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Formation control of second-order nonlinear multi-agent systems under sensor and actuator attacks using adaptive neural network

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Abstract

In this paper, we present an adaptive leader-follower formation control strategy using neural networks (NN) for a category of second-order nonlinear Multi-Agent Systems with unmodeled dynamics, this approach tackles the complexities arising from sensor and actuator disruptions. Second-order formation control involves the simultaneous regulation of both the positions and velocities of multiple agents in a formation, which inherently introduces additional complexity compared to first-order control. The primary objective is to achieve asymptotic consensus among control system while significantly reducing inter-agent communication and ensuring security against sensor and actuator attacks. The suggested control technique achieves the necessary leader-follower formation through the utilization of Lyapunov stability analysis to ensure that all system errors stay contained inside ultimate boundedness that is semi-global and uniform even under difficult circumstances. Finally, numerical simulations further validate the robustness and effectiveness of the control framework, demonstrating successful formation control despite sensor and actuator attacks.

Keywords: Adaptive neural network, multi-Agent System (MAS), Lyapunov stability, sensor and actuator attack, framework for leader-follower, second-order nonlinear dynamics.

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1. Preliminaries

Formation control plays a vital role in the coordinated management of multi-agent systems (MAS), largely due to its extensive range of real-world applications [1, 10, 17]. MAS comprises multiple intelligent agents that interact to create sophisticated group behaviors by sharing information within their local networks. A key focus in formation control is developing robust control protocols that enable agents to

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reach and maintain specified geometric configurations necessary for accomplishing specific tasks. Among the diverse approaches to formation control in (MAS), the leader-follower strategy [4] has emerged as a widely favored method. This approach is valued for its straightforward design, dependable performance, and ease of scalability making it a popular and effective choice for researchers and practitioners. More recently, neighbor-based formation control methods [7] have gained traction. These approaches enhance efficiency by requiring each agent to communicate with only a few nearby agents, reducing overall communication needs while maintaining effective coordination. In recent years, considerable research has focused on identifying and addressing the vulnerabilities of networked multi-agent systems in the presence of diverse cyber threats. These threats include denial-of-service (DoS) attacks [9], which aim to overwhelm the system to hinder normal operations; replay attacks, where attackers maliciously resend authentic messages; false-data injection (FDI) attacks [14], designed to introduce deceptive information; and camouflage attacks, which mask malicious actions as standard operational behavior. Among these, sensor and actuator attacks [19], stand out as a particularly subtle and dangerous type of threat. Numerous fractional-order (non-integer order) systems still exist in nature and engineering applications [3, 18], primarily due to the inherent material properties of physical systems and other complex characteristics [5, 15]. Examples include bacteria and other microorganisms in aquatic ecosystems, as well as unmanned aerial vehicles operating in environments with high spatial complexity.

Malicious cyber-physical assaults pose significant obstacles to MAS [6], jeopardizing system dependability. Senor and physical assaults are especially pernicious because they enable adversaries to introduce hidden flaws, using agent cooperation and trust to manipulate the system's operation. The challenge is made worse by DoS [8], a problem in real-world MAS because they prevent the exact sharing of information needed for coordinated control. A backstepping-based approach incorporates a single parameter learning method for simplified design, leveraging neural networks to address the unknown dynamics in MAS while ensuring resilience to sensor attacks and actuator faults [12]. Agents exhibit behavior governed by non-linear functions, which are either well-defined or satisfy a Lipschitz-type condition to ensure specific smoothness characteristics. Recently, there has been significant progress in developing adaptive consensus methods for non-linear systems, utilizing neural networks (NN) [13] to harness their capacity for managing complex, uncertain interactions. These advanced approaches aim to help agents reach consensus or shared objectives, even amidst the challenges of non-linear dynamics.

Taking distributed cyber-attack detection methods into consideration, this study addresses these issues by proposing a robust adaptive formation control [11] approach for non-linear multi-agent systems having double-integral unknowing dynamics. Utilizing neural networks (NNs) versatility, the approach successfully tackles the unidentified dynamics while integrating a strong framework to manage time delays and lessen the impact of vicious attackers. A numerical simulation demonstrates the suggested method's capacity to sustain robust formation control in the face of deception assaults and actuator faults based on single parameter.

To address the cyber attacks complex-valued neural networks (CVNNs) using an event-triggered (ET) approach is used to Secure Communication Protocols [16], ensure that the information shared between agents remains accurate and trustworthy, thereby preventing malicious agents from introducing deceptive data into the system. Additionally, Robust Control Techniques are applied to create control laws capable of maintaining system stability despite the presence of compromised agents. By incorporating these approaches into the adaptive formation control method, the system becomes more resilient to cyber-attacks, while sustaining its performance and stability. This research [2] investigates a leader-follower MAS subjected to communication failures due to sensor and actuator attack. To address these challenges, introduce a robust consensus protocol using neural network. The proposed controller is designed to mitigate the effects of sensor and actuator attacks, enabling the followers to effectively track the virtual leader's behavior.

This study presents a novel adaptive formation control technique that combines targeted methods with neural network-based approximation to handle cyber-attacks. The approach's efficacy in accomplishing intended formation objectives despite these obstacles is demonstrated through numerical simulations.

The following are this study's main contributions.

1) We propose a formation control framework for a second-order nonlinear multi-agent system, with stability validated through the Lyapunov stability theorem. This approach is designed to maintain system stability even when faced with sensor and actuator attacks.
2) The core methodology uses adaptive neural network approximation to accurately compensate for unknown nonlinear dynamics. This technique mitigates uncertainties that often affect the stability of multi-agent systems, thus improving their reliability and overall performance.

The content is structured as follows. Section 1 presents the research background and key challenges tackled in this work. Section 2 outlines fundamental concepts, covering neural networks, graph theory, and essential lemmas. Section 3 describes the system model, the design of the control protocol, and supporting theoretical proofs. Section 4 offers a simulation example to demonstrate the effectiveness of the proposed approach. Section 5 offers a concise overview of the key findings and proposes potential avenues for future exploration. Figure 1 illustrates the detailed content architecture, detailing its structure and components.

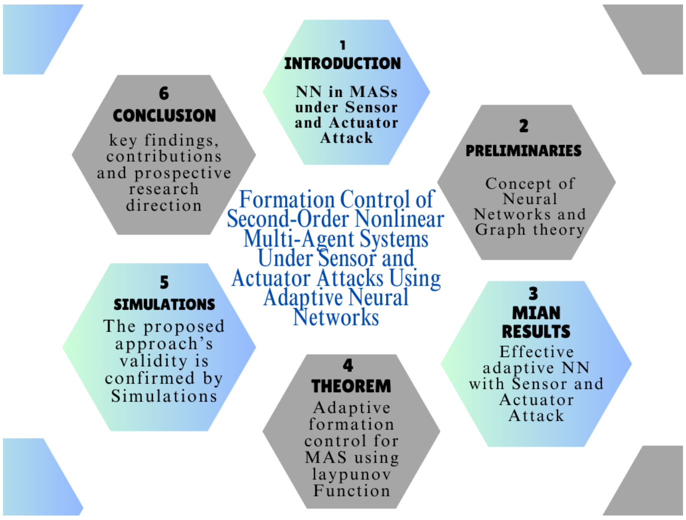


Figure 1: Procedural flow and system workflow diagram.

2. Fundamental concepts

2.1. Neural network

It has been shown that neural networks (NNs) can estimate universal functions. Providing a continuous function $\phi(p) : \mathbb{R}^m \times \mathbb{R}^n$, the function over a compact set $\vee$ can be approximated by NN in the following form:

$$\hat{\phi}(p) = \mathbf{G}^T Q(p),$$

where the weight matrix with neuron number c is $\mathbf{G} \in \mathbb{R}^{c\times n}$ and $Q(\mathbf{p}) = \left[q_1 p, \dots, q_{cp}\right]^T$ is the vector of basis function $q_{\mu=1}, \dots, c(p) = E^{-\frac{(p-\nu_\alpha)^T(\theta-\nu_\alpha)}{2}}$, $\nu_\alpha = \left[\nu_{\alpha 1}, \dots, \nu_{\alpha m}\right]^T$ the center of receptive field. The ideal neural weight $\mathbf{G}^* \in \mathbb{R}^{c\times n}$ for the continuous function $\phi(p)$ is defined as

$$G^* := \arg \min_{G\in\mathbb{R}^{c\times b}} \left\{ \sup_{p\in\vee} \|\phi(p) - G^T Q(p)\| \right\}$$

in order to rewrite $\phi(p)$ as

$$\phi(\theta) = {G^*}^T Q(p) + \varepsilon(p),$$

where the approximation error is represented by $\varepsilon(p) \in \mathbb{R}^n$ and there exists a constant r that is positive so $\|\varepsilon(p)\| \leqslant r$.

The optimal NN weight G^* aims to guarantee the smallest possible difference between $G^TQ(p)$ and $\phi(p)$. However, it is merely an "artificial" quantity for a survey, it is not feasible to utilize it as a basis for developing the control scheme directly. Instead, the actual control is generally formulated using estimates obtained through adaptive tuning.

2.2. Graph theory

The interconnected graph considered in this research pertains to the studied multi-agent framework is an undirected connected graph $E = (O, D, \zeta)$, where $D = \{1, 2, \dots, m\}$ is the label set of all nodes, $\zeta \in \mathbb{D} \times \mathbb{D}$ is the edge set, and $O = [o_{\alpha\beta}] \in \mathbb{R}^{m\times m}$ is the adjacency matrix, with an entry $o_{\alpha\beta} \geqslant 0$ indicating the interaction weight shared between agents α and β. The neighbor of node α is considered to be node β if the edge $\chi_{\alpha\beta}$ holds $\chi_{\alpha\beta} = (\alpha, \beta) \in \zeta$ the adjacency element $o_{\alpha\beta} = 1$ and $\lambda\alpha = \{\beta \mid (\alpha, \beta) \in \zeta\}$ indicates the neighbor label set. Assuming $\chi_{\alpha\beta} \notin \zeta$, then $o_{\alpha\beta} = 0$ the graph E can be referred to undirected graph whether matrix B adjacency elements meet the requirement $o_{\alpha\beta} = o_{\alpha\beta}$, $\alpha, \beta = 1, \dots, m$ implying that $\chi_{\alpha\beta} \in \zeta \iff \chi_{\alpha\beta} \in \zeta$. When there is an undirected path $(\alpha, \alpha_1), \dots, (\alpha_\alpha, \beta)$, for any two distinguishable nodes α and β, consequently, the graph without a direction E is regarded as linked. Laplacian matrix by which graph E is connected is,

$$\Upsilon = \operatorname{diag}\left(\sum_{\beta=1}^{m} o_{1\beta}, \dots, \sum_{\beta=1}^{m} o_{n\beta}\right) - O,$$

$\mathbb{S} = \operatorname{diag}\{s_1, \dots, s_m\}$ is the matrix that describes the communication weights across agents and the leader. $s_\alpha = 1$ if the agent α is able to link with the leader, $s_\alpha = 0$ otherwise. While $s_\alpha + \cdots + s_m \geqslant 1$, the leader must be associated with a minimum of one agent.

2.3. Auxiliary lemmas

Lemma 2.1. *The irreducibility of the Laplacian matrix of an undirected graph Z is an essential and adequate requirement for its connectivity.*

Lemma 2.2. *Consider an irreducible matrix $\Upsilon = [\ell_{\alpha\beta}] \in \mathbb{R}^{m\times m}$ whereby $\ell_{\alpha\beta} = \ell_{\alpha\beta} \leqslant 0$ and $\ell_{\alpha\alpha} = -\sum_{\beta=1}^{m} \ell_{\alpha\beta}$. Then, all of the eigenvalues of $\tilde{\Upsilon} = \begin{bmatrix} \ell_{11}+s_1 & \cdots & \ell_{1m} \\ \vdots & \ddots & \vdots \\ \ell_{m1} & \cdots & \ell_{mm}+s_m \end{bmatrix}$ are positive, where $s_1, \dots, s_m$ are non negative constants stipulated that $s_1 + \cdots + s_m > 0$.*

Lemma 2.3. *When $\mathbb{C}_1(p) = \mathbb{C}_1^T(p)$ and $\mathbb{C}_2(p) = \mathbb{C}_2^T(p)$, the matrix inequality that*

$$\begin{bmatrix} \mathbb{C}_1(p) & \mathbb{C}_3(p) \\ \mathbb{C}_3^T(p) & \mathbb{C}_2(p) \end{bmatrix} > 0,$$

applies to any of the two inequalities listed below, at least one of the conditions must be met.

1) $\mathbb{C}_1(p) > 0$, $\mathbb{C}_2(p) - \mathbb{C}_3^T(p)\mathbb{C}_1^{-1}(p)\mathbb{C}_3(p) > 0$;
2) $\mathbb{C}_2(p) > 0$, $\mathbb{C}_1(p) - \mathbb{C}_3(p)\mathbb{C}_2^{-1}(p)\mathbb{C}_3^T(p) > 0$.

Lemma 2.4. *The initial condition of the continuous function $\mathcal{M}(t) \geqslant 0$ is bounded. The following inequalities can be preserved when this condition occurs $\dot{\mathcal{M}}(t) \leqslant -g\mathcal{M}(t) + s$, whereas g and s are two positive constants are present.*

$$\mathcal{M}(t) \leqslant \mathcal{M}(0)\epsilon^{-gt} + \frac{s}{g}(1 - \epsilon^{-gt}).$$

3. Important results

3.1. Formulation of the problem

An overview of this double integral dynamics model of a non-linear multi-agent system that includes m agents,

$$\frac{d}{dt}p_\alpha(t) = q_\alpha(t), \quad \frac{d}{dt}q_\alpha(t) = h_\alpha + \delta^a_\alpha \Delta h_\alpha + \phi_\alpha(p_\alpha + \delta^s_\alpha \Delta p^\alpha, q_\alpha + \delta^s_\alpha \Delta q^\alpha), \quad \alpha = 1, \dots, m. \tag{3.1}$$

The position state is described as $p_\alpha(t) = \left[p_{\alpha 1}, \dots, p_{\alpha n}\right]^T \in \mathbb{R}^n$ while the velocity state is $q_\alpha(t) = \left[q_{\alpha 1}, \dots, q_{\alpha n}\right]^T \in \mathbb{R}^n$. We assume that the attacker has access to the agent communication channels, and it can change the actuator and sensor channels data of the agents. The system output is transmitted to the controller through the sensor channel. An attacker can compromise this channel by injecting false data, effectively altering the sensor readings. Assuming the system state is measurable through the sensor, the attack on the sensor channel can be mathematically modeled as

$$\tilde{p}_\alpha = p_\alpha + \delta^s_\alpha \Delta p_\alpha, \quad \tilde{q}_\alpha = q_\alpha + \delta^s_\alpha \Delta q_\alpha,$$

where $\tilde{p}_\alpha$ and $\tilde{q}_\alpha$ are indicating the corrupted sensor data of position and velocity, respectively. δ^s_α is a binary variable indicating the presence of an attack on sensor channel ($\delta^s_\alpha = 1$ when there is an attack on sensor channel $\delta^s_\alpha = 0$ otherwise). Δp_α and Δq_α denote attack signals injected into the position and velocity data, respectively. Each agent transmits its control input to the plant via the actuator channel. However, this channel is susceptible to attacks, where an adversary can manipulate the transmitted control input by injecting malicious signals. The attack on the actuator channel can be rigorously modeled as

$$\tilde{h}_\alpha = h_\alpha + \delta^a_\alpha \Delta h_\alpha,$$

where $\tilde{h}_\alpha$ represents corrupted control input. δ^a_α is a binary indicator of the attack state ($\delta^a_\alpha = 0$ when actuator channel is attack free and $\delta^a_\alpha = 1$ when the channel is under attack). The undefined non-linear dynamic function is $\phi(\cdot) \in \mathbb{R}^n$. The following dynamics, which are seen as independent leader agents, describe the required reference signals as

$$\frac{d}{dt}\bar{p}(t) = \bar{q}(t), \quad \frac{d}{dt}\bar{q}(t) = \xi(t), \tag{3.2}$$

where the reference position is denoted by $\bar{p} \in \mathbb{R}^n$, while the reference velocity is $\bar{q} \in \mathbb{R}^n$, and the smooth bounded function is $\xi(\cdot) \in \mathbb{R}^n$. The attack scenario is described in Figure 2.

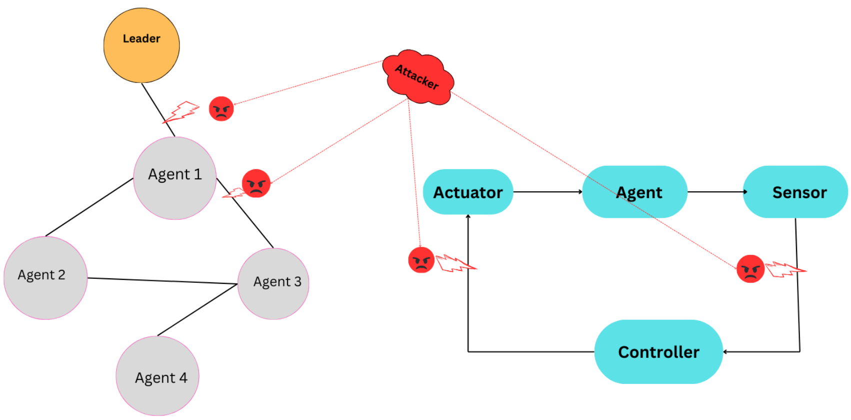


Figure 2: The leader-following MASs subjected to sensor and actuator attacks.

Definition 3.1. In the event the system's results (3.1) involving multiple agents be determined, the second-order leader-follower formation is accomplished, and fulfill the following conditions $\lim_{t\to\infty}\|\tilde{p}_\alpha(t)-\bar{p}_\alpha(t)-\omega_\alpha\|=0$, $\lim_{t\to\infty}\|\tilde{q}_\alpha(t)-\bar{q}(t)\|=0$, $\alpha=1,\dots,m$. The desired location of the agent α is demonstrated through the constant vector $\omega_\alpha=\left[\omega_{\alpha 1},\dots,\omega_{\alpha n}\right]^T\in\mathbb{R}^n$.

Objective of control. The objective is to develop an adaptive control method to facilitate formation in nonlinear multi-agent systems (3.1), ensuring that when the sensor and actuator assaults alter the states of the agents, all deviation indicators maintain their ultimate boundedness that is semi-global and uniform. Despite the interfering assaults, the system develops second-order leader-follower relationships.

3.2. Designing the control scheme

Transformations of coordinates should be represented by

$$\aleph_{p\alpha}(t)=p_\alpha+\delta_\alpha^s\Delta p_\alpha(t)-\bar{p}(t)-\omega_\alpha,\quad \aleph_{q\alpha}(t)=q_\alpha+\delta_\alpha^s\Delta q_\alpha(t)-\bar{q}(t),\quad \alpha=1,\dots,m. \tag{3.3}$$

The error dynamics that follow are generated by (3.1) and (3.2)

$$\begin{aligned}&\frac{d}{dt}\aleph_{p\alpha}(t)=\aleph_{q\alpha}(t),\\&\frac{d}{dt}\aleph_{q\alpha}(t)=h_\alpha(t)+\delta_\alpha^a\Delta h_\alpha+\phi_\alpha(p_\alpha+\delta_\alpha^s\Delta p_\alpha,q_\alpha+\delta_\alpha^s\Delta q_\alpha)-\xi(t),\\&\qquad k=1,\dots,m.\end{aligned} \tag{3.4}$$

The following rewriting of error dynamics (3.4) is necessary to make it simpler

$$\frac{d}{dt}\aleph(t)=\begin{bmatrix}\aleph_q(t)\\ h(t)+\delta^a\Delta h+\boldsymbol{\Phi}(\aleph+\delta^s\Delta\aleph)-\xi(t)\otimes 1_m\end{bmatrix}, \tag{3.5}$$

in which $\aleph(t)=\left[\aleph_p^T(t),\aleph_q^T(t)\right]^T\in\mathbb{R}^{2mn}$ when $\aleph_p(t)=\left[\aleph_{p1}^T(t),\dots,\aleph_{pm}^T(t)\right]^T\in\mathbb{R}^{mn}$ while $\aleph_q(t)=\left[\aleph_{q1}^T(t),\dots,\aleph_q^T(t)\right]^T\in\mathbb{R}^{mn}$, $\mathbf{h}=\left[\mathbf{h}_1^T,\dots,\mathbf{h}_m^T\right]^T\in\mathbb{R}^{mn}$, and $\boldsymbol{\Phi}(\aleph+\delta^s\Delta\aleph)=\left[\phi_1^T,\dots,\phi_m^T\right]^T\in\mathbb{R}^{mn}$ $1_m=\left[1,\dots,1\right]^T\in\mathbb{R}^m$, δ^a and δ^s are binary attack indicators for sensor and actuator, respectively, and $\otimes$ is Kronecker product. Define the formation discrepancies in terms of position and velocity as

$$\begin{aligned}\epsilon_{p\alpha}(t)&=\sum_{l\in\lambda_\alpha}\eta_{\alpha\beta}\left(\tilde{p}_\alpha(t)-\omega_\alpha-\tilde{p}_\beta(t)+\omega_\beta\right)+d_\alpha\left(\tilde{p}_\alpha(t)-\bar{p}(t)-\omega_\alpha\right),\\ \epsilon_{p\alpha}(t)&=\sum_{l\in\lambda_\alpha}\eta_{\alpha\beta}\left(\tilde{q}_\alpha(t)-\tilde{q}_\beta(t)\right)+d_\alpha\left(\tilde{q}_\alpha(t)-\bar{q}(t)\right),\\ \alpha&=1,\dots,m.\end{aligned} \tag{3.6}$$

In this context $\eta_{\alpha\beta}$ and s_α denote element of $\mathcal{O}$ and $\mathcal{S}$ matrices, Subsection 2.2 with λ_α represents the neighbor labels for agent α. Using (3.3) and (3.6), we can re-express the formation error terms as

$$\begin{aligned}\epsilon_{q_\alpha}(t)&=\sum_{\beta\in\lambda\alpha}\eta_{\alpha\beta}(\aleph_{p\alpha}(t)-\aleph_{p\beta}(t))+d_\alpha\aleph_{p\alpha}(t),\\ \epsilon_{q_\alpha}(t)&=\sum_{\beta\in\lambda\alpha}\eta_{\alpha\beta}(\aleph_{q\alpha}(t)-\aleph_{q\beta}(t))+d_\alpha\aleph_{q\alpha}(t),\\ \alpha&=1,\dots,m.\end{aligned}$$

For the unknown nonlinear function $\phi_\alpha(\tilde{p}_\alpha,\tilde{q}_\alpha)$ (3.5), consider a compact set $\Theta_\alpha\subset\mathbb{R}^{2n}$, for $\left[\tilde{p}_\alpha^T,\tilde{q}_\alpha^T\right]^T\in\Theta_\alpha$. To incorporate the sensor and actuator attacks the expression of $\phi_\alpha(\tilde{p}_\alpha,\tilde{q}_\alpha)$ under the attack would be modified to include an adversarial impact. The function can be approximated by an NN model, yielding as

$$\phi_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha) = {G^*_\alpha}^T Q_\alpha(\tilde{p}_\alpha(t), \tilde{q}_\alpha(t) + \varepsilon_\alpha(\tilde{p}_\alpha(t), \tilde{q}_\alpha(t)), \tag{3.7}$$

consider the ideal neural weight matrix $G^*_\alpha \in \mathbb{R}^{c_\alpha \times n}$ with neuron number c_α and corresponding basis function vector $Q_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha) \in \mathbb{R}^{c_\alpha}$ with approximation error $\epsilon_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha)$ bounded by $\|\varepsilon_\alpha(\tilde{p}_\alpha(t), \tilde{q}_\alpha(t)\| \leqslant \gamma_\alpha$, where γ_α is a predetermined constants. (3.7), the optimal matrix of weight G^*_α can be expressed as a constant that is not known, creating challenges to implement in practical control designs. To overcome this limitation, we replace the ideal neural network weight G^*_α with the estimated weight $\hat{G}_\alpha(t)$. The general formation control is subsequently formulated using this estimation as follows

$$h_\alpha(t) = h_n(t) + h_a(t) + h_r(t),$$

where $h_n(t) = -\sigma_p \epsilon_{p\alpha} - \sigma_q \epsilon_{q\alpha}$ is the nominal controller that handles the standard tracking. $\sigma_p > 0, \sigma_q > 0$ are two design constants. $h_a(t) = -{\hat{G}_\alpha}^T Q_\alpha(\tilde{p}_\alpha(t), \tilde{q}_\alpha(t)$ is adaptive component to handle unknown system non-linearities using neural networks. $h_r(t) = \kappa_r \epsilon_q(t)$ is robust term, which counters the actuator attack and directly impacts the control input, where κ_r is a robust gain. The complete control law under sensor and actuator attack becomes

$$h_\alpha(t) = -\sigma_p \epsilon_{p\alpha}(t) - \sigma_q \epsilon_{q_\alpha}(t) - \hat{G}^T_\alpha(t) \times Q_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha) - \kappa_r \epsilon_{q\alpha}, \quad \alpha = 1, \ldots, m. \tag{3.8}$$

In which $\hat{G}_\alpha \in \mathbb{R}^{c_\alpha \times n}$ is the approximate value of G^*_α. By introducing an adversarial disturbance in the sensor and actuator channel, it affects the nonlinear dynamics. Even when adversarial interruptions occur, control robustness is maintained by incorporating this change. In order to tune $\hat{G}_\alpha(t)$, the NN updating law is as follows

$$\frac{d}{dt}\hat{G}_\alpha(t) = \Gamma_\alpha \left(Q_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha)\big(\epsilon_{p\alpha}(t) + \epsilon_{q\alpha}(t)\big)^T - \tau_\alpha \hat{G}_\alpha(t) \right), \quad \alpha = 1, \ldots, m, \tag{3.9}$$

where the design constant is $\tau_\alpha > 0$ while the positive definite constant matrix is $\Gamma_\alpha \in \mathbb{R}^{c_\alpha \times c_\alpha}$.

Observation 1. In the control law described in (3.8) the terms related to the position and velocity errors are formulated as specified in (3.6). These error terms are incorporated to ensure effective compensation for deviations in both positional and velocity dynamics, facilitating the system's adherence to the desired formation trajectory, and expertly designed to enable the agents within the network to successfully achieve their objectives. The neural network expression $\hat{G}^T_\alpha(t) Q_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha)$ plays a crucial role in compensating for unknown dynamics by dynamically tuning the neural network weight $\hat{G}_\alpha(t)$ with sensor and actuator attacks according to the updating law (3.9). This means an adversary can influence how the system operates. Adding false data in the agents also makes it harder to keep the formation stable. This unique control strategy addresses formation management in intricate second-rank nonlinear structures.

3.3. *Proof-based conjecture*

Conjecture 1. The nonlinear dynamics of a second-order agent-based system described in (3.1) operates under an undirected connected graph E and maintains bounded initial conditions. By implementing the adaptive formation control law (3.8), the neural network weight updating rule (3.9), and selecting the design constants σ_p and σ_q, the desired formation can be achieved. Furthermore, incorporating sensor and actuator attack into the nonlinear dynamics introduces significant challenges in the state. This approach enables a robust evaluation of control performance ensuring resilient system operation. The control objectives are achieved for a sufficiently smooth movement trajectory,

$$\sigma_p > 1, \quad \sigma_q > 1\frac{1}{2} + \frac{1}{2(\theta^{\tilde{\Upsilon}}_{min})^2}, \quad \sigma_p + \sigma_q > \frac{1}{\theta^{\tilde{\Upsilon}}_{min}}. \tag{3.10}$$

Let $\theta^{\tilde{\Upsilon}}_{min}$ denote the smallest matrix's eigenvalue $\tilde{\Upsilon}$. This will lead to the achievement of the intended control goals.

1. Each error will be semi-globally uniformly ultimately bounded.
2. Even with sensor and actuator assault, the multi-agent creation will be effectively maintained with uniform reference trajectories.

Proof. We select the following Lyapunov function candidate

$$\mathcal{M}(t) = \frac{1}{2}\aleph^T(t)\left(\begin{bmatrix}(\sigma_p+\sigma_q)\tilde{\Upsilon}\tilde{\Upsilon} & \tilde{\Upsilon}\\ \tilde{\Upsilon} & \tilde{\Upsilon}\end{bmatrix}\otimes I_m\right)\times\aleph(t)+\frac{1}{2}\sum_{\alpha=1}^{m}T_r\left\{\tilde{G}_\alpha^T(t)\Upsilon_\alpha^{-1}\tilde{G}_\alpha(t)\right\}.$$

It is definitely positive. The following outcome is achieved when the design parameters fulfill the condition (3.10), where $\tilde{\Upsilon}=\Upsilon+\Pi$ by Lemma 2.2, positive definiteness of the symmetrical matrix $\tilde{\Upsilon}$ is verified. The following result is assured as long as the design parameters meet condition (3.10), $(\sigma_p+\sigma_q)\tilde{\Upsilon}\tilde{\Upsilon}-\tilde{\Upsilon}>0$. Consequently, the matrix $\begin{bmatrix}(\sigma_p+\sigma_q)\tilde{\Upsilon}\tilde{\Upsilon} & \tilde{\Upsilon}\\ \tilde{\Upsilon} & \tilde{\Upsilon}\end{bmatrix}$ is also positive definite, as indicated by Lemma 2.3. Therefore, the function $\mathcal{M}(t)$ can be regarded as a candidate for a Lyapunov function. The time derivative of $\mathcal{H}(t)$ concerning the dynamics described in (3.5) and (3.9) is given by

$$\begin{aligned}\frac{d}{dt}\mathcal{M}(t) &= \aleph^T(t)\left(\begin{bmatrix}(\sigma_p+\sigma_q)\tilde{\Upsilon}\tilde{\Upsilon} & \tilde{\Upsilon}\\ \tilde{\Upsilon} & \tilde{\Upsilon}\end{bmatrix}\otimes I_m\right)\times\begin{bmatrix}\aleph_q(t)\\ h(t)+\delta^a\Delta h+\mathbf{\Phi}(\aleph+\delta^s\Delta\aleph)-\xi(t)\otimes\mathbf{1}_m\end{bmatrix}\\ &\quad+\sum_{\alpha=1}^{m}T_r\left\{\tilde{G}_\alpha^T(t)\left(Q_k(\tilde{p}_\alpha,\tilde{q}_\alpha)\times(\epsilon_{p\alpha}(t)+\epsilon_{q\alpha}(t))^T-\tau_\alpha\hat{G}_\alpha(t)\right)\right\}.\end{aligned}\tag{3.11}$$

Given that $\epsilon_p(t)=\tilde{\Upsilon}\aleph_p(t)$ and $\epsilon_q(t)=\tilde{\Upsilon}\aleph_q(t)$, where $\epsilon_p(t)=\left[\epsilon_{p1}^T(t),\dots,\epsilon_{pm}^T(t)\right]^T\in\mathbb{R}^{mn}$, $\epsilon_q(t)=\left[\epsilon_{q1}^T(t),\dots,\epsilon_{qm}^T(t)\right]^T\in\mathbb{R}^{mn}$ and $\Delta h(t)=\left[\Delta {h_1}^T(t),\ \Delta {h_2}^T(t)\ \cdots\ \Delta {h_m}^T(t)\right]^T$ represents the actuator attack vector, so (3.11) can be reformulated accordingly

$$\begin{aligned}\frac{d}{dt}\mathcal{M}(t) &= \left[(\sigma_p+\sigma_q)\epsilon_p^T(t)\tilde{\Upsilon}+\epsilon_q^T(t),\epsilon_p^T(t)+\epsilon_q^T(t)\right]\begin{bmatrix}\aleph_q(t)\\ h(t)+\delta^a\Delta h+\mathbf{\Phi}(\aleph+\delta^s\Delta\aleph)-\xi(t)\otimes\mathbf{1}_m\end{bmatrix}\\ &\quad+\sum_{\alpha=1}^{m}T_r\left\{\tilde{G}_\alpha^T(t)\left(Q_\alpha(\tilde{p}_\alpha,\tilde{q}_\alpha)\times(\epsilon_{p\alpha}(t)+\epsilon_{q\alpha}(t))^T-\tau_\alpha\hat{G}_k(t)\right)\right\}.\end{aligned}\tag{3.12}$$

After straightforward steps, the following expression can be derived from (3.12):

$$\begin{aligned}\frac{d}{dt}\mathcal{M}(t) &= \sum_{\alpha=1}^{m}\left((\sigma_p+\sigma_q)\epsilon_{p\alpha}^T(t)\epsilon_{q\alpha}(t)+\epsilon_{q\alpha}^T(t)\times\aleph_{q\alpha}(t)\right)\\ &\quad+\sum_{\alpha=1}^{m}\left(\epsilon_{p\alpha}^T(t)+\epsilon_{q\alpha}^T(t)\right)\times\left(h_\alpha+\delta_\alpha^a\Delta h_\alpha+\phi_\alpha(\tilde{p}_\alpha,\tilde{q}_\alpha)-\xi(t)\right)\\ &\quad+\sum_{\alpha=1}^{m}T_r\left\{\tilde{G}_\alpha^T(t)\left(Q_\alpha(\tilde{p}_\alpha,\tilde{q}_\alpha)\times(\epsilon_{p\alpha}(t)+\epsilon_{q\alpha}(t))^T-\tau_\alpha\hat{G}_\alpha(t)\right)\right\}.\end{aligned}\tag{3.13}$$

By substituting the neural network approximation from (3.7) and the controller from (3.8) into (3.13), we obtain the following result

$$\begin{aligned}\frac{d}{dt}\mathcal{M}(t) &= \sum_{\alpha=1}^{m}\left((\sigma_p+\sigma_q)\epsilon_{p\alpha}^T(t)\epsilon_{q\alpha}(t)+\epsilon_{q\alpha}^T(t)\aleph_{q\alpha}(t)\right)+\sum_{\alpha=1}^{m}\left(\epsilon_{p\alpha}^T(t)+\epsilon_{q\alpha}^T(t)\right)\left(-\sigma_p\epsilon_{p_\alpha}(t)\right.\\ &\quad-\sigma_q\epsilon_{q_\alpha}(t)+\delta_\alpha^a\Delta h_\alpha-\kappa_r\epsilon_{q\alpha}-\hat{G}_\alpha^T(t)Q_\alpha(\tilde{p}_\alpha,\tilde{q}_\alpha)+{G_\alpha^*}^TQ_\alpha(\tilde{p}_\alpha,\tilde{q}_\alpha)+\varepsilon_\alpha(\tilde{p}_\alpha,\tilde{q}_\alpha)\\ &\quad\left.-\xi(t)\right)+\sum_{\alpha=1}^{m}T_r\left\{\tilde{G}_\alpha^T(t)\left(Q_\alpha(\tilde{p}_\alpha,\tilde{q}_\alpha)\times(\epsilon_{p\alpha}(t)+\epsilon_{q\alpha}(t))^T-\tau_\alpha\hat{G}_\alpha(t)\right)\right\}.\end{aligned}\tag{3.14}$$

Considering the equation $\tilde{G}_\alpha(t) = \hat{G}_\alpha(t) - G^*_\alpha$, the following is an alternative expression for (3.14):

$$\begin{aligned}\frac{d}{dt}\mathcal{M}(t) = &-\sum_{\alpha=1}^{m} \sigma_p \epsilon^T_{p\alpha}(t)\epsilon_{p\alpha}(t) - \sum_{\alpha=1}^{m} \sigma_q \times \epsilon^T_{q\alpha}(t)\epsilon_{q\alpha}(t) - \sum_{\alpha=1}^{m} \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \times \kappa_r \epsilon_{q\alpha}(t)\\ &+ \sum_{\alpha=1}^{m} \epsilon^T_{q\alpha}(t)\aleph_{q\alpha}(t) - \sum_{\alpha=1}^{m} \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \tilde{G}^T_\alpha(t) \times Q_k(\tilde{p}_\alpha, \tilde{q}_\alpha)\\ &+ \sum_{\alpha=1}^{m} \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \times \varepsilon_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha) - \sum_{\alpha=1}^{m} \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \times (\xi(t)\\ &+ \sum_{\alpha=1}^{m} \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \times \delta^a_\alpha \Delta h_\alpha - \sum_{\alpha=1}^{m} T_r\left\{\tau_\alpha \times \tilde{G}^T_\alpha(t)\hat{G}_\alpha(t)\right\}\\ &+ \sum_{\alpha=1}^{m} T_r\left\{\tilde{G}^T_\alpha(t) Q_k(\tilde{p}_\alpha, \tilde{q}_\alpha) \times \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right)\right\}.\end{aligned} \tag{3.15}$$

Based on the properties of the trace operation, we have $u^T v = T_r(uv^T) = T_r(vu^T)$, $\forall u, v \in \mathbb{R}^b$, which leads to the following conclusion

$$\left(\epsilon_{p\alpha}(t) + \epsilon_{q\alpha}(t)\right)^T \tilde{G}^T_\alpha(t) Q_k(\tilde{p}_\alpha, \tilde{q}_\alpha) = T_r\left\{\tilde{G}^T_\alpha(t) Q_k(\tilde{p}_\alpha, \tilde{q}_\alpha) \left(\epsilon_{p\alpha}(t) + \epsilon_{q\alpha}(t)\right)^T\right\}. \tag{3.16}$$

By applying (3.16), the following is a modification of (3.15):

$$\begin{aligned}\frac{d}{dt}\mathcal{M}(t) = &-\sum_{\alpha=1}^{m} \sigma_p \epsilon^T_{p\alpha}(t)\epsilon_{p\alpha}(t) - \sum_{\alpha=1}^{m} \sigma_q \times \epsilon^T_{q\alpha}(t)\epsilon_{q\alpha}(t)\\ &+ \sum_{\alpha=1}^{m} \epsilon^T_{q\alpha}(t)\aleph_{q\alpha}(t) + \sum_{\alpha=1}^{m} \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \varepsilon_k(\tilde{p}_\alpha, \tilde{q}_\alpha)\\ &- \sum_{\alpha=1}^{m} \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \times \xi(t) + \sum_{\alpha=1}^{m} \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \times \delta^a_\alpha \Delta h_\alpha\\ &- \sum_{\alpha=1}^{m} \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \times \kappa_r \epsilon_{q\alpha}(t) - \sum_{\alpha=1}^{m} T_r\left\{\tau_\alpha \tilde{G}^T_\alpha(t)\hat{G}_\alpha(t)\right\}.\end{aligned} \tag{3.17}$$

The following outcomes can be obtained via Young's inequality and the Cauchy-Bunyakovsky-Schwarz inequality:

$$\begin{aligned}\epsilon^T_{q\alpha}(t)\aleph_{q\alpha}(t) &\leqslant \frac{1}{2}\epsilon^T_{q\alpha}(t)\epsilon_{q\alpha}(t) + \frac{1}{2}\aleph^T_{q\alpha}(t)\aleph_{q\alpha}(t), \left(\epsilon_{p\alpha}(t) + \epsilon_{q\alpha}(t)\right)^T \varepsilon_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha)\\ &\leqslant \frac{1}{2}\epsilon^T_{p\alpha}(t)\epsilon_{p\alpha}(t) + \frac{1}{2}\epsilon^T_{q\alpha}(t)\epsilon_{q\alpha}(t) + \|\varepsilon_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha)\|^2, \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \xi(t)\\ &\leqslant \frac{1}{2}\epsilon^T_{p\alpha}(t)\epsilon_{p\alpha}(t) + \frac{1}{2}\epsilon^T_{q\alpha}(t)\epsilon_{q\alpha}(t) + \|\xi(t)\|^2, \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \delta^a_\alpha \Delta h_\alpha\\ &\leqslant \frac{1}{2}\epsilon^T_{p\alpha}(t)\epsilon_{p\alpha}(t) + \frac{1}{2}\epsilon^T_{q\alpha}(t)\epsilon_{q\alpha}(t) + \|\delta^a_\alpha \Delta h_\alpha\|^2, \left(\epsilon^T_{p\alpha}(t) + \epsilon^T_{q\alpha}(t)\right) \kappa_r \epsilon_{q\alpha}(t)\\ &\leqslant \frac{1}{2}\epsilon^T_{p\alpha}(t)\epsilon_{p\alpha}(t) + \frac{1}{2}\epsilon^T_{q\alpha}(t)\epsilon_{q\alpha}(t) + \|\kappa_r \epsilon_{q\alpha}(t)\|^2.\end{aligned} \tag{3.18}$$

By inserting the inequalities from (3.18) into (3.17), the following results obtained

$$\begin{aligned}\frac{d}{dt}\mathcal{M}(t) \leqslant &-\aleph^T(t)\left(\begin{bmatrix}(\sigma_p - 1)\tilde{\Upsilon}\tilde{\Upsilon} & 0\\ 0 & \left(\sigma_q - 1\frac{1}{2}\right)\tilde{\Upsilon}\tilde{\Upsilon} - \frac{1}{2}I_n\end{bmatrix} \otimes I_m\right)\aleph(t) - \sum_{\alpha=1}^{m} T_r\left\{\tau_\alpha \tilde{G}^T_\alpha(t)\hat{G}_\alpha(t)\right\}\\ &+ \sum_{\alpha=1}^{m} \|\varepsilon_\alpha(\tilde{p}_\alpha, q_\alpha)\|^2 + n\|\xi(t)\|^2 + n\|\delta^a_\alpha \Delta h_\alpha\|^2 + n\|\kappa_r \epsilon_{q\alpha}(t)\|^2.\end{aligned} \tag{3.19}$$

By applying the relation $\tilde{G}_\alpha(t) = \hat{G}_\alpha(t) - G^*_\alpha$, the resulting equation can be derived

$$T_r\left\{\tau_\alpha \tilde{G}^T_\alpha(t)\hat{G}_\alpha(t)\right\} = \frac{\tau_\alpha}{2}T_r\left\{\tilde{G}^T_\alpha(t)\tilde{G}_\alpha(t)\right\} + \frac{\tau_\alpha}{2}T_r\left\{\hat{G}^T_\alpha(t)\hat{G}_\alpha(t)\right\} - \frac{\tau_\alpha}{2}T_r\left\{G^{*T}_\alpha G^*_\alpha\right\}. \tag{3.20}$$

Let $\nabla(t)$ denote a combined bound term defined as the sum of all relevant bounded quantities arising from disturbance, approximation error, and adaptive estimation, used in the Lyapunov-based inequality. $\nabla(t) = \sum_{\alpha=1}^m \frac{\tau_\alpha}{2}T_r\left\{G^{*T}_\alpha G^*_\alpha\right\} + \sum_{\alpha=1}^m \|\varepsilon_\alpha(\tilde{p}_\alpha, \tilde{q}_\alpha)\|^2 + n\|\xi(t)\|^2 + n\|\delta^a_\alpha \Delta h_\alpha\|^2 + n\|\kappa_r \epsilon_{q\alpha}(t)\|^2$. By substituting (3.20) into (3.19) we obtain the following result

$$\frac{d}{dt}\mathcal{M}(t) \leqslant -\aleph^T(t)\left(\begin{bmatrix}(\sigma_p-1)\tilde{\Upsilon}\tilde{\Upsilon} & 0\\ 0 & \left(\sigma_q - 1\frac{1}{2}\right)\tilde{\Upsilon}\tilde{\Upsilon} - \frac{1}{2}I_n\end{bmatrix}\otimes I_m\right)\aleph(t) - \frac{\tau_\alpha}{2}T_r\left\{\tilde{G}^T_\alpha(t)\tilde{G}_\alpha(t)\right\} + \nabla(t).$$

Since all the component of $\nabla(t)$ are bounded, there exist a constant o such that $\|\nabla(t)\| \leqslant d_1$. Let θ^g_{min} represent the smallest value of the matrix's eigen spectrum $\begin{bmatrix}(\sigma_p-1)\tilde{\Upsilon}\tilde{\Upsilon} & 0\\ 0 & \left(\sigma_q - 1\frac{1}{2}\right)\tilde{\Upsilon}\tilde{\Upsilon} - \frac{1}{2}I_n\end{bmatrix}$, and θ^l_{max} express the largest value in the matrix's eigen spectrum $\begin{bmatrix}(\sigma_p+\sigma_q)\tilde{\Upsilon}^T\tilde{\Upsilon} & \tilde{\Upsilon}\\ \tilde{\Upsilon} & \tilde{\Upsilon}\end{bmatrix}$, and $\theta^{\Gamma^{-1}_\alpha}_{max}$ be the maximum eigenvalue of Γ^{-1}_α. From (3.18) the following can be derived

$$\frac{d}{dt}\mathcal{M}(t) \leqslant -\frac{\theta^g_{min}}{\theta^l_{max}}\aleph^T(t)\left(\begin{bmatrix}(\sigma_p+\sigma_q)\tilde{\Upsilon}^T\tilde{\Upsilon} & \tilde{\Upsilon}\\ \tilde{\Upsilon} & \tilde{\Upsilon}\end{bmatrix}\otimes I_m\right)\aleph(t) - \frac{1}{2}\sum_{\alpha=1}^m \frac{\tau_\alpha}{\theta^{\Gamma^{-1}_\alpha}_{max}}T_r\left\{\tilde{G}^T_\alpha(t)\Gamma^{-1}_\alpha\tilde{G}_\alpha(t)\right\} + s. \tag{3.21}$$

Define g as the minimum eigenvalue of $g = \min\left\{2\frac{\theta^g_{min}}{\theta^l_{max}}, \frac{\tau_1}{\theta^{\Gamma^{-1}_1}_{max}} \dots, \frac{\tau_m}{\tau^{\Gamma^{-1}_m}_{max}}\right\}$ so the inequality (3.21) becomes

$$\frac{d}{dt}\mathcal{M}(t) \leqslant -g\mathcal{M}(t) + s. \tag{3.22}$$

By applying Lemma 2.4 to (3.22), the following inequality can be obtained

$$\mathcal{M}(t) \leqslant \epsilon^{-gt}\mathcal{M}(0) + \frac{s}{g}\left(1 - \epsilon^{-gt}\right).$$

Based on the inequality provided, it can be demonstrated that, first, the errors $\aleph_{p\alpha}(t), \aleph_{q\alpha}(t), \tilde{G}_\alpha(t)$, and $\alpha = 1, \dots, n$ are characterized as Semi-Globally Uniformly Ultimately Bounded (SGUUB). This is true even in the presence of the sensor and the actuator attack which impact both the system's dynamics and the control law. Consequently, the system exhibits stability despite adversarial disturbances. Second, the tracking errors $\aleph_{p\alpha}(t)$ and $\aleph_{q\alpha}(t)$ can attain the desired accuracy through the appropriate selection of sufficiently large design parameters. This implies that the multi-agent formation can still be successfully realized in the context of the sensor and actuator attacks, provided that the design constants are suitably chosen to address these challenges. Overall, the formation control protocol is resilient to sensor and actuator attacks while ensuring the desired tracking performance is achieved. □

4. Simulation

To validate the effectiveness of the proposed control strategy against sensor and actuator attacks, numerical simulations are performed in a 2-D plane using MATLAB. A system consisting of four agents functions on a two-dimensional plane is outlined as below:

$$\frac{d}{dt}p_\alpha(t) = q_\alpha(t), \quad \frac{d}{dt}q(t) = h_\alpha(t) + \delta^a_\alpha \Delta h_\alpha + \begin{bmatrix}p_{\alpha 1} + \rho_\alpha \cos^2(p_{\alpha 1}, q_{\alpha 1})\\ q_{\alpha 2} + \varphi_\alpha \sin^2(p_{\alpha 2}, q_{\alpha 2})\end{bmatrix}, \quad \alpha = 1,2,3,4,$$

where $p_\alpha(t) = [p_{\alpha 1}, p_{\alpha 2}]$, $q_\alpha(t) = [q_{\alpha 1}, q_{\alpha 2}]$, $\delta_\alpha^a = 1$ and $\Delta h_\alpha = 0.5$. The values of ρ_α for $\alpha = 1,2,3,4$ are set to $-0.25, 0.3, -0.2$, and 0.1, respectively, and the values of φ_α for $\alpha = 1,2,3,4$ are $0.3, 0.1, -0.8$, and -0.6, respectively. The starting positions are as follows $p_{\alpha=1,2,3,4}(0) = [3,3]^T$, $[-3,-3]^T$, $[2.5,-2.5]^T$, $[-2.5,2.5]^T$, respectively. The desired trajectory for the formation movement is defined by the following dynamic function, with starting values $\bar{p}(0) = [0,0]^T$,

$$\frac{d}{dt}\bar{p}(t) = \bar{q}(t), \quad \frac{d}{dt}\bar{q}(t) = [3\cos(0.3t), 3\sin(0.3t)] .$$

The following are the reference signals and the required relative locations for the agents $\omega_\alpha = 1,2,3,4 = [1.5,1.5]^T, [1.5,-1.5]^T, [-1.5,1.5]^T, [-1.5,-1.5]^T$. According to Figure 3 the adjacency matrix describes the

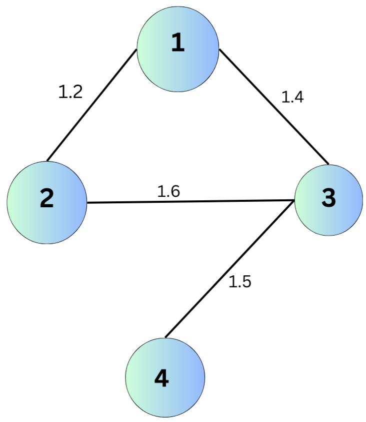


Figure 3: Communication topology.

connections between agents O given by

$$O = \begin{bmatrix} 0 & 1.2 & 1.4 & 0 \\ 1.2 & 0 & 1.6 & 0 \\ 1.4 & 1.6 & 0 & 1.5 \\ 0 & 0 & 1.5 & 0 \end{bmatrix},$$

while the communication links between the agents and the leader are characterized by the diagonal matrix $\mathcal{S}$, defined as $\mathcal{S} = \mathrm{diag}(1,0,0,0)$, showing that only agent 1 receives direct information from the leader.

The formation control approach obtained from (3.8) establishes the design parameters based on the control situations in (3.10) that $\sigma_p = 10$ and $\sigma_q = 25$. With 12 neurons in the neural network configuration, the centers are uniformly spaced from -3 and 3. In (3.9), the design parameters are defined by the update rule $\Upsilon_\alpha = 0.7\alpha_{12}$ for $\alpha = 1,2,3,4$ and $\tau_\alpha = 0.25$ for the same indices, while the initial weights are set as $\hat{G}_{\alpha=1,2,3,4}(0) = [0.3]_{12\times 2}$. The simulation results are further detailed in Figures 4 and 5, which visually depict the agents' trajectories $q_{\alpha 1}$ and $q_{\alpha 2}$ are expected to converge to the desired trajectory over time. In Figures 6 and 7 the position and velocity error (ϵ_p, ϵ_q) plots reveal persistent deviations caused by sensor and actuator attacks, emphasizing the need for robust control strategies and mitigation mechanisms to ensure system stability and resilience under adverse conditions. In Figure 8 solid lines represent the error for the first component and the dashed line represents the error for the second component of position and velocity of each agent based on control law (3.8). The position graph Figure 9 tracks the evolution of each agent's trajectory, showing how their positions adapt over time to converge toward desired path under the influence of controller and the velocity graph highlighting how agents adjust their speed to align with desired motion despite disturbances. Figure 10 reflects consistent performance by showing that the NN weights stay within the defined limits. These figures highlight the system's reliability and efficiency across different operational scenarios. The simulation results demonstrate that the proposed formation control method is both reliable and effective in meeting the desired control objectives.

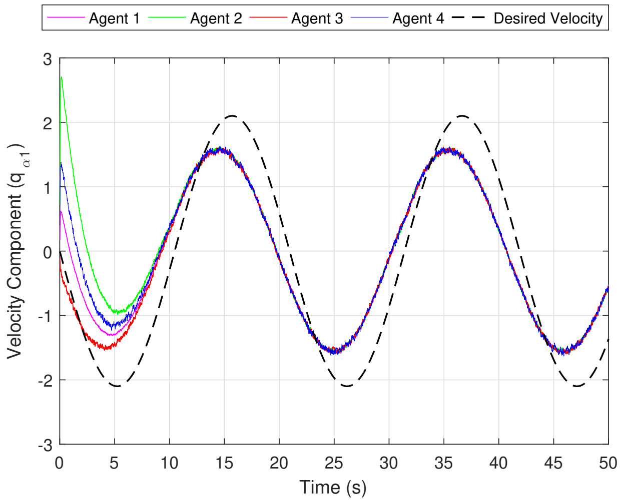


Figure 4: Accurate tracking of velocity for the initial coordinate is essential for achieving precise results.

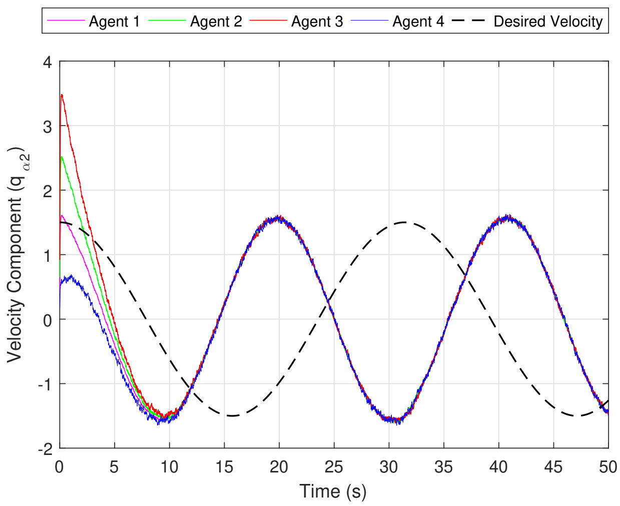
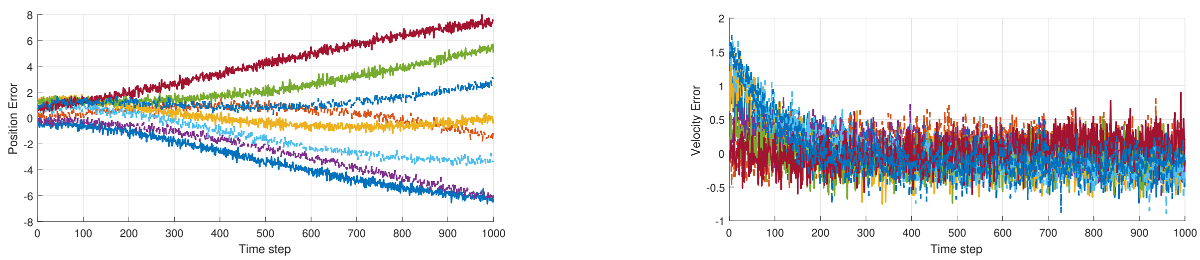


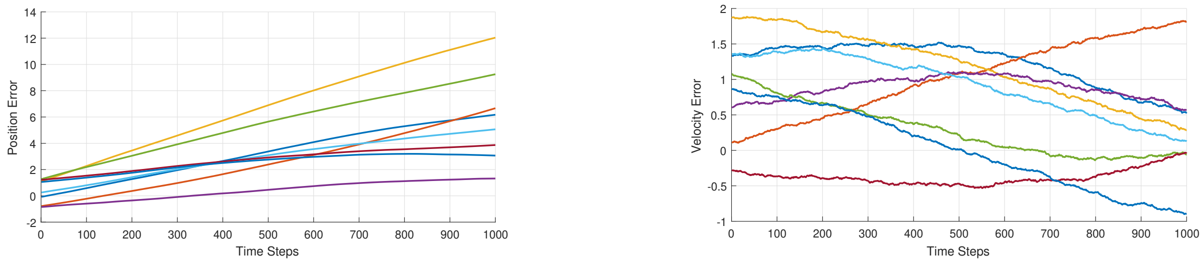
Figure 5: Accurate tracking of velocity for the second coordinate is essential for achieving precise results.



(a) Sensor attack on position error.

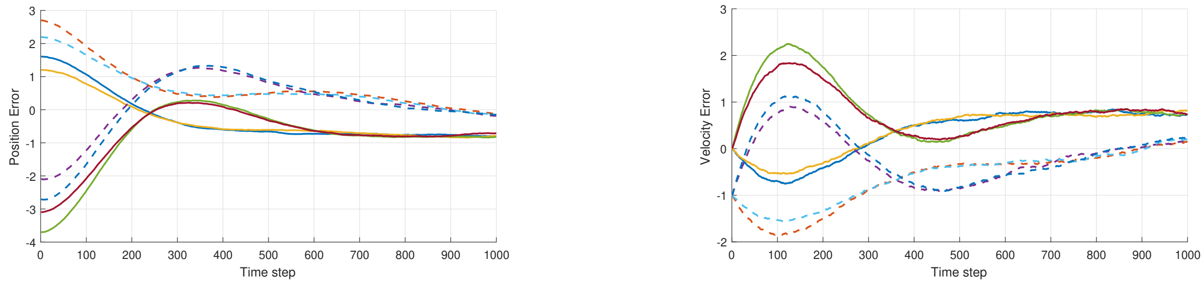
(b) Sensor attack on velocity error.

Figure 6: Position and velocity error dynamics under sensor attack without controller.



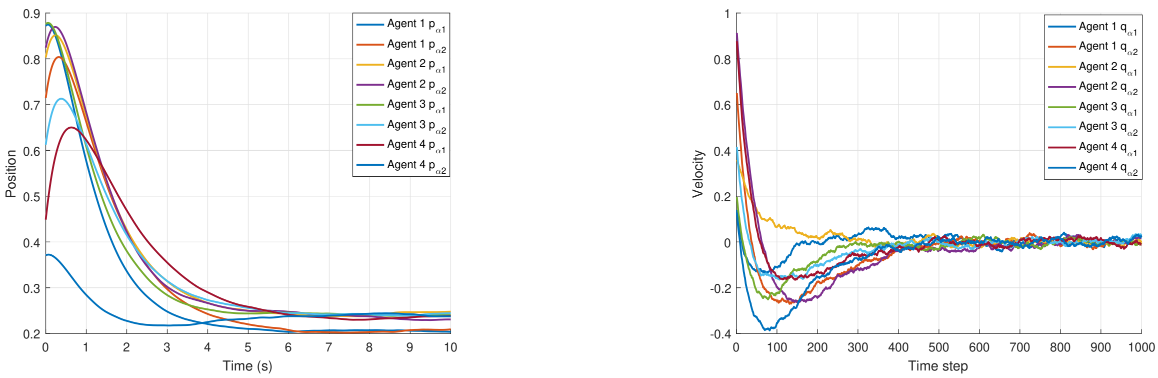
(a) Actuator attack on position error. **(b)** Actuator attack on velocity error.

Figure 7: Position and velocity error dynamics under actuator attack without controller.



(a) Position error update. **(b)** Velocity error update.

Figure 8: Position and velocity error updates for each agent based on control law.



(a) Position update in 2D plane. **(b)** Velocity update in 2D plane.

Figure 9: Position and velocity updates for each agent in 2D plane based on control law for different values of control gain.

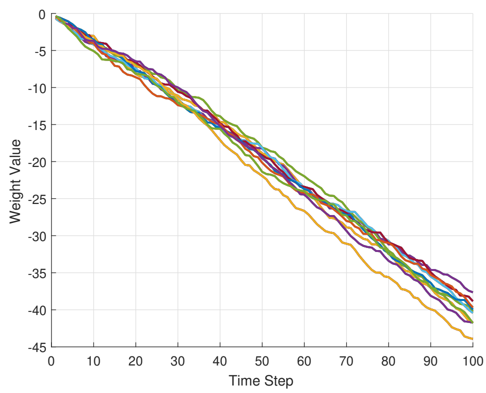


Figure 10: Time-series visualization of neural network weight updates for agent position and velocity.

5. Conclusion

This research proposes a novel formation control method for second-order multi-agent systems with unknown nonlinear dynamics that use leader-follower adaptive neural networks. In this approach, Neural networks are leveraged to estimate the unknown dynamic functions, facilitating the construction of a resilient framework that effectively manages the uncertainties. Under typical circumstances, simulations validate the adaptive NN formation control scheme's efficacy, and Lyapunov stability analysis shows that it achieves its goals. However, the resilience and stability of the adaptive NN control system may be jeopardized by Sensor and Actuator assaults. Such attacks can potentially cause synchronization issues or misleading data injections, which might impair performance or prevent control objectives from being met. The main contribution is the design of a robust stability model that ensures resilience of the adaptive NN control system under such adversarial conditions. By incorporating position and velocity gains (σ_p ,σ_q) along with a robust gain (κ_r), the proposed adaptive control framework significantly enhances trajectory tracking accuracy and maintains system stability despite the presence of sensor and actuator assaults. Furthermore, this project will investigate artificial potential field (APF) method to enable cooperative control and coordination among agents for second-order nonlinear multi-agent systems using reinforcement learning to develop an attack-resilient control framework, in contrast, the majority of earlier studies related to APF in multi-agent systems have concentrated on systems of the first order. Future work could also focus on enhancing the resilience of the proposed control framework by incorporating time-delay robustness and event-triggered control. These directions will enhance applicability in real-world distributed systems with limited sensing and dynamic network conditions.

Author contributions

Conceptualization, Adnan Burhan Rajab; software, Taoufik Saidani; validation, Naveed Iqbal; formal analysis, Taoufik Saidani; resources, Naveed Iqbal; data curation, Nabila Jahangir; writing original draft, Nabila Jahangir and Azmat Ullah Khan Niazi; writing-review & editing, Nabila Jahangir, Azmat Ullah Khan Niazi; supervision, Azmat Ullah Khan Niazi; project administration, Naveed Iqbal.

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Code availability

The code is considered an intellectual property of the University of Lahore, Sargodha, and therefore not publicly available.

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