs. Moreover, DDFIC gains were formulated by LMI solutions. Finally, some numerical illustrations were presented to substantiate the obtained results, and the efficiency of DDFIC is depicted with diagrammatic representations.

It is important to note that the findings that were reported in this study are original and completely unique, as noted in both Remarks 2.5 and 3.4. This research has a positive outlook for the future, which is a very promising factor. Under the effect of adaptive impulsive control, our primary target for the foreseeable future is to explore the behavior of CVNNs that have unbounded transmission delay and unlimited distribution delay settings. This will include both the transmission delay and the distribution delay.

Authors' contributions

The authors declare that the study was realized in collaboration with equal responsibility. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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