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Bipolar linear Diophantine fuzzy EDAS for multi-attribute group decision-making

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Abstract

The formation of new properties for Bipolar Linear Diophantine Fuzzy Set ($\mathcal{BLDFS}$) enhances the evaluation procedure by including control parameters and meticulously accumulating positive as well as negative opinions. While previous approaches encounter difficulties in accurately grasping uncertainties, the design and development of a novel Bipolar Linear Diophantine Fuzzy Evaluation based on Distance from Average Solution ($\mathcal{BLDFEDAS}$) approach aim to bridge the gap in the existing research. An application of implementing this approach is presented by case analysis on choosing the best Forensic Decision Intelligence ($\mathcal{FDI}$) system, as the significance of forensic science in the court system has yet to be demonstrated. Further, a novel Bipolar Linear Diophantine Fuzzy Multi-Attributive Border Approximation Area Comparison ($\mathcal{BLDFMABAC}$) methodology is provided to uplift the effectiveness of results by $\mathcal{BLDFEDAS}$. By rigorously incorporating control parameters and bipolarity, the field substantially progresses and establishes a robust framework for tackling ambiguity in decision support systems.

Keywords: Bipolar linear Diophantine fuzzy set, evaluation based on distance from average solution, multi-attributive border approximation area comparison, multi-attribute group decision-making.

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1. Introduction

The complexity of Decision-making problems grows in this competitive world. Therefore, it becomes harder for an individual decision expert to make a well-informed choice. Multi-Attribute Decision-Making($\mathcal{MADM}$) and Multi-Criteria Decision-Making($\mathcal{MCDM}$) have wider approaches in many fields of science, which has made researchers work more on it [51]. These $\mathcal{MADM}$ methods were developed with many operators to handle vagueness. In the field of digital transformation, aggregation operator

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was discussed for linear diophantine fuzzy [27]. Jana et al. discussed Dombi aggregation operators [24] and Dombi prioritized aggregation operators [25] in a bipolar fuzzy set. Lee developed operations based on bipolar-valued fuzzy sets [33]. Asif et al. examined Pythagorean fuzzy set through Hamacher aggregation operator [6]. Frank Choquet Bonferroni mean operators were deliberated by Wang et al. [52]. Aggregation operator named Sugeno-Weber was explored for spherical fuzzy by Hussain and Ullah [17]. To provide satisfying results, it is necessary to integrate the opinions of various skilled experts. As a result, Multi-Attribute Group Decision-making to choose ($\mathcal{MAGDM}$) and Multi-Criteria Group Decision-making ($\mathcal{MCGDM}$) offer a high potential and structured approach for evaluating multiple conflicting criteria. Many decision-making strategies coped with uncertainty were addressed with $\mathcal{MAGDM}$ and $\mathcal{MCGDM}$ tools like green supplier selection [2], disaster control [10], pattern classification [11], cybersecurity [37], and best airline during pandemic [50]. These tools were widely conferred through interval fuzzy [18], bipolar fuzzy [21], q-rung orthopair fuzzy [35, 51], and trapezoidal fuzzy numbers [42]. Similarity functions on fuzzy sets were well deliberated by previous studies [30, 34, 48]. In the contemporary world, there is a need for the best $\mathcal{FDI}$ System to be developed into an application for resolving critical cases and to lead an excellent path for finding the right decision at the earliest. Many researchers discussed well about $\mathcal{FDI}$ through intelligent systems [38], its effectiveness in criminal justice [28], and efficacy in investigation [46] . $\mathcal{MAGDM}$ strategies can be beneficial for assessing and alleviating threats concerning criminal investigation, which comprises procedural inaccuracies and legal complications, as such, the problem is set up. A $\mathcal{MADM}$ problem based on Krushkal's algorithm is well discussed [31]. Robot selection [19], smog mitigation [32], and material selection [47] were analyzed through $\mathcal{MCDM}$ methods. Also, frameworks related to multi-fuzzy were examined by recent researches [49].

1.1. Literature review

Zadeh [54] initiated Fuzzy Set ($\mathcal{FS}$) with belongingness area in the interval $[0, 1]$. It was then evolved into an Intuitionistic Fuzzy Set ($\mathcal{IFS}$) [7, 8], which demanded the total belongingness area and non-belongingness area be in $[0, 1]$. Due to the loss of the condition, it proceeds to develop into Pythagorean Fuzzy Set ($\mathcal{PFS}$) [53] with a square sum of belongingness area and non-belongingness area lies in $[0, 1]$. To conquer the inability of earlier ailments, it was transitioned into q-Rung Orthopair Fuzzy Set (q-$\mathcal{ROFS}$) [5] with the q^{th} sum of the condition. Upon all inefficiency to handle unpredictability, Linear Diophantine Fuzzy Set ($\mathcal{LDFS}$) [45] was integrated by incorporating control parameters. Bipolar Fuzzy Set ($\mathcal{BFS}$) [55, 56] is an interpretation of $\mathcal{FS}$ to assist the mind of humans to take the most advantageous judgments in every instance with the analyzation of equally beneficial and detrimental consequences [9, 56]. It was further improved into Bipolar Pythagorean Fuzzy Set ($\mathcal{BPFS}$) [1] besides having its implications for tackling $\mathcal{MADM}$ in various fuzzy like q-rung [36], plithogenic [43], and complex linear diophantine fuzzy [26]; $\mathcal{MCDM}$ [40, 52]; and group decision making problems [18]. Many researchers had beneficial discussions regarding decision-making in a bipolar environment [20, 22]. An $\mathcal{EDAS}$ [23] approach in a bipolar context was well explored by Chiranjibe Jana and Madhumangal Pal. Jana [21] also evaluated the bipolar fuzzy $\mathcal{MABAC}$ process.

Amongst many conventional decision-making approaches, $\mathcal{EDAS}$ is a beneficial technique in resolving multi-attribute group decision-making ($\mathcal{MAGDM}$). It was advanced by Ghorabee [16] and further extended by other researchers [23]. Unlike other approaches like $\mathcal{VIKOR}$ [12, 42], $\mathcal{TOPSIS}$ [3, 12, 15], $\mathcal{ELECTRE}$ [3, 4], $\mathcal{SWARA}$-$\mathcal{COPRAS}$ [13], $\mathcal{CIFS}$-$\mathcal{COPRAS}$ [44], $\mathcal{CODAS}$ [29], etc., $\mathcal{EDAS}$ [35] excels well at choosing optimal options. It is extremely effective when there are competing criteria in $\mathcal{MAGDM}$ problems. On comparing to those others, $\mathcal{EDAS}$ is based on the average solution from positive and negative distances (PDA & NDA) instead of ideal solutions. It offers possibilities to broaden this strategy with numerous tactics. These enable $\mathcal{EDAS}$ to serve as a suitable auxiliary and provide credible decision-making outputs. $\mathcal{MABAC}$ was originally presented by Pamucar and Irovic [41]. It considers competing characteristics when making decisions. A substantially more tangible and effective clumping of data can be acquired by proactively taking into account the benefit of Border Approximation Region ($\mathcal{BAR}$) to consider the decision maker's ineffability and fuzziness of the preference context. $\mathcal{MABAC}$ was applied to various applications as it can be easily paired with other strategies. Table 1 shows the comparison of literature based on different components.

Table 1: Literature-Comparative table.

Component	$\mathcal{LDFS}$ [45]	$\mathcal{BFS}$ [55, 56]	$\mathcal{BLDFS}$(proposed)
Novelty	Generalized fuzzy model with control parameters	Incorporation of both positive and negative information	Bipolar fuzzy model with control parameters
Methodology	Linear Diophantine fuzzy set	Bipolar fuzzy	Bipolar linear diophantine fuzzy
Strengths	Simplicity	Enhanced uncertainty management	Flexible, robust
Scalability	Limited	Moderate	High

1.2. Motivation and contribution

A fascinating approach to the idea of dealing with ambiguity is that, with the advancement of pairing bipolarity and control parameters, a developed methodology named $\mathcal{BLDFS}$ [39] is designed new operations and properties. This tool will be valuable in addressing difficulties in the real world by acquiring the benefits and drawbacks of any circumstances and incorporating control parameters influencing our choice to make the best option. Weighted Average Operator is proposed in a novel way which can be further developed into many operators, like Hamacher Operators or Einstein Operators. Even there are many Decision-making approaches exist in the literature, there is a wide scarcity of research on fuzzy $\mathcal{EDAS}$. To bridge the research disparity, a classic decision-making approach $\mathcal{EDAS}$ is indulged with $\mathcal{BLDFS}$ based on an analogous $\mathcal{EDAS}$ in bipolar surroundings and utilized to an $\mathcal{MAGDM}$ problem. The proposed methodology contributes by enriching the understanding of the method. The aim of making the right choice can be dealt with by our technique. When compared to other $\mathcal{MADM}$ or $\mathcal{MCDM}$ methods, this novel technique can deal with any conflicting issues and provides us a prioritizing option because of the PDA and NDA. Also, it has the ability to handle both quantitative as well as qualitative ambiguous information. In today's modern world, critical issues show the significance of bipolarity. The positive and negative evaluations simultaneously enable a flexible and nuanced analysis of conflicting criteria.

Beyond fuzzy, the ambiguous circumstances in deciding upon an appropriate $\mathcal{FDI}$ system exist, and no previous work has addressed them. So, the proposed new technique significantly uplifts as a best decision support tool in $\mathcal{FDI}$ systems. The imperative need to prioritize the selection of best $\mathcal{FDI}$ is motivated by its crucial role in criminal operations. Inefficiencies, delays, and accuracies in the case can intensify the effects of crime scenes leading to more difficulties like losing of crucial evidence, wrongful convictions, wastage of resources, misjudgment, etc. The major reason for choosing the application is that accurate and timely decisions are vital in increasing complex criminal investigations. With the integrated positive and negative evaluations it is more flexible in selecting $\mathcal{FDI}$. The proposed technology streamlines the decision process thereby reducing delays which could lead to loss of critical evidence, delayed justice, and wrongful convictions.

The major novelty of the study is the innovative introduction of $\mathcal{EDAS}$ and $\mathcal{MABAC}$ methods from a $\mathcal{BLDFS}$. A specific development of complex problems in the field of $\mathcal{FDI}$, which has never been addressed before under uncertain conditions. The incorporation of control parameters offers a better theoretical framework in the field of fuzzy. The advanced algorithm offers effective and timely solutions to practical problems and serves as a robust decision support tool, which represents an important step forward in both the forensic department and fuzzy set theory. There are many conflicting and uncertain factors in the challenging field of forensics. The proposed approach addresses these challenges more extensively. It not only improves the robustness and flexibility of the forensic department but also bridges a crucial gap in the application of fuzzy set theory to forensic decision intelligence. A combination of strategic planning, and resource allocation through the proposed methods can minimize the impact of delays in judgments. The nomenclature of this study is given in Table 2.

Table 2: Nomenclature.

Notation	Notion
$\mathcal{FS}$	Fuzzy set
$\mathcal{IFS}$	Intuitionistic fuzzy set
$\mathcal{PFS}$	Pythagorean fuzzy set
$q-\mathcal{ROFS}$	q-Rung orthopair fuzzy set
$\mathcal{LDFS}$	Linear Diophantine fuzzy set
$\mathcal{BFS}$	Bipolar fuzzy set
$\mathcal{BPFS}$	Bipolar pythagorean fuzzy set
$\mathcal{BLDFS}$	Bipolar linear Diophantine fuzzy Set
$\mathcal{BLDFEDAS}$	Bipolar linear Diophantine fuzzy evaluation based on distance from average solution
$\mathcal{BLDFMABAC}$	Bipolar linear Diophantine fuzzy multi-attributive border approximation area comparison
$\mathcal{MAGDM}$	Multi-attribute group decision-making
$\mathcal{FDI}$	Forensic decision intelligence
G	Discourse universe
H	Belongingness area
R	Non-belongingness area
E, L	Control parameters

1.3. *Flow of manuscript*

- In Section 2, we reexamine some essential concepts to fabricate our model.
- Section 3 comprises of our implemented postulation with definitions, fundamental operations, properties, and propositions.
- In Section 4, $\mathcal{BLDFEDAS}$ approach is developed in the form of an algorithm.
- Section 5 and its subsections consist of the problematic utilization of the proposed methodology by opting for an effectual $\mathcal{FDI}$ system.
- An corroboration of $\mathcal{BLDFEDAS}$ by $\mathcal{BLDFMABAC}$ is given in Section 6.
- Section 7 contains the discussion and conclusion part with future research plans.

2. Preliminaries

We recall certain fundamental descriptions that will be beneficial for subsequent concepts. G is the discourse universe throughout this manuscript.

Definition 2.1 ([54])**.** On G, $\mathcal{FS}$ is defined as $\mathcal{B} = (i, \mathsf{H}_{\mathcal{B}}(i) : i \in \mathsf{G})$, where $\mathsf{H}_{\mathcal{B}}(i)$ is the belongingness area of $\mathcal{B} \in [0, 1]$.

Example 2.2. Let $\mathsf{G} = \{i_1, i_2\}$ be two routes. A person wants to choose the best route to his destination based on the path. Then an $\mathcal{FS}$ is given as

$$\mathcal{B} = ((i_1, 0.8), (i_2, 0.4) : i_1, i_2 \in \mathsf{G}).$$

Definition 2.3 ([45])**.** On G, $\mathcal{LDFS}$ is defined as,

$$\mathcal{S} = (i, \mathsf{H}_{\mathcal{S}}(i), \mathsf{R}_{\mathcal{S}}(i), \mathsf{E}_{\mathcal{S}}(i), \mathsf{L}_{\mathcal{S}}(i) : i \in \mathsf{G}),$$

where $\mathsf{H}_{\mathcal{S}}(i), \mathsf{R}_{\mathcal{S}}(i)$, and $\mathsf{E}_{\mathcal{S}}(i), \mathsf{L}_{\mathcal{S}}(i)$ are the belongingness area, non-belongingness area, and control parameters of $\mathcal{S} \in [0, 1]$. It has a requirement that $0 \preceq \mathsf{H}_{\mathcal{S}}(i)\ \mathsf{E}_{\mathcal{S}}(i) + \mathsf{R}_{\mathcal{S}}(i)\ \mathsf{L}_{\mathcal{S}}(i) \preceq 1$ and $0 \preceq \mathsf{E}_{\mathcal{S}}(i) + \mathsf{L}_{\mathcal{S}}(i) \preceq 1$.

Definition 2.4 ([55, 56]). On G, $\mathcal{BFS}$ is defined as

$$\mathcal{H} = \left(i, \mathsf{H}^{\mathcal{P}}_{\mathcal{H}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{H}}(i) : i \in \mathsf{G}\right),$$

where $\mathsf{H}^{\mathcal{P}}_{\mathcal{H}}(i)$ is the positive belongingness area and $\mathsf{H}^{\mathcal{N}}_{\mathcal{H}}(i)$ is the negative belongingness area of $\mathcal{H}$ lies in $[0,1]$ and $[-1,0]$ subsequently.

Example 2.5. Consider Example 2.2. A $\mathcal{BFS}$ is given as

$$\mathcal{B} = ((i_1, 0.8, -0.2), (i_2, 0.4, -0.6) : i_1, i_2 \in \mathsf{G}).$$

Definition 2.6 ([1]). On G, $\mathcal{BPFS}$ is defined as

$$\mathcal{L} = \left(i, \mathsf{H}^{\mathcal{P}}_{\mathcal{L}}(i), \mathsf{R}^{\mathcal{P}}_{\mathcal{L}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{L}}(i), \mathsf{R}^{\mathcal{N}}_{\mathcal{L}}(i) : i \in \mathsf{G}\right),$$

where $\mathsf{H}^{\mathcal{P}}_{\mathcal{L}}(i), \mathsf{R}^{\mathcal{P}}_{\mathcal{L}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{L}}(i)$, and $\mathsf{R}^{\mathcal{N}}_{\mathcal{L}}(i)$ are the positive belongingness area, non-belongingness area, negative belongingness area, and non-belongingness area of $\mathcal{L}$ lies in $[0,1]$ and $[-1,0]$ subsequently. It is mandatory that $0 \preceq (\mathsf{H}^{\mathcal{P}}_{\mathcal{L}}(i))^2 + (\mathsf{R}^{\mathcal{P}}_{\mathcal{L}}(i))^2 \preceq 1$ and $-1 \preceq (\mathsf{H}^{\mathcal{N}}_{\mathcal{L}}(i))^2 + (\mathsf{R}^{\mathcal{N}}_{\mathcal{L}}(i))^2 \preceq 0$.

Definition 2.7 ([1]). Let a group of $\mathcal{BPFV}$, $\mathfrak{J}_s$, be defined by $\mathfrak{J}_s = \left(\mathsf{H}^{\mathcal{P}}_s, \mathsf{R}^{\mathcal{P}}_s, \mathsf{H}^{\mathcal{N}}_s, \mathsf{R}^{\mathcal{N}}_s\right)$, where $s = 1, 2, \ldots, \mathrm{j}$. Then

$$\text{Ł}_{\mathfrak{f}}(\mathfrak{J}_1, \mathfrak{J}_2, \ldots, \mathfrak{J}_s) = \sum_{s=1}^{\mathrm{j}} \mathfrak{f}_s \mathfrak{J}_s = \left[\left(\sqrt{1 - \prod_{s=1}^{\mathrm{j}} (1 - (\mathsf{H}^{\mathcal{P}}_s)^2)^{\mathfrak{f}_s}}\right), \left(\prod_{s=1}^{\mathrm{j}} (\mathsf{R}^{\mathcal{P}}_s)^{\mathfrak{f}_s}\right), -\left(\prod_{s=1}^{\mathrm{j}} (-\mathsf{H}^{\mathcal{N}}_s)^{\mathfrak{f}_s}\right), -\left(\sqrt{1 - \prod_{s=1}^{\mathrm{j}} (1 - ((-\mathsf{R}^{\mathcal{N}}_s)^2))^{\mathfrak{f}_s}}\right)\right],$$

where the weight for $\mathfrak{J}_s$, $s = 1, 2, \ldots, \mathrm{j}$ is given as $\mathfrak{f} = [\mathfrak{f}_1, \mathfrak{f}_2, \ldots, \mathfrak{f}_{\mathrm{j}}]^{\mathcal{T}}$ with $\mathfrak{f}_s \succ 0$ and $\sum_{s=1}^{\mathrm{j}} \mathfrak{f}_s = 1$ is a Bipolar Pythagorean Fuzzy Weighted Average ($\mathcal{BPFWA}$) operator.

Definition 2.8 ([45]). Consider a Linear Diophantine Fuzzy Value ($\mathcal{LDFV}$), $\mathcal{O}$. The scoring value, $\mathcal{SV}(\mathcal{O})$, is given as

$$\mathcal{SV}(\mathcal{O}) = \frac{[(\mathsf{H}_{\mathcal{O}} - \mathsf{R}_{\mathcal{O}}) + (\mathsf{E}_{\mathcal{O}} - \mathsf{L}_{\mathcal{O}})]}{2}, \quad \forall \mathcal{SV}(\mathcal{O}) \in [-1, 1].$$

Definition 2.9 ([45]). Consider a $\mathcal{LDFV}$, $\mathcal{W}$. The accuracy value, $\mathcal{AV}(\mathcal{W})$, is given as

$$\mathcal{AV}(\mathcal{W}) = \frac{\left[\frac{(\mathsf{H}_{\mathcal{W}} + \mathsf{R}_{\mathcal{W}})}{2} + (\mathsf{E}_{\mathcal{W}} + \mathsf{L}_{\mathcal{W}})\right]}{2}, \quad \forall \mathcal{AV}(\mathcal{W}) \in [0, 1].$$

Definition 2.10 ([21]). Consider two $\mathcal{BFS}$, then the $\mathcal{BF}$ normalized hamming distance is given by

$$d_{\mathcal{BFNHD}} = \frac{((|\mathsf{H}^{\mathcal{P}}_1 - \mathsf{H}^{\mathcal{P}}_2| + |\mathsf{H}^{\mathcal{N}}_1 - \mathsf{H}^{\mathcal{N}}_2|)}{2}.$$

3. Some operators and functions on $\mathcal{BLDFS}$

An emerging concept $\mathcal{BLDFS}$ is implemented, and some operations & functions describing $\mathcal{BLDFS}$ environment are offered in this part.

Definition 3.1. $\mathcal{BLDFS}$ w is a portion on the discourse universe G,

$$\mathsf{w} = \left\{\left(i, (\mathsf{H}^{\mathcal{P}}_{\mathsf{w}}(i), \mathsf{R}^{\mathcal{P}}_{\mathsf{w}}(i), \mathsf{H}^{\mathcal{N}}_{\mathsf{w}}(i), \mathsf{R}^{\mathcal{N}}_{\mathsf{w}}(i)), (\mathsf{E}^{\mathcal{P}}_{\mathsf{w}}(i), \mathsf{L}^{\mathcal{P}}_{\mathsf{w}}(i), \mathsf{E}^{\mathcal{N}}_{\mathsf{w}}(i), \mathsf{L}^{\mathcal{N}}_{\mathsf{w}}(i))\right) : i \in \mathsf{G}\right\},$$

where

- $\mathsf{H}^{\mathcal{P}}_{\mathsf{w}}(i)$ and $\mathsf{R}^{\mathcal{P}}_{\mathsf{w}}(i)$ are positive belongingness and non-belongingness area, $\mathsf{E}^{\mathcal{P}}_{\mathsf{w}}(i)$ and $\mathsf{L}^{\mathcal{P}}_{\mathsf{w}}(i)$ are the control parameters in $[0,1]$ to a $\mathcal{BLDFS}$ w.
- $\mathsf{H}^{\mathcal{N}}_{\mathsf{w}}(i)$ and $\mathsf{R}^{\mathcal{N}}_{\mathsf{w}}(i)$ are negative belongingness and non-belongingness area, $\mathsf{E}^{\mathcal{N}}_{\mathsf{w}}(i)$ and $\mathsf{L}^{\mathcal{N}}_{\mathsf{w}}(i)$ are the control parameters in $[-1,0]$ to a $\mathcal{BLDFS}$ w.

The succeeding prerequisites are ought to be met. For all $i \in \mathsf{G}$:

(i) $0 \preceq \mathsf{E}^{\mathcal{P}}_{\mathsf{w}}(i)\mathsf{H}^{\mathcal{P}}_{\mathsf{w}}(i) + \mathsf{L}^{\mathcal{P}}_{\mathsf{w}}(i)\mathsf{R}^{\mathcal{P}}_{\mathsf{w}}(i) \preceq 1$;
(ii) $0 \preceq \mathsf{E}^{\mathcal{N}}_{\mathsf{w}}(i)\mathsf{H}^{\mathcal{N}}_{\mathsf{w}}(i) + \mathsf{L}^{\mathcal{N}}_{\mathsf{w}}(i)\mathsf{R}^{\mathcal{N}}_{\mathsf{w}}(i) \preceq 1$;
(iii) $0 \preceq \mathsf{E}^{\mathcal{P}}_{\mathsf{w}}(i) + \mathsf{L}^{\mathcal{P}}_{\mathsf{w}}(i) \preceq 1$;
(iv) $-1 \preceq \mathsf{E}^{\mathcal{N}}_{\mathsf{w}}(i) + \mathsf{L}^{\mathcal{N}}_{\mathsf{w}}(i) \preceq 0$.

$\mathcal{BLDFS}$ is represented in the form of graph in Figure 1, through which the broader scope of the extended fuzzy set is seen.

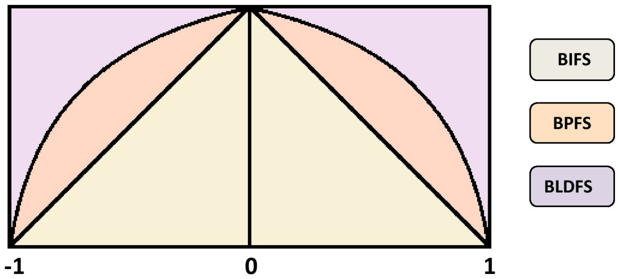

Figure 1: Graphical representation of the $\mathcal{BLDFS}$.

A Portion

$$\mathsf{w} = \left\{\left((\mathsf{H}^{\mathcal{P}}_{\mathsf{w}}, \mathsf{R}^{\mathcal{P}}_{\mathsf{w}}, \mathsf{H}^{\mathcal{N}}_{\mathsf{w}}, \mathsf{R}^{\mathcal{N}}_{\mathsf{w}}), (\mathsf{E}^{\mathcal{P}}_{\mathsf{w}}, \mathsf{L}^{\mathcal{P}}_{\mathsf{w}}, \mathsf{E}^{\mathcal{N}}_{\mathsf{w}}, \mathsf{L}^{\mathcal{N}}_{\mathsf{w}})\right)\right\},$$

represents bipolar linear Diophantine fuzzy values ($\mathcal{BLDFV}$).

Example 3.2. Consider a squad of boutique $\mathsf{G} = \{i_1, i_2\}$. A $\mathcal{BLDFV}$ can be given as $\mathsf{w} = \{(i_1, (0.7, 0.1, -0.4, -0.3), (0.6, 0.4, -0.8, -0.1)), (i_2, (0.8, 0.2, -0.6, -0.1), (0.5, 0.3, -0.3, -0.2))\}$, where the control parameters are considered to be $\mathsf{E}^{\mathcal{P}}_{\mathsf{w}}$ = *quality cloth*, $\mathsf{L}^{\mathcal{P}}_{\mathsf{w}}$ = *not a quality cloth*, $\mathsf{E}^{\mathcal{N}}_{\mathsf{w}}$ = *inferiority cloth* and $\mathsf{L}^{\mathcal{N}}_{\mathsf{w}}$ = *not an inferiority cloth*.

Definition 3.3. Consider two $\mathcal{BLDFS}$

$$\mathcal{K} = \left\{\left(i, (\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i)), (\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i))\right) | i \in \mathsf{G}\right\},$$
$$\mathcal{M} = \left\{\left(i, (\mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i), \mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i), \mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i)), (\mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i), \mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i), \mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i), \mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i))\right) : i \in \mathsf{G}\right\}.$$

Then, the subsequent operations are valid.

(a) $\mathcal{K} \subseteq \mathcal{M} \iff \mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i) \preceq \mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i)$, $\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i) \succeq \mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i)$, $\mathsf{E}^{\mathcal{P}}_{\mathcal{K}} \preceq \mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i)$, $\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i) \succeq \mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i)$, $\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i) \succeq \mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i)$, $\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i) \preceq \mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i)$, $\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i) \succeq \mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i)$, $\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i) \preceq \mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i)$.
(b) $\mathcal{K} = \mathcal{M} \iff \mathcal{K} \subseteq \mathcal{M}$ and $\mathcal{M} \subseteq \mathcal{K}$.
(c) $\mathcal{K}^{\mathfrak{C}} = \left\{\left(i, (\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i)), (\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i)\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i))\right) : i \in \mathsf{G}\right\}$.
(d) $\mathcal{K} \cap \mathcal{M} = \{(i, (\mathfrak{min}(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i)), \mathfrak{max}(\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i)), \mathfrak{max}(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i)), \mathfrak{min}(\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i))), (\mathfrak{min}(\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i)), \mathfrak{max}(\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i)), \mathfrak{max}(\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i)), \mathfrak{min}(\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i))))\}$.
(e) $\mathcal{K} \cup \mathcal{M} = \{(i, (\mathfrak{max}(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i)), \mathfrak{min}(\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i)), \mathfrak{min}(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i)), \mathfrak{max}(\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i))), (\mathfrak{max}(\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i)), \mathfrak{min}(\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i)), \mathfrak{min}(\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i)), \mathfrak{max}(\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i))))\}$.

Definition 3.4. A $\mathcal{BLDFS}$ on G is called

(i) Null: $\mathcal{K}_{\mathcal{N}} = \{(i,(0,1,0,-1),(0,1,0,-1)) : i \in \mathsf{G}\}$;
(ii) Absolute: $\mathcal{K}_{\mathcal{A}} = \{(i,(1,0,-1,0),(1,0,-1,0)) : i \in \mathsf{G}\}$.

Proposition 3.5. *Consider a $\mathcal{BLDFS}$, $\mathcal{K}$. Then*

(a) $(\mathcal{K}^{\mathfrak{C}})^{\mathfrak{C}} = \mathcal{K}$;
(b) $(\mathcal{K}_{\mathcal{A}})^{\mathfrak{C}} = \mathcal{K}_{\mathcal{N}}$;
(c) $(\mathcal{K}_{\mathcal{N}}) \subseteq \mathcal{K}$;
(d) $\mathcal{K} \subseteq (\mathcal{K}_{\mathcal{A}})$.

Proposition 3.6. *Let*

$$\mathcal{K}_{\mathfrak{r}} = \left\{\left(i,(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}_{\mathfrak{r}}}(i),\mathsf{R}^{\mathcal{P}}_{\mathcal{K}_{\mathfrak{r}}}(i),\mathsf{H}^{\mathcal{N}}_{\mathcal{K}_{\mathfrak{r}}}(i),\mathsf{R}^{\mathcal{N}}_{\mathcal{K}_{\mathfrak{r}}}(i)),(\mathsf{E}^{\mathcal{P}}_{\mathcal{K}_{\mathfrak{r}}}(i),\mathsf{L}^{\mathcal{P}}_{\mathcal{K}_{\mathfrak{r}}}(i),\mathsf{E}^{\mathcal{N}}_{\mathcal{K}_{\mathfrak{r}}}(i),\mathsf{L}^{\mathcal{N}}_{\mathcal{K}_{\mathfrak{r}}}(i))\right) : i \in \mathsf{G}\right\},\ \mathfrak{r} = 1,2,3,$$

be a three $\mathcal{BLDFS}$. Then

(a) $\mathcal{K}_1 \cup \mathcal{K}_2 = \mathcal{K}_2 \cup \mathcal{K}_1$;
(b) $\mathcal{K}_1 \cap \mathcal{K}_2 = \mathcal{K}_2 \cap \mathcal{K}_1$;
(c) $\mathcal{K}_1 \cup (\mathcal{K}_2 \cup \mathcal{K}_3) = (\mathcal{K}_1 \cup \mathcal{K}_2) \cup \mathcal{K}_3$;
(d) $\mathcal{K}_1 \cap (\mathcal{K}_2 \cap \mathcal{K}_3) = (\mathcal{K}_1 \cap \mathcal{K}_2) \cap \mathcal{K}_3$.

Proof. Since the $\mathfrak{max}$ and $\mathfrak{min}$ functions are associative and commutative, the proof holds simply. □

Proposition 3.7. *Let a two $\mathcal{BLDFS}$ on G be, $\mathcal{K}$ and $\mathcal{M}$. Then De Morgan's law holds, i.e.,*

(a) $(\mathcal{K} \cap \mathcal{M})\mathfrak{C} = \mathcal{K}^{\mathfrak{C}} \cup \mathcal{M}^{\mathfrak{C}}$;
(b) $(\mathcal{K} \cup \mathcal{M})^{\mathfrak{C}} = \mathcal{K}^{\mathfrak{C}} \cap \mathcal{M}^{\mathfrak{C}}$.

Proof.

$$\begin{aligned}(\mathfrak{a})(\mathcal{K}\cap\mathcal{M})^{\mathfrak{C}} &= \Big\{\Big(i,(\mathfrak{min}(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i))),\\ &\qquad(\mathfrak{min}(\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i)))\Big)\Big\}^{\mathfrak{C}}\\ (\mathcal{K}\cap\mathcal{M})^{\mathfrak{C}} &= \Big\{\Big(i,(\mathfrak{max}(\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i))),\\ &\qquad(\mathfrak{max}(\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i)))\Big)\Big\}.\end{aligned} \tag{3.1}$$

By Definition 3.3 (c),

$$\begin{aligned}\mathcal{K}^{\mathfrak{C}} &= \Big\{\Big(i,(\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i)),(\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i)\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i))\Big) : i\in\mathsf{G}\Big\},\\ \mathcal{M}^{\mathfrak{C}} &= \Big\{\Big(i,(\mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i),\mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i),\mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i),\mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i)),(\mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i),\mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i),\mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i)\mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i))\Big) : i\in\mathsf{G}\Big\}.\end{aligned}$$

By Definition 3.3 (e),

$$\begin{aligned}\mathcal{K}^{\mathfrak{C}}\cup\mathcal{M}^{\mathfrak{C}} = \Big\{\Big(i,&(\mathfrak{max}(\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i)),\ \mathfrak{min}(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i)),\ \mathfrak{min}(\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i)),\ \mathfrak{max}(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i))),\\ &(\mathfrak{max}(\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i)),\ \mathfrak{min}(\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i),\ \mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i)),\ \mathfrak{min}(\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i)))\Big)\Big\}.\end{aligned} \tag{3.2}$$

From equations (3.1) and (3.2), we have $(\mathcal{K}\cap\mathcal{M})^{\mathfrak{C}} = \mathcal{K}^{\mathfrak{C}}\cup\mathcal{M}^{\mathfrak{C}}$. Hence the proof.

$$\begin{aligned}(\mathfrak{b})(\mathcal{K}\cup\mathcal{M})^{\mathfrak{C}} &= \Big\{\Big(i,(\mathfrak{max}(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i)),\ \mathfrak{min}(\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i))),\\ &\qquad(\mathfrak{max}(\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i),\ \mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i)))\Big)\Big\}^{\mathfrak{C}}\\ (\mathcal{K}\cup\mathcal{M})^{\mathfrak{C}} &= \Big\{\Big(i,(\mathfrak{min}(\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i)),\ \mathfrak{max}(\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i)),\ \mathfrak{min}(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i))),\\ &\qquad(\mathfrak{min}(\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i)),\mathfrak{max}(\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i),\ \mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i)),\ \mathfrak{max}(\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i)),\mathfrak{min}(\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i),\mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i)))\Big)\Big\}.\end{aligned} \tag{3.3}$$

By Definition 3.3 (c),

$$\mathcal{K}^{\mathfrak{C}} = \left\{ \left(i, (\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i)), (\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i)\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i)) \right) : i \in \mathsf{G} \right\},$$
$$\mathcal{M}^{\mathfrak{C}} = \left\{ \left(i, (\mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i), \mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i), \mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i)), (\mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i), \mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i), \mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i)\mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i)) \right) : i \in \mathsf{G} \right\}.$$

By Definition 3.3 (d),

$$\begin{aligned}\mathcal{K}^{\mathfrak{C}} \cap \mathcal{M}^{\mathfrak{C}} = \Big\{ \Big(i, (&\min(\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{P}}_{\mathcal{M}}(i)), \max(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{P}}_{\mathcal{M}}(i)), \ \max(\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{R}^{\mathcal{N}}_{\mathcal{M}}(i)), \ \min(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{H}^{\mathcal{N}}_{\mathcal{M}}(i))), \\ &(\min(\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{P}}_{\mathcal{M}}(i)), \max(\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}(i), \ \mathsf{E}^{\mathcal{P}}_{\mathcal{M}}(i)), \ \max(\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{L}^{\mathcal{N}}_{\mathcal{M}}(i)), \min(\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}(i), \mathsf{E}^{\mathcal{N}}_{\mathcal{M}}(i))) \Big) \Big\}.\end{aligned} \tag{3.4}$$

From equation (3.3) & (3.4), we have $(\mathcal{K} \cup \mathcal{M})^{\mathfrak{C}} = \mathcal{K}^{\mathfrak{C}} \cap \mathcal{M}^{\mathfrak{C}}$. Hence the proof. □

Definition 3.8. Let us consider a two $\mathcal{BLDFV}$, $\mathcal{K}$ and $\mathcal{M}$. Let $\pi \geqslant 0$, then some fundamental operators are given below:

$$\begin{aligned}
\mathcal{K}^{\pi} &= \Big\{ \Big((\mathsf{H}^{\mathcal{P}}_{\mathcal{K}})^{\pi}, 1-(1-\mathsf{R}^{\mathcal{P}}_{\mathcal{K}})^{\pi}, -(1-(1-(-\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}))^{\pi}), -(-\mathsf{R}^{\mathcal{N}}_{\mathcal{K}})^{\pi} \Big), \Big((\mathsf{E}^{\mathcal{P}}_{\mathcal{K}})^{\pi}, 1-(1 \\
&\quad -\mathsf{L}^{\mathcal{P}}_{\mathcal{K}})^{\pi}, -(1-(1-(-\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}))^{\pi}), -(-\mathsf{L}^{\mathcal{N}}_{\mathcal{K}})^{\pi} \Big) \Big\}, \\
\pi\mathcal{K} &= \Big\{ \Big((1-(1-\mathsf{H}^{\mathcal{P}}_{\mathcal{K}})^{\pi}, (\mathsf{R}^{\mathcal{P}}_{\mathcal{K}})^{\pi}, -(-\mathsf{H}^{\mathcal{N}}_{\mathcal{K}})^{\pi}, -(1-(1-(-\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}))^{\pi}) \Big) \Big(1-(1-\mathsf{E}^{\mathcal{P}}_{\mathcal{K}})^{\pi}, \\
&\quad (\mathsf{L}^{\mathcal{P}}_{\mathcal{K}})^{\pi}, -(-\mathsf{E}^{\mathcal{N}}_{\mathcal{K}})^{\pi}, -(1-(1-(-\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}))^{\pi}) \Big) \Big\}, \\
\mathcal{K} \boxtimes \mathcal{M} &= \Big\{ \Big(((\mathsf{H}^{\mathcal{P}}_{\mathcal{K}}).(\mathsf{H}^{\mathcal{P}}_{\mathcal{M}})), (\mathsf{R}^{\mathcal{P}}_{\mathcal{K}} + \mathsf{R}^{\mathcal{P}}_{\mathcal{M}} - \mathsf{R}^{\mathcal{P}}_{\mathcal{K}}.\mathsf{R}^{\mathcal{P}}_{\mathcal{M}}), -((-\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}) + (-\mathsf{H}^{\mathcal{N}}_{\mathcal{M}}) - (-\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}.\mathsf{H}^{\mathcal{N}}_{\mathcal{M}})), -(\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}.\mathsf{R}^{\mathcal{N}}_{\mathcal{M}}) \Big) \\
&\quad \Big(((\mathsf{E}^{\mathcal{P}}_{\mathcal{K}}).(\mathsf{E}^{\mathcal{P}}_{\mathcal{M}})), (\mathsf{L}^{\mathcal{P}}_{\mathcal{K}} + \mathsf{L}^{\mathcal{P}}_{\mathcal{M}} - \mathsf{L}^{\mathcal{P}}_{\mathcal{K}}.\mathsf{L}^{\mathcal{P}}_{\mathcal{M}}), -((-\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}) + (-\mathsf{E}^{\mathcal{N}}_{\mathcal{M}}) - (-\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}.\mathsf{E}^{\mathcal{N}}_{\mathcal{M}})), -(\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}.\mathsf{L}^{\mathcal{N}}_{\mathcal{M}}) \Big) \Big\}, \\
(iv)\mathcal{K} \boxplus \mathcal{M} &= \Big\{ \Big((\mathsf{H}^{\mathcal{P}}_{\mathcal{K}} + \mathsf{H}^{\mathcal{P}}_{\mathcal{M}} - \mathsf{H}^{\mathcal{P}}_{\mathcal{K}}.\mathsf{H}^{\mathcal{P}}_{\mathcal{M}}), ((\mathsf{R}^{\mathcal{P}}_{\mathcal{K}}).(\mathsf{R}^{\mathcal{P}}_{\mathcal{M}})), -(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}}.\mathsf{H}^{\mathcal{N}}_{\mathcal{M}}), -((-\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}) + (-\mathsf{R}^{\mathcal{N}}_{\mathcal{M}}) - (-\mathsf{R}^{\mathcal{N}}_{\mathcal{K}}.\mathsf{R}^{\mathcal{N}}_{\mathcal{M}})) \Big) \\
&\quad \Big((\mathsf{E}^{\mathcal{P}}_{\mathcal{K}} + \mathsf{E}^{\mathcal{P}}_{\mathcal{M}} - \mathsf{E}^{\mathcal{P}}_{\mathcal{K}}.\mathsf{E}^{\mathcal{P}}_{\mathcal{M}}), ((\mathsf{L}^{\mathcal{P}}_{\mathcal{K}}).(\mathsf{L}^{\mathcal{P}}_{\mathcal{M}})), -(\mathsf{E}^{\mathcal{N}}_{\mathcal{K}}.\mathsf{E}^{\mathcal{N}}_{\mathcal{M}}), -((-\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}) + (-\mathsf{L}^{\mathcal{N}}_{\mathcal{M}}) - (-\mathsf{L}^{\mathcal{N}}_{\mathcal{K}}.\mathsf{L}^{\mathcal{N}}_{\mathcal{M}})) \Big) \Big\}.
\end{aligned}$$

3.1. *Some fundamental functions on $\mathcal{BLDFS}$*

The scoring value, accuracy value and $\mathcal{BLDFWA}$ operator are established in this section.

Definition 3.9. Consider a $\mathcal{BLDFV}$, $\mathcal{K}$.

(i) The scoring value $\mathcal{SV}(\mathcal{K})$ is given as

$$\mathcal{SV}(\mathcal{K}) = \frac{\{((\mathsf{H}^{\mathcal{P}}_{\mathcal{K}} - \mathsf{R}^{\mathcal{P}}_{\mathcal{K}}) + (\mathsf{E}^{\mathcal{P}}_{\mathcal{K}} - \mathsf{L}^{\mathcal{P}}_{\mathcal{K}})) - ((\mathsf{H}^{\mathcal{N}}_{\mathcal{K}} - \mathsf{R}^{\mathcal{N}}_{\mathcal{K}}) + (\mathsf{E}^{\mathcal{N}}_{\mathcal{K}} - \mathsf{L}^{\mathcal{N}}_{\mathcal{K}}))\}}{4}, \forall \mathcal{SV}(\mathcal{K}) \in [-1, 1].$$

(ii) The accuracy value $\mathcal{AV}(\mathcal{K})$ is given as

$$\mathcal{AV}(\mathcal{K}) = \frac{\{((\frac{1}{2}(\mathsf{H}^{\mathcal{P}}_{\mathcal{K}} + \mathsf{R}^{\mathcal{P}}_{\mathcal{K}})) + (\mathsf{E}^{\mathcal{P}}_{\mathcal{K}} + \mathsf{L}^{\mathcal{P}}_{\mathcal{K}})) - ((\frac{1}{2}(\mathsf{H}^{\mathcal{N}}_{\mathcal{K}} + \mathsf{R}^{\mathcal{N}}_{\mathcal{K}})) + (\mathsf{E}^{\mathcal{N}}_{\mathcal{K}} + \mathsf{L}^{\mathcal{N}}_{\mathcal{K}}))\}}{4}, \forall \mathcal{AV}(\mathcal{K}) \in [0, 1].$$

Consider two $\mathcal{BLDFV}$, $\mathcal{K}$ and $\mathcal{M}$. Then if

(1) $\mathcal{SV}(\mathcal{M}) \prec \mathcal{SV}(\mathcal{K})$, then $\mathcal{M} \prec \mathcal{K}$;
(2) $\mathcal{SV}(\mathcal{M}) \succ \mathcal{SV}(\mathcal{K})$, then $\mathcal{M} \succ \mathcal{K}$;
(3) $\mathcal{SV}(\mathcal{M}) = \mathcal{SV}(\mathcal{K})$, then
  (a) $\mathcal{AV}(\mathcal{M}) \prec \mathcal{AV}(\mathcal{K})$, then $\mathcal{M} \prec \mathcal{K}$;
  (b) $\mathcal{AV}(\mathcal{M}) \succ \mathcal{AV}(\mathcal{K})$, then $\mathcal{M} \succ \mathcal{K}$;

(c) $\mathcal{AV}(\mathcal{M}) = \mathcal{AV}(\mathcal{K})$, then $\mathcal{M} = \mathcal{K}$.

Definition 3.10. Let a group of $\mathcal{BLDFV}$, $\mathfrak{J}_s$ be defined by, $\mathfrak{J}_s = \{(\mathsf{H}_s^{\mathcal{P}}, \mathsf{R}_s^{\mathcal{P}}, \mathsf{H}_s^{\mathcal{N}}, \mathsf{R}_s^{\mathcal{N}}), (\mathsf{E}_s^{\mathcal{P}}, \mathsf{L}_s^{\mathcal{P}}, \mathsf{E}_s^{\mathcal{N}}, \mathsf{L}_s^{\mathcal{N}})\}$, where $s = 1, 2, \ldots, \mathrm{j}$. Then

$$
\begin{aligned}
Ł_{\mathfrak{f}}(\mathfrak{J}_1, \mathfrak{J}_2, \ldots, \mathfrak{J}_s) &= \sum_{s=1}^{\mathrm{j}} \mathfrak{f}_s \mathfrak{J}_s \\
&= \Bigg[\Bigg(\left(1 - \prod_{s=1}^{\mathrm{j}} (1 - \mathsf{H}_s^{\mathcal{P}})^{\mathfrak{f}_s}\right), \left(\prod_{s=1}^{\mathrm{j}} (\mathsf{R}_s^{\mathcal{P}})^{\mathfrak{f}_s}\right), -\left(\prod_{s=1}^{\mathrm{j}} (-\mathsf{H}_s^{\mathcal{N}})^{\mathfrak{f}_s}\right), -\left(1 - \prod_{s=1}^{\mathrm{j}} \left(1 - (-\mathsf{R}_s^{\mathcal{N}})\right)^{\mathfrak{f}_s}\right) \Bigg), \\
&\quad \Bigg(\left(1 - \prod_{s=1}^{\mathrm{j}} (1 - \mathsf{E}_s^{\mathcal{P}})^{\mathfrak{f}_s}\right), \left(\prod_{s=1}^{\mathrm{j}} (\mathsf{L}_s^{\mathcal{P}})^{\mathfrak{f}_s}\right), -\left(\prod_{s=1}^{\mathrm{j}} (-\mathsf{E}_s^{\mathcal{N}})^{\mathfrak{f}_s}\right), -\left(1 - \prod_{s=1}^{\mathrm{j}} \left(1 - (-\mathsf{L}_s^{\mathcal{N}})\right)^{\mathfrak{f}_s}\right) \Bigg) \Bigg],
\end{aligned}
$$

where the weight for $\mathfrak{J}_s$, $s = 1, 2, \ldots, \mathrm{j}$ is given as $\mathfrak{f} = [\mathfrak{f}_1, \mathfrak{f}_2, \ldots, \mathfrak{f}_{\mathrm{j}}]^{\mathcal{T}}$ with $\mathfrak{f}_s \succ 0$ and $\sum_{s=1}^{\mathrm{j}} \mathfrak{f}_s = 1$ is a $\mathcal{BLDFWA}$ operator.

Example 3.11. Let $\mathfrak{J}_1 = \{(0.3, 0.8, -0.3, -0.4)(0.6, 0.1, -0.4, -0.3)\}$ and $\mathfrak{J}_2 = \{(0.5, 0.1, -0.7, -0.7)(0.6, 0.2, -0.3, -0.4)\}$. Then,

$$
Ł_{\mathfrak{f}} = \{(0.6534, 0.1860, -0.5729, -0.6592)(0.6, 0.1614, -0.3391, -0.3591)\}.
$$

4. Bipolar linear Diophantine fuzzy $\mathcal{EDAS}$ approach

This part contains an approach for addressing an $\mathcal{MAGDM}$ issue by means of Bipolar Linear Diophantine Fuzzy Weighted Average ($\mathcal{BLDFWA}$) operator through $\mathcal{EDAS}$.

Let us consider a group of η alternatives $\{\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_\eta\}$ and a group of t attributes $\{\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_t\}$ associated with $\{\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_t\}$ weight vectors. Let $\{\mathcal{Q}_1, \mathcal{Q}_2, \ldots, \mathcal{Q}_E\}$ experts with corresponding vector to be $\{\mathcal{U}_1, \mathcal{U}_2, \ldots, \mathcal{U}_E\}$, then the evaluation matrix of the $\mathcal{BLDFEDAS}$ is given by

$$
\xi = [\mathcal{L}_{\mathfrak{a}\mathfrak{s}}^{E}]_{\eta \times t} = \left\{ ((\mathsf{H}_{\mathfrak{a}\mathfrak{s}}^{\mathcal{P}})^E, (\mathsf{R}_{\mathfrak{a}\mathfrak{s}}^{\mathcal{P}})^E, (\mathsf{H}_{\mathfrak{a}\mathfrak{s}}^{\mathcal{N}})^E, (\mathsf{R}_{\mathfrak{a}\mathfrak{s}}^{\mathcal{N}})^E), ((\mathsf{E}_{\mathfrak{a}\mathfrak{s}}^{\mathcal{P}})^E, (\mathsf{L}_{\mathfrak{a}\mathfrak{s}}^{\mathcal{P}})^E, (\mathsf{E}_{\mathfrak{a}\mathfrak{s}}^{\mathcal{N}})^E, (\mathsf{L}_{\mathfrak{a}\mathfrak{s}}^{\mathcal{N}})^E) \right\}_{\eta \times t},
$$

$\mathfrak{a} = 1, 2, \ldots, \eta$, $\mathfrak{s} = 1, 2, \ldots, t$, $E = 1, 2, \ldots, \mathcal{F}$.

An algorithmic approach on $\mathcal{BLDFEDAS}$ is as follows.

Step 1. The $\mathcal{BLDF}$ matrix formulation evaluated by $\xi = [\mathcal{L}_{\mathfrak{a}\mathfrak{s}}^{E}]_{\eta \times t}$ is given by

$$
\xi = \begin{array}{c} \\ \mathcal{L}_1 \\ \\ \mathcal{L}_2 \\ \\ \vdots \\ \mathcal{L}_\eta \\ \\ \end{array}
\begin{array}{c}
\begin{array}{cccc} \quad\mathcal{T}_1 & \quad\quad\mathcal{T}_2 & \cdots & \quad\mathcal{T}_t \end{array} \\
\begin{pmatrix}
\begin{matrix}\{((\mathsf{H}_{11}^{\mathcal{P}})^E, (\mathsf{R}_{11}^{\mathcal{P}})^E, (\mathsf{H}_{11}^{\mathcal{N}})^E, (\mathsf{R}_{11}^{\mathcal{N}})^E), \\ ((\mathsf{E}_{11}^{\mathcal{P}})^E, (\mathsf{L}_{11}^{\mathcal{P}})^E, (\mathsf{E}_{11}^{\mathcal{N}})^E, (\mathsf{L}_{11}^{\mathcal{N}})^E)\}\end{matrix} & \begin{matrix}\{((\mathsf{H}_{12}^{\mathcal{P}})^E, (\mathsf{R}_{12}^{\mathcal{P}})^E, (\mathsf{H}_{12}^{\mathcal{N}})^E, (\mathsf{R}_{12}^{\mathcal{N}})^E), \\ ((\mathsf{E}_{12}^{\mathcal{P}})^E, (\mathsf{L}_{12}^{\mathcal{P}})^E, (\mathsf{E}_{12}^{\mathcal{N}})^E, (\mathsf{L}_{12}^{\mathcal{N}})^E)\}\end{matrix} & \begin{matrix}\cdots \\ \cdots\end{matrix} & \begin{matrix}\{((\mathsf{H}_{1t}^{\mathcal{P}})^E, (\mathsf{R}_{1t}^{\mathcal{P}})^E, (\mathsf{H}_{1t}^{\mathcal{N}})^E, (\mathsf{R}_{1t}^{\mathcal{N}})^E), \\ ((\mathsf{E}_{1t}^{\mathcal{P}})^E, (\mathsf{L}_{1t}^{\mathcal{P}})^E, (\mathsf{E}_{1t}^{\mathcal{N}})^E, (\mathsf{L}_{1t}^{\mathcal{N}})^E)\}\end{matrix} \\
\begin{matrix}\{((\mathsf{H}_{21}^{\mathcal{P}})^E, (\mathsf{R}_{21}^{\mathcal{P}})^E, (\mathsf{H}_{21}^{\mathcal{N}})^E, (\mathsf{R}_{21}^{\mathcal{N}})^E), \\ ((\mathsf{E}_{21}^{\mathcal{P}})^E, (\mathsf{L}_{21}^{\mathcal{P}})^E, (\mathsf{E}_{21}^{\mathcal{N}})^E, (\mathsf{L}_{21}^{\mathcal{N}})^E)\}\end{matrix} & \begin{matrix}\{((\mathsf{H}_{22}^{\mathcal{P}})^E, (\mathsf{R}_{22}^{\mathcal{P}})^E, (\mathsf{H}_{22}^{\mathcal{N}})^E, (\mathsf{R}_{22}^{\mathcal{N}})^E), \\ ((\mathsf{E}_{22}^{\mathcal{P}})^E, (\mathsf{L}_{22}^{\mathcal{P}})^E, (\mathsf{E}_{22}^{\mathcal{N}})^E, (\mathsf{L}_{22}^{\mathcal{N}})^E)\}\end{matrix} & \begin{matrix}\cdots \\ \cdots\end{matrix} & \begin{matrix}\{((\mathsf{H}_{2t}^{\mathcal{P}})^E, (\mathsf{R}_{2t}^{\mathcal{P}})^E, (\mathsf{H}_{2t}^{\mathcal{N}})^E, (\mathsf{R}_{2t}^{\mathcal{N}})^E), \\ ((\mathsf{E}_{2t}^{\mathcal{P}})^E, (\mathsf{L}_{2t}^{\mathcal{P}})^E, (\mathsf{E}_{2t}^{\mathcal{N}})^E, (\mathsf{L}_{2t}^{\mathcal{N}})^E)\}\end{matrix} \\
\vdots & \vdots & \vdots & \vdots \\
\begin{matrix}\{((\mathsf{H}_{\eta1}^{\mathcal{P}})^E, (\mathsf{R}_{\eta1}^{\mathcal{P}})^E, (\mathsf{H}_{\eta1}^{\mathcal{N}})^E, (\mathsf{R}_{\eta1}^{\mathcal{N}})^E), \\ ((\mathsf{E}_{\eta1}^{\mathcal{P}})^E, (\mathsf{L}_{\eta1}^{\mathcal{P}})^E, (\mathsf{E}_{\eta1}^{\mathcal{N}})^E, (\mathsf{L}_{\eta1}^{\mathcal{N}})^E)\}\end{matrix} & \begin{matrix}\{((\mathsf{H}_{\eta2}^{\mathcal{P}})^E, (\mathsf{R}_{\eta2}^{\mathcal{P}})^E, (\mathsf{H}_{\eta2}^{\mathcal{N}})^E, (\mathsf{R}_{\eta2}^{\mathcal{N}})^E), \\ ((\mathsf{E}_{\eta2}^{\mathcal{P}})^E, (\mathsf{L}_{\eta2}^{\mathcal{P}})^E, (\mathsf{E}_{\eta2}^{\mathcal{N}})^E, (\mathsf{L}_{\eta2}^{\mathcal{N}})^E)\}\end{matrix} & \begin{matrix}\cdots \\ \cdots\end{matrix} & \begin{matrix}\{((\mathsf{H}_{\eta t}^{\mathcal{P}})^E, (\mathsf{R}_{\eta t}^{\mathcal{P}})^E, (\mathsf{H}_{\eta t}^{\mathcal{N}})^E, (\mathsf{R}_{\eta t}^{\mathcal{N}})^E), \\ ((\mathsf{E}_{\eta t}^{\mathcal{P}})^E, (\mathsf{L}_{\eta t}^{\mathcal{P}})^E, (\mathsf{E}_{\eta t}^{\mathcal{N}})^E, (\mathsf{L}_{\eta t}^{\mathcal{N}})^E)\}\end{matrix}
\end{pmatrix}
\end{array},
$$

which is an expression of $\mathcal{BLDF}$ information. This $\mathcal{L}_\eta$ alternative information is dependent on $\mathcal{T}_t$ attributes of $\mathcal{Q}_E$ experts.

Step 2. Aggregate the matrix $\mathcal{L}_{\mathfrak{as}}$ using the proposed $\mathcal{BLDFWA}$ operator and is given by

$$\xi = [\mathcal{L}_{\mathfrak{as}}]_{\eta\times t} = \begin{array}{c} \\ \mathcal{L}_1 \\ \\ \mathcal{L}_2 \\ \\ \vdots \\ \mathcal{L}_\eta \\ \end{array} \begin{pmatrix} \mathcal{T}_1 & \mathcal{T}_2 & \cdots & \mathcal{T}_t \\ \{((H^{\mathcal{P}}_{11}),(R^{\mathcal{P}}_{11}),(H^{\mathcal{N}}_{11}),(R^{\mathcal{N}}_{11})), & \{((H^{\mathcal{P}}_{12}),(R^{\mathcal{P}}_{12}),(H^{\mathcal{N}}_{12}),(R^{\mathcal{N}}_{12})), & \cdots & \{((H^{\mathcal{P}}_{1t}),(R^{\mathcal{P}}_{1t}),(H^{\mathcal{N}}_{1t}),(R^{\mathcal{N}}_{1t})), \\ ((E^{\mathcal{P}}_{11}),(L^{\mathcal{P}}_{11}),(E^{\mathcal{N}}_{11}),(L^{\mathcal{N}}_{11}))\} & ((E^{\mathcal{P}}_{12}),(L^{\mathcal{P}}_{12}),(E^{\mathcal{N}}_{12}),(L^{\mathcal{N}}_{12}))\} & \cdots & ((E^{\mathcal{P}}_{1t}),(L^{\mathcal{P}}_{1t}),(E^{\mathcal{N}}_{1t}),(L^{\mathcal{N}}_{1t}))\} \\ \{((H^{\mathcal{P}}_{21}),(R^{\mathcal{P}}_{21}),(H^{\mathcal{N}}_{21}),(R^{\mathcal{N}}_{21})), & \{((H^{\mathcal{P}}_{22}),(R^{\mathcal{P}}_{22}),(H^{\mathcal{N}}_{22}),(R^{\mathcal{N}}_{22})), & \cdots & \{((H^{\mathcal{P}}_{2t}),(R^{\mathcal{P}}_{2t}),(H^{\mathcal{N}}_{2t}),(R^{\mathcal{N}}_{2t})), \\ ((E^{\mathcal{P}}_{21}),(L^{\mathcal{P}}_{21}),(E^{\mathcal{N}}_{21}),(L^{\mathcal{N}}_{21}))\} & ((E^{\mathcal{P}}_{22}),(L^{\mathcal{P}}_{22}),(E^{\mathcal{N}}_{22}),(L^{\mathcal{N}}_{22}))\} & \cdots & ((E^{\mathcal{P}}_{2t}),(L^{\mathcal{P}}_{2t}),(E^{\mathcal{N}}_{2t}),(L^{\mathcal{N}}_{2t}))\} \\ \vdots & \vdots & \vdots & \vdots \\ \{((H^{\mathcal{P}}_{\eta 1}),(R^{\mathcal{P}}_{\eta 1}),(H^{\mathcal{N}}_{\eta 1}),(R^{\mathcal{N}}_{\eta 1})), & \{((H^{\mathcal{P}}_{\eta 2}),(R^{\mathcal{P}}_{\eta 2}),(H^{\mathcal{N}}_{\eta 2}),(R^{\mathcal{N}}_{\eta 2})), & \cdots & \{((H^{\mathcal{P}}_{\eta t}),(R^{\mathcal{P}}_{\eta t}),(H^{\mathcal{N}}_{\eta t}),(R^{\mathcal{N}}_{\eta t})), \\ ((E^{\mathcal{P}}_{\eta 1}),(L^{\mathcal{P}}_{\eta 1}),(E^{\mathcal{N}}_{\eta 1}),(L^{\mathcal{N}}_{\eta 1}))\} & ((E^{\mathcal{P}}_{\eta 2}),(L^{\mathcal{P}}_{\eta 2}),(E^{\mathcal{N}}_{\eta 2}),(L^{\mathcal{N}}_{\eta 2}))\} & \cdots & ((E^{\mathcal{P}}_{\eta t}),(L^{\mathcal{P}}_{\eta t}),(E^{\mathcal{N}}_{\eta t}),(L^{\mathcal{N}}_{\eta t}))\} \end{pmatrix},$$

where the alternative information is based on the attributes by all the experts.

Step 3. Average solution ($\mathcal{AS}$) is computed based on the attributes proposed,

$$\begin{aligned} \mathcal{AS} = \frac{[\sum_{\mathfrak{a}=1}^{\eta} \mathcal{I}_{\mathfrak{s}} \mathcal{L}_{\mathfrak{as}}]}{\eta} = \Bigg[\Bigg(\Bigg(1-\prod_{\mathfrak{a}=1}^{\eta}((1-H^{\mathcal{P}}_{\mathcal{K}})^{\mathcal{I}_{\mathfrak{s}}})^{1/\eta}\Bigg), \Bigg(\prod_{\mathfrak{a}=1}^{\eta}((R^{\mathcal{P}}_{\mathcal{K}})^{\mathcal{I}_{\mathfrak{s}}})^{1/\eta}\Bigg), -\Bigg(\prod_{\mathfrak{a}=1}^{\eta}((-H^{\mathcal{N}}_{\mathcal{K}})^{\mathcal{I}_{\mathfrak{s}}})^{1/\eta}\Bigg), \\ -\Bigg(1-\prod_{\mathfrak{a}=1}^{\eta}((1-(-R^{\mathcal{N}}_{\mathcal{K}}))^{\mathcal{I}_{\mathfrak{s}}})^{1/\eta}\Bigg)\Bigg), \Bigg(\Bigg(1^{\eta}_{\mathfrak{a}=1}((1-E^{\mathcal{P}}_{\mathcal{K}})^{\mathcal{I}_{\mathfrak{s}}})^{1/\eta}\Bigg), \Bigg(\prod_{\mathfrak{a}=1}^{\eta}((L^{\mathcal{P}}_{\mathcal{K}})^{\mathcal{I}_{\mathfrak{s}}})^{1/\eta}\Bigg), \\ -\Bigg(\prod_{\mathfrak{a}=1}^{\eta}((-E^{\mathcal{N}}_{\mathcal{K}})^{\mathcal{I}_{\mathfrak{s}}})^{1/\eta}\Bigg), -\Bigg(1-\prod_{\mathfrak{a}=1}^{\eta}((1-(-L^{\mathcal{N}}_{\mathcal{K}}))^{\mathcal{I}_{\mathfrak{s}}})^{1/\eta}\Bigg)\Bigg)\Bigg]. \end{aligned}$$

Step 4. With the use of $\mathcal{AS}$, PDA and NDA are computed with the help of the below formula:

$$[\mathsf{PDA}_{\mathfrak{s}}]_{\eta\chi t} = \frac{\mathfrak{max}(0,(\mathcal{L}_{\mathfrak{as}}-\mathcal{AS}_{\mathfrak{s}}))}{\mathcal{AS}_{\mathfrak{s}}}, \qquad [\mathsf{NDA}_{\mathfrak{s}}]_{\eta\chi t} = \frac{\mathfrak{max}(0,(\mathcal{AS}_{\mathfrak{s}}-\mathcal{L}_{\mathfrak{as}})}{\mathcal{AS}_{\mathfrak{s}}}.$$

For flexibility, we employ scoring value as below:

$$[\mathsf{PDA}_{\mathfrak{s}}]_{\eta\chi t} = \frac{\mathfrak{max}(0,(\mathcal{SV}(\mathcal{L}_{\mathfrak{as}})-\mathcal{SV}(\mathcal{AS}_{\mathfrak{s}})))}{\mathcal{SV}(\mathcal{AS}_{\mathfrak{s}})}, \qquad [\mathsf{NDA}_{\mathfrak{s}}]_{\eta\chi t} = \frac{\mathfrak{max}(0,(\mathcal{SV}(\mathcal{AS}_{\mathfrak{s}})-\mathcal{SV}(\mathcal{L}_{\mathfrak{as}})))}{\mathcal{SV}(\mathcal{AS}_{\mathfrak{s}})}.$$

Step 5. Determine the weighted sums SPDA & SNDA of PDA & NDA by using the formula:

$$\mathsf{SPDA}_{\mathfrak{a}} = \sum_{\mathfrak{s}=1}^{t} \mathcal{I}_{\mathfrak{s}} \mathsf{PDA}_{\mathfrak{as}}, \quad \mathsf{SNDA}_{\mathfrak{s}} = \sum_{\mathfrak{s}=1}^{t} \mathcal{I}_{\mathfrak{s}} \mathsf{NDA}_{\mathfrak{as}}.$$

Step 6. Normalize the equations of Step 5 as follows:

$$\mathsf{NSPDA}_{\mathfrak{a}} = \frac{\mathsf{SPDA}_{\mathfrak{a}}}{\mathfrak{max}_{\mathfrak{a}}(\mathsf{SPDA}_{\mathfrak{a}})}, \quad \mathsf{NSNDA}_{\mathfrak{a}} = \frac{\mathsf{SNDA}_{\mathfrak{a}}}{\mathfrak{max}_{\mathfrak{a}}(\mathsf{SNDA}_{\mathfrak{a}})}.$$

Step 7. Calculate the evaluation score ($\mathcal{ES}$) for each equations of Step 6 as

$$\mathcal{ES}_{\mathfrak{a}} = \frac{(\mathsf{NSPDA}_{\mathfrak{a}})-(1-\mathsf{NSNDA}_{\mathfrak{a}})}{2}.$$

Step 8. The alternatives are ordered based on the $\mathcal{ES}$, which paves to choose the best one. Figure 2 represents the diagramatic outline of the proposed $\mathcal{BLDFEDAS}$ algorithm.

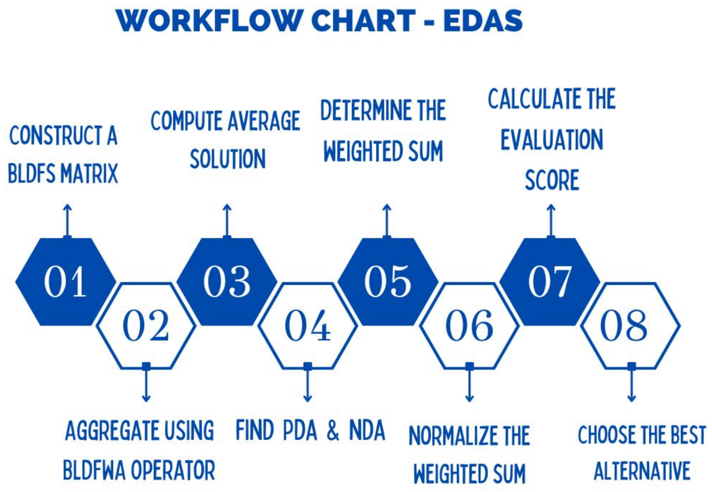

Figure 2: Algorithmic approach - $\mathcal{BLDFEDAS}$

5. $\mathcal{MAGDM}$ problem based on $\mathcal{FDI}$ system

This part gives an in-depth overview of $\mathcal{FDI}$ system, its technologies, and benefactors in extremely crucial situations. An advantageous technology is analyzed in choosing one of the best $\mathcal{FDI}$ among several systems through an $\mathcal{MAGDM}$ problem.

5.1. Outline of $\mathcal{FDI}$ system

The forensic department plays a major role in the justice system for crimes as they offer scientific support to help with inquiries and judicial actions. Generally, these departments have different teams to analyze from different perspectives. Special teams of the department investigate crime scenes to collect samples and document proofs. They have multiple responsibilities, like gathering, conserving, and evaluating virtual and real evidence from crime sites, which include forensic photography, weapons, analyzing DNA, and recognizing fingerprints. In contemporary society, forensics is a much-needed department to be developed into artificial intelligence, despite its vital role in solving sophisticated criminal probes, protecting justice, and guarding the safety of the public. In an era of evolving criminal tactics and increased complexity of offenses, $\mathcal{FDI}$ is a crucial component in fighting crimes. Developing an application with the concept of ordering, and prioritizing cases, along with the steps to follow up to solve any critical cases easily and in justice, is essential to upholding the law in the complex world. The technologies of an $\mathcal{FDI}$ system should be fed up with innovative and knowledgable ideas like task prioritizing, reconstructing a reality crime scene into a virtual one, making decisions with provided shreds of evidence, identification through suspicion with accuracy, and visualizing 3-D video development to smoothen workflow process. Linking through social media data provides vast data to the system for updating itself with location and behaviors. It can easily integrate data, visualize crime patterns, predict crime spots, and track crime movements, offering a secure collection of evidence and enhancing trustworthiness in legal proceedings. Applications developed with the best of such technology reduce time, especially in complex cases, cost, the availability of multiple crews of experts, and separate laboratories for different purposes like DNA tests, imaging, etc. Some of the key features & benefits of the technology are as below.

* Based on urgency, resource availability, and solvability, cases are prioritized and evidence is hierarchized.
* With real-time updates, it processes data and provides visualizations for jurors and investigators to visit crime sites virtually, and assessment reports will be generated to guide and predict decisions.

Also, the process of the cases is updated automatically, and final reports are developed automatically to be submitted clearly in court.

* Supports investigation strategies with consideration of human-like intelligence, risks, and time constraints. Shares safe communication among the investigators to enhance coordination.
* Metrics depending on performance and feedback will be incorporated for the effective leading of cases.
* Recovers deleted data easily and Saves long-term data securely so that maintenance at risk of fraud can be avoided, reducing costs.
* Provide decisions that are far more accountable and transparent with geospatial analysis.
* Identify individuals with high accuracy through advanced technologies like iris scanning, facial recognition, and foot recognition, and by analyzing voices, which are all crucial in modern crimes.
* Upholds ethical and legal conviction in forensic practice.

By harnessing such technology, it affords a powerful toolset for resolving the intricate issues of contemporary law enforcement and forensics investigations.

5.2. *Numerical procedure*

The forensic department confronts two teams of experts, $\mathcal{F}_1$ = *Forensic and Data Scientists* and $\mathcal{F}_2$ = *Ethicists and Legal Experts* to develop an $\mathcal{FDI}$ system into an application for solving critical complex issues. So, five different $\mathcal{FDI}$ systems $\mathcal{E}_1$, $\mathcal{E}_2$, $\mathcal{E}_3$, $\mathcal{E}_4$, and $\mathcal{E}_5$ are developed by different forensic teams and are analyzed to opt for the best among them. Four kinds of factors, notably, $\mathcal{I}_1$ = *Real-time Data Processing*, $\mathcal{I}_2$ = *Predictive Analysis*, $\mathcal{I}_3$ = *Cognitive Computing* and $\mathcal{I}_4$ = *Geospatial Analysis* are considered. The weights of the attributes are chosen to be (0.4, 0.2, 0.3, 0.1) correspondingly. In a similar manner, the weights of the experts are chosen as (0.4, 0.6). The linguistic scale of the issue detection is given in Table 3.

Table 3: Linguistic scale

	$\mathsf{E}^{\mathcal{P}}$	$\mathsf{L}^{\mathcal{P}}$	$\mathsf{E}^{\mathcal{N}}$	$\mathsf{L}^{\mathcal{N}}$
$\mathcal{I}_1$	Effective	Not effective	Weak	Not weak
$\mathcal{I}_2$	Accurate	Not accurate	Erraneous	Not erraneous
$\mathcal{I}_3$	Minimum	Not minimum	Maximum	Not maximum
$\mathcal{I}_4$	More efficient	Not more efficient	Less efficient	Not less efficient

Step 1. The Matrix based on the decision of $\mathcal{F}_1$ and $\mathcal{F}_2$ are given in Tables 4 and 5.

Table 4: Decision grid of expert $\mathcal{F}_1$.

	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{I}_4$
$\mathcal{E}_1$	$\{(0.7,0.4,-0.5,-0.8),$ $(0.8,0.2,-0.3,-0.6)\}$	$\{(0.5,0.2,-0.3,-0.6),$ $(0.7,0.2,-0.2,-0.5)\}$	$\{(0.6,0.4,-0.5,-0.7),$ $(0.7,0.2,-0.3,-0.6)$	$\{(0.5,0.3,-0.4,-0.6),$ $(0.7,0.1,-0.4,-0.4)\}$
$\mathcal{E}_2$	$\{(0.8,0.3,-0.5,-0.7),$ $(0.8,0.2,-0.4,-0.5)\}$	$\{(0.6,0.2,-0.5,-0.7),$ $(0.8,0.2,-0.4,-0.5)\}$	$\{(0.6,0.3,-0.5,-0.7),$ $(0.7,0.2,-0.3,-0.5)$	$\{(0.7,0.4,-0.6,-0.6),$ $(0.5,0.2,-0.5,-0.4)\}$
$\mathcal{E}_3$	$\{(0.6,0.5,-0.2,-0.9),$ $(0.8,0.2,-0.2,-0.7)\}$	$\{(0.4,0.3,-0.3,-0.7),$ $(0.6,0.3,-0.2,-0.6)\}$	$\{(0.6,0.4,-0.4,-0.6),$ $(0.6,0.3,-0.3,-0.6)$	$\{(0.5,0.4,-0.3,-0.7),$ $(0.6,0.2,-0.4,-0.5)\}$
$\mathcal{E}_4$	$\{(0.5,0.3,-0.2,-0.6),$ $(0.5,0.2,-0.3,-0.6)\}$	$\{(0.3,0.2,-0.2,-0.5),$ $(0.4,0.3,-0.2,-0.5)\}$	$\{(0.3,0.8,-0.3,-0.4),$ $(0.6,0.1,-0.4,-0.3)$	$\{(0.7,0.3,-0.5,-0.6),$ $(0.7,0.2,-0.3,-0.6)\}$
$\mathcal{E}_5$	$\{(0.7,0.4,-0.4,-0.8),$ $(0.4,0.5,-0.2,-0.7)\}$	$\{(0.6,0.2,-0.3,-0.4),$ $(0.3,0.4,-0.3,-0.6)\}$	$\{(0.5,0.8,-0.7,-0.8),$ $(0.5,0.5,-0.1,-0.8)$	$\{(0.6,0.5,-0.4,-0.8),$ $(0.7,0.1,-0.5,-0.3)\}$

Table 5: Decision grid of expert $\mathcal{F}_2$.

	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{I}_4$
$\mathcal{E}_1$	$\{(0.8, 0.2, -0.6, -0.9), (0.8, 0.1, -0.5, -0.3)\}$	$\{(0.6, 0.2, -0.4, -0.7), (0.8, 0.1, -0.3, -0.6)\}$	$\{(0.4, 0.3, -0.4, -0.2), (0.6, 0.2, -0.3, -0.1)$	$\{(0.5, 0.4, -0.6, -0.7), (0.7, 0.3, -0.4, -0.5)\}$
$\mathcal{E}_2$	$\{(0.8, 0.2, -0.7, -0.8), (0.8, 0.1, -0.6, -0.2)\}$	$\{(0.6, 0.2, -0.4, -0.6), (0.8, 0.1, -0.3, -0.5)\}$	$\{(0.6, 0.2, -0.5, -0.6), (0.7, 0.1, -0.3, -0.1)$	$\{(0.7, 0.4, -0.6, -0.7), (0.8, 0.2, -0.4, -0.5)\}$
$\mathcal{E}_3$	$\{(0.7, 0.3, -0.4, -0.9), (0.7, 0.1, -0.3, -0.3)\}$	$\{(0.5, 0.3, -0.6, -0.7), (0.7, 0.2, -0.4, -0.6)\}$	$\{(0.5, 0.3, -0.3, -0.2), (0.6, 0.2, -0.3, -0.1)$	$\{(0.6, 0.4, -0.5, -0.6), (0.6, 0.4, -0.3, -0.4)\}$
$\mathcal{E}_4$	$\{(0.4, 0.2, -0.6, -0.7), (0.6, 0.3, -0.5, -0.4)\}$	$\{(0.4, 0.3, -0.5, -0.6), (0.5, 0.3, -0.4, -0.5)\}$	$\{(0.5, 0.1, -0.7, -0.7), (0.6, 0.2, -0.3, -0.4)$	$\{(0.4, 0.2, -0.6, -0.7), (0.4, 0.4, -0.3, -0.5)\}$
$\mathcal{E}_5$	$\{(0.6, 0.2, -0.4, -0.7), (0.8, 0.2, -0.3, -0.6)\}$	$\{(0.5, 0.2, -0.3, -0.6), (0.7, 0.3, -0.2, -0.6)\}$	$\{(0.7, 0.1, -0.4, -0.7), (0.8, 0.1, -0.2, -0.7)$	$\{(0.8, 0.1, -0.3, -0.8), (0.9, 0.1, -0.2, -0.5)\}$

Step 2. The aggregated matrix with the use of $\mathcal{BLDFWA}$ operator is given in Table:6.

Table 6: Aggregated matrix using $\mathcal{BLDFWA}$ operator.

	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{I}_4$
$\mathcal{E}_1$	$\{(0.7648, 0.2640, -0.5579, -0.8680), (0.8, 0.1320, -0.4075, -0.4403)\}$	$\{(0.5627, 0.2, -0.3565, -0.6634), (0.7648, 0.1320, -0.2550, -0.5627)\}$	$\{(0.4899, 0.3365, -0.4373, -0.4597), (0.6434, 0.2, -0.3, -0.3493)\}$	$\{(0.5, 0.3618, -0.5167, -0.6592), (0.7, 0.2259, -0.4, -0.4591)\}$
$\mathcal{E}_2$	$\{(0.8, 0.2352, -0.6118, -0.7648), (0.8, 0.1320, -0.5101, -0.3371)\}$	$\{(0.6, 0.2, -0.4373, -0.6434), (0.8, 0.1320, -0.3365, -0.5)\}$	$\{(0.6, 0.2352, -0.5, -0.6434), (0.3365, 0.5, -0.3, -0.2886)\}$	$\{(0.7, 0.4, -0.6, -0.6592), (0.7114, 0.2, -0.4391, -0.4591)\}$
$\mathcal{E}_3$	$\{(0.6634, 0.3680, -0.3031, -0.9), (0.7450, 0.1320, -0.2550, -0.5012)\}$	$\{(0.4621, 0.3, -0.4548, -0.7), (0.6634, 0.2352, -0.3031, -0.6)\}$	$\{(0.6, 0.3366, -0.3366, -0.3937), (0.6, 0.2352, -0.4547, -0.3493)\}$	$\{(0.5626, 0.4, -0.4165, -0.6392), (0.6, 0.3268, -0.3391, -0.4391)\}$
$\mathcal{E}_4$	$\{(0.4421, 02416, -0.4260, -0.6592), (0.5626, 0.2615, -0.4165, -0.4768)\}$	$\{(0.3618, 0.2615, -0.3719, -0.5592), (0.4621, 0.3, -0.3162, -0.5)\}$	$\{(0.6534, 0.1860, -0.5729, -0.6592), (0.6, 0.1614, -0.3391, -0.3591)\}$	$\{(0.5452, 0.2416, -0.5592, -0.6592), (0.5452, 0.3268, -0.3, -0.5392)\}$
$\mathcal{E}_5$	$\{(0.6434, 0.2869, -0.4, -0.7393), (0.6896, 0.3371, -0.2590, -0.6392)\}$	$\{(0.5426, 0.2, -0.3, -0.5167), (0.5789, 0.3418, -0.2390, -0.6)\}$	$\{(0.6319, 0.5068, -0.5130, -0.7393), (0.7114, 0.2885, -0.1589, -0.7393)\}$	$\{(0.7360, 0.2885, -0.3391, -0.8), (0.8448, 0.1, -0.3120, -0.4165)\}$

Step 3. The $\mathcal{AS}$ is computed as

$$\begin{aligned}
\mathcal{AS}_1 &= (0.6840, 0.2753, -0.4459, -0.8058, 0.7319, 0.1825, -0.3559, -0.4887)\,,\\
\mathcal{AS}_2 &= (0.5127, 0.2288, -0.3798, -0.6225, 0.6757, 0.2112, -0.2875, -0.5547)\,,\\
\mathcal{AS}_3 &= (0.5986, 0.3019, -0.4645, -0.5988, 0.5947, 0.2557, -0.2942, -0.5405)\,,\\
\mathcal{AS}_4 &= (0.6204, 0.3320, -0.4761, -0.6901, 0.6996, 0.2171, -0.3541, -0.4642)\,.
\end{aligned}$$

Step 4. The scoring value is employed in Table:7.

Table 7: Table of scoring value.

$\mathcal{SV}$	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{I}_4$
$\mathcal{E}_1$	0.2064	0.0952	0.0853	0.1026
$\mathcal{E}_2$	0.3132	0.1746	0.2002	0.1830
$\mathcal{E}_3$	0.0163	0.0120	0.1691	0.0282
$\mathcal{E}_4$	0.0520	-0.0217	0.1999	0.0457
$\mathcal{E}_5$	-0.0026	0.0005	-0.0646	0.1567
$\mathcal{AS}$	0.1163	0.0596	0.0637	0.1117

With the use of Table:7, PDA and NDA are computed in Tables 8 and 9.

Table 8: Table of PDA.

	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{I}_4$
$\mathcal{E}_1$	0.7747	0.5973	0.3390	0.0000
$\mathcal{E}_2$	1.6930	1.9295	2.1428	0.6383
$\mathcal{E}_3$	0.0000	0.0000	1.6546	0.0000
$\mathcal{E}_4$	0.0000	0.0000	2.1381	0.0000
$\mathcal{E}_5$	0.0000	0.0000	0.0000	0.4028

Table 9: Table of NDA.

	$\mathcal{I}_1$	$\mathcal{I}_2$	$\mathcal{I}_3$	$\mathcal{I}_4$
$\mathcal{E}_1$	0.0000	0.0000	0.0000	0.0814
$\mathcal{E}_2$	0.0000	0.0000	0.0000	0.0000
$\mathcal{E}_3$	0.8598	0.7986	0.0000	0.7475
$\mathcal{E}_4$	0.5528	1.4546	0.0000	0.5908
$\mathcal{E}_5$	1.0223	0.9916	2.0141	0.0000

Step 5. The SPDA and SNDA values are given as $\text{SPDA}_1 = 0.5310$, $\text{SNDA}_1 = 0.0081$, $\text{SPDA}_2 = 1.7697$, $\text{SNDA}_2 = 0$, $\text{SPDA}_3 = 0.4963$, $\text{SNDA}_3 = 0.5783$, $\text{SPDA}_4 = 0.6414$, $\text{SNDA}_4 = 0.5711$, $\text{SPDA}_5 = 0.0402$, $\text{SNDA}_5 = 1.2114$.

Step 6. The NSPDA and NSNDA values are given as $\text{NSPDA}_1 = 0.3$, $\text{NSNDA}_1 = 0.0066$, $\text{NSPDA}_2 = 1$, $\text{NSNDA}_2 = 0$, $\text{NSPDA}_3 = 0.2804$, $\text{NSNDA}_3 = 0.4773$, $\text{NSPDA}_4 = 0.3624$, $\text{NSNDA}_4 = 0.4714$, $\text{NSPDA}_5 = 0.0227$, $\text{NSNDA}_5 = 1$.

Step 7. The $\mathcal{AS}$ value is given as $\mathcal{ES}_1 = -0.3467$, $\mathcal{ES}_2 = 0$, $\mathcal{ES}_3 = -0.1211$, $\mathcal{ES}_4 = -0.0831$, $\mathcal{ES}_5 = -0.4386$.

Step 8. Hence, $\mathcal{E}_2 \succ \mathcal{E}_4 \succ \mathcal{E}_3 \succ \mathcal{E}_1 \succ \mathcal{E}_5$ (Figure 3).

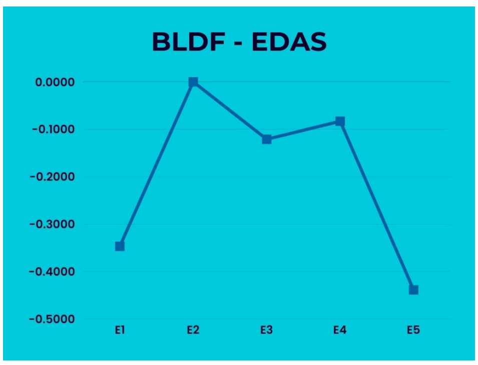

Figure 3: Graphical representation of the obtained result through $\mathcal{BLDFEDAS}$.

6. Bipolar linear Diophantine fuzzy $\mathcal{MABAC}$ approach

An $\mathcal{MABAC}$ approach for a $\mathcal{BLDF}$ is developed and compared to the proposed approach of $\mathcal{BLDFEDAS}$ to check the validity of the implemented method.

Definition 6.1. Consider two $\mathcal{BLDFS}$, then the $\mathcal{BLDF}$ normalized hamming distance is given by

$$d_{\mathcal{BLDFNHD}} = \frac{\left((|H_1^{\mathcal{P}} - H_2^{\mathcal{P}}| + |R_1^{\mathcal{P}} - R_2^{\mathcal{P}}| + |H_1^{\mathcal{N}} - H_2^{\mathcal{N}}| + |R_1^{\mathcal{N}} - R_2^{\mathcal{N}}|) + (|E_1^{\mathcal{P}} - E_2^{\mathcal{P}}| + |L_1^{\mathcal{P}} - L_2^{\mathcal{P}}| + |E_1^{\mathcal{N}} - E_2^{\mathcal{N}}| + |L_1^{\mathcal{N}} - L_2^{\mathcal{N}}|)\right)}{8}.$$

An algorithmic approach on $\mathcal{BLDFMABAC}$ is as follows.

Step 1. The $\mathcal{BLDF}$ matrix formulation evaluated by

$$\xi = [\mathcal{L}^{E}_{\mathfrak{a}\mathfrak{s}}]_{\eta \times t} = \left\{((H^{\mathcal{P}}_{\mathfrak{a}\mathfrak{s}})^E, (R^{\mathcal{P}}_{\mathfrak{a}\mathfrak{s}})^E, (H^{\mathcal{N}}_{\mathfrak{a}\mathfrak{s}})^E, (R^{\mathcal{N}}_{\mathfrak{a}\mathfrak{s}})^E), ((E^{\mathcal{P}}_{\mathfrak{a}\mathfrak{s}})^E, (L^{\mathcal{P}}_{\mathfrak{a}\mathfrak{s}})^E, (E^{\mathcal{N}}_{\mathfrak{a}\mathfrak{s}})^E, (L^{\mathcal{N}}_{\mathfrak{a}\mathfrak{s}})^E)\right\}_{\eta \times t},$$

$\mathfrak{a} = 1, 2, \dots, \eta$, $\mathfrak{s} = 1, 2, \dots, t$, $E = 1, 2, \dots, \digamma$, is given by

$$\xi = \begin{array}{c} \\ \mathcal{L}_1 \\ \\ \mathcal{L}_2 \\ \\ \vdots \\ \mathcal{L}_\eta \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{T}_1 & \mathcal{T}_2 & \cdots & \mathcal{T}_t \end{array} \\ \begin{pmatrix} \{((H^{\mathcal{P}}_{11})^E, (R^{\mathcal{P}}_{11})^E, (H^{\mathcal{N}}_{11})^E, (R^{\mathcal{N}}_{11})^E), & \{((H^{\mathcal{P}}_{12})^E, (R^{\mathcal{P}}_{12})^E, (H^{\mathcal{N}}_{12})^E, (R^{\mathcal{N}}_{12})^E), & \cdots & \{((H^{\mathcal{P}}_{1t})^E, (R^{\mathcal{P}}_{1t})^E, (H^{\mathcal{N}}_{1t})^E, (R^{\mathcal{N}}_{1t})^E), \\ ((E^{\mathcal{P}}_{11})^E, (L^{\mathcal{P}}_{11})^E, (E^{\mathcal{N}}_{11})^E, (L^{\mathcal{N}}_{11})^E)\} & ((E^{\mathcal{P}}_{12})^E, (L^{\mathcal{P}}_{12})^E, (E^{\mathcal{N}}_{12})^E, (L^{\mathcal{N}}_{12})^E)\} & \cdots & ((E^{\mathcal{P}}_{1t})^E, (L^{\mathcal{P}}_{1t})^E, (E^{\mathcal{N}}_{1t})^E, (L^{\mathcal{N}}_{1t})^E)\} \\ \{((H^{\mathcal{P}}_{21})^E, (R^{\mathcal{P}}_{21})^E, (H^{\mathcal{N}}_{21})^E, (R^{\mathcal{N}}_{21})^E), & \{((H^{\mathcal{P}}_{22})^E, (R^{\mathcal{P}}_{22})^E, (H^{\mathcal{N}}_{22})^E, (R^{\mathcal{N}}_{22})^E), & \cdots & \{((H^{\mathcal{P}}_{2t})^E, (R^{\mathcal{P}}_{2t})^E, (H^{\mathcal{N}}_{2t})^E, (R^{\mathcal{N}}_{2t})^E), \\ ((E^{\mathcal{P}}_{21})^E, (L^{\mathcal{P}}_{21})^E, (E^{\mathcal{N}}_{21})^E, (L^{\mathcal{N}}_{21})^E)\} & ((E^{\mathcal{P}}_{22})^E, (L^{\mathcal{P}}_{22})^E, (E^{\mathcal{N}}_{22})^E, (L^{\mathcal{N}}_{22})^E)\} & \cdots & ((E^{\mathcal{P}}_{2t})^E, (L^{\mathcal{P}}_{2t})^E, (E^{\mathcal{N}}_{2t})^E, (L^{\mathcal{N}}_{2t})^E)\} \\ \vdots & \vdots & \vdots & \vdots \\ \{((H^{\mathcal{P}}_{\eta 1})^E, (R^{\mathcal{P}}_{\eta 1})^E, (H^{\mathcal{N}}_{\eta 1})^E, (R^{\mathcal{N}}_{\eta 1})^E), & \{((H^{\mathcal{P}}_{\eta 2})^E, (R^{\mathcal{P}}_{\eta 2})^E, (H^{\mathcal{N}}_{\eta 2})^E, (R^{\mathcal{N}}_{\eta 2})^E), & \cdots & \{((H^{\mathcal{P}}_{\eta t})^E, (R^{\mathcal{P}}_{\eta t})^E, (H^{\mathcal{N}}_{\eta t})^E, (R^{\mathcal{N}}_{\eta t})^E), \\ ((E^{\mathcal{P}}_{\eta 1})^E, (L^{\mathcal{P}}_{\eta 1})^E, (E^{\mathcal{N}}_{\eta 1})^E, (L^{\mathcal{N}}_{\eta 1})^E)\} & ((E^{\mathcal{P}}_{\eta 2})^E, (L^{\mathcal{P}}_{\eta 2})^E, (E^{\mathcal{N}}_{\eta 2})^E, (L^{\mathcal{N}}_{\eta 2})^E)\} & \cdots & ((E^{\mathcal{P}}_{\eta t})^E, (L^{\mathcal{P}}_{\eta t})^E, (E^{\mathcal{N}}_{\eta t})^E, (L^{\mathcal{N}}_{\eta t})^E)\} \end{pmatrix} \end{array},$$

which is an expression of $\mathcal{BLDF}$ information. This $\mathcal{L}_\eta$ alternative information is dependent on $\mathcal{T}_t$ attributes of $\mathcal{Q}_E$ experts.

Step 2. Aggregate the matrix $\mathcal{L}_{\mathfrak{a}\mathfrak{s}}$ using the proposed $\mathcal{BLDFWA}$ operator and is given by

$$\xi = [\mathcal{L}_{\mathfrak{a}\mathfrak{s}}]_{\eta \times t} = \begin{array}{c} \\ \mathcal{L}_1 \\ \\ \mathcal{L}_2 \\ \\ \vdots \\ \mathcal{L}_\eta \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{T}_1 & \mathcal{T}_2 & \cdots & \mathcal{T}_t \end{array} \\ \begin{pmatrix} \{((H^{\mathcal{P}}_{11}), (R^{\mathcal{P}}_{11}), (H^{\mathcal{N}}_{11}), (R^{\mathcal{N}}_{11})), & \{((H^{\mathcal{P}}_{12}), (R^{\mathcal{P}}_{12}), (H^{\mathcal{N}}_{12}), (R^{\mathcal{N}}_{12})), & \cdots & \{((H^{\mathcal{P}}_{1t}), (R^{\mathcal{P}}_{1t}), (H^{\mathcal{N}}_{1t}), (R^{\mathcal{N}}_{1t})), \\ ((E^{\mathcal{P}}_{11}), (L^{\mathcal{P}}_{11}), (E^{\mathcal{N}}_{11}), (L^{\mathcal{N}}_{11}))\} & ((E^{\mathcal{P}}_{12}), (L^{\mathcal{P}}_{12}), (E^{\mathcal{N}}_{12}), (L^{\mathcal{N}}_{12}))\} & \cdots & ((E^{\mathcal{P}}_{1t}), (L^{\mathcal{P}}_{1t}), (E^{\mathcal{N}}_{1t}), (L^{\mathcal{N}}_{1t}))\} \\ \{((H^{\mathcal{P}}_{21}), (R^{\mathcal{P}}_{21}), (H^{\mathcal{N}}_{21}), (R^{\mathcal{N}}_{21})), & \{((H^{\mathcal{P}}_{22}), (R^{\mathcal{P}}_{22}), (H^{\mathcal{N}}_{22}), (R^{\mathcal{N}}_{22})), & \cdots & \{((H^{\mathcal{P}}_{2t}), (R^{\mathcal{P}}_{2t}), (H^{\mathcal{N}}_{2t}), (R^{\mathcal{N}}_{2t})), \\ ((E^{\mathcal{P}}_{21}), (L^{\mathcal{P}}_{21}), (E^{\mathcal{N}}_{21}), (L^{\mathcal{N}}_{21}))\} & ((E^{\mathcal{P}}_{22}), (L^{\mathcal{P}}_{22}), (E^{\mathcal{N}}_{22}), (L^{\mathcal{N}}_{22}))\} & \cdots & ((E^{\mathcal{P}}_{2t}), (L^{\mathcal{P}}_{2t}), (E^{\mathcal{N}}_{2t}), (L^{\mathcal{N}}_{2t}))\} \\ \vdots & \vdots & \vdots & \vdots \\ \{((H^{\mathcal{P}}_{\eta 1}), (R^{\mathcal{P}}_{\eta 1}), (H^{\mathcal{N}}_{\eta 1}), (R^{\mathcal{N}}_{\eta 1})), & \{((H^{\mathcal{P}}_{\eta 2}), (R^{\mathcal{P}}_{\eta 2}), (H^{\mathcal{N}}_{\eta 2}), (R^{\mathcal{N}}_{\eta 2})), & \cdots & \{((H^{\mathcal{P}}_{\eta t}), (R^{\mathcal{P}}_{\eta t}), (H^{\mathcal{N}}_{\eta t}), (R^{\mathcal{N}}_{\eta t})), \\ ((E^{\mathcal{P}}_{\eta 1}), (L^{\mathcal{P}}_{\eta 1}), (E^{\mathcal{N}}_{\eta 1}), (L^{\mathcal{N}}_{\eta 1}))\} & ((E^{\mathcal{P}}_{\eta 2}), (L^{\mathcal{P}}_{\eta 2}), (E^{\mathcal{N}}_{\eta 2}), (L^{\mathcal{N}}_{\eta 2}))\} & \cdots & ((E^{\mathcal{P}}_{\eta t}), (L^{\mathcal{P}}_{\eta t}), (E^{\mathcal{N}}_{\eta t}), (L^{\mathcal{N}}_{\eta t}))\} \end{pmatrix} \end{array},$$

where the alternative information is based on the attributes by all the experts.

Step 3. The $\mathcal{BAR}$ $\mathcal{R} = [\mathcal{V}_{\mathfrak{s}}]_{\eta X t}$ is computed and its matrix can be evaluated as

$$\mathcal{V}_{\mathfrak{s}} = \left[\sum_{\mathfrak{a}=1}^{\eta} \mathcal{J}_{\mathfrak{s}} \mathcal{L}_{\mathfrak{a}\mathfrak{s}}\right]^{1/\eta}.$$

Step 4. Evaluate the distance $\mathcal{D} = [d_{\mathfrak{a}\mathfrak{s}}]_{\eta \times t}$ between the alternative and $\mathcal{BAR}$. Here $d(\mathcal{J}\mathcal{L}_{\mathfrak{a}\mathfrak{s}}, \mathcal{V}_{\mathfrak{s}})$ represents the mean distance from $\mathcal{J}\mathcal{L}_{\mathfrak{a}\mathfrak{s}}$ to $\mathcal{V}_{\mathfrak{s}}$,

$$d_{\mathfrak{a}\mathfrak{s}} = \begin{cases} d(\mathcal{J}\mathcal{L}_{\mathfrak{a}\mathfrak{s}}, \mathcal{V}_{\mathfrak{s}}), & \text{if } \mathcal{J}\mathcal{L}_{\mathfrak{a}\mathfrak{s}} \succ \mathcal{V}_{\mathfrak{s}}, \\ 0, & \text{if } \mathcal{J}\mathcal{L}_{\mathfrak{a}\mathfrak{s}} \approx \mathcal{V}_{\mathfrak{s}}, \\ -d(\mathcal{J}\mathcal{L}_{\mathfrak{a}\mathfrak{s}}, \mathcal{V}_{\mathfrak{s}}), & \text{if } \mathcal{J}\mathcal{L}_{\mathfrak{a}\mathfrak{s}} \prec \mathcal{V}_{\mathfrak{s}}, \end{cases}$$

which can be calculated with the help of Definition 6.1. With values of $d_{\mathfrak{a}\mathfrak{s}}$, the following are considered. If

1. $d_{as} \prec O \Rightarrow$ Lower Approximation Region ($\mathcal{L}\tilde{\mathcal{A}}\mathcal{R}$) owns the alternative;
2. $d_{as} \approx O \Rightarrow$ Border Approximation Region ($\mathcal{B}\tilde{\mathcal{A}}\mathcal{R}$) owns the alternative;
3. $d_{as} \succ O \Rightarrow$ Upper Approximation Region ($\mathcal{U}\tilde{\mathcal{A}}\mathcal{R}$) owns the alternative.

It is clear that the best one belongs to $(\mathcal{V}^{\mathcal{P}}(\mathcal{U}\tilde{\mathcal{A}}\mathcal{R}))$ and the worst one belongs to $(\mathcal{V}^{\mathcal{N}}(\mathcal{L}\tilde{\mathcal{A}}\mathcal{R}))$.

Step 5. The value sum of the alternative can be derived as $\mathcal{S}_a = \sum_{s=1}^{t} d_{as}$.

Step 6. The alternatives are ordered based on the $\mathcal{S}$, which paves to choose the optimal one.

Figure 4 represents the diagramatic outline of the proposed $\mathcal{BLDF}$MABAC algorithm.

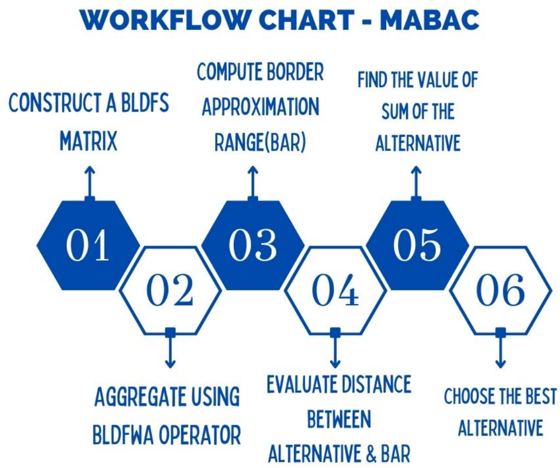

Figure 4: Algorithmic approach - $\mathcal{BLDF}$MABAC.

6.1. *Analyzation of comparision*

The proposed problem in Section 5.2 is solved using $\mathcal{BLDF}$MABAC approach to check the validity.

Step 1. The Matrix besed on the decision of $\mathcal{F}_1$ and $\mathcal{F}_2$ are given in Tables 4 and 5.

Step 2. The aggregated matrix with the use of $\mathcal{BLDFWA}$ operator is given in Table 6.

Step 3. The $\mathcal{BAR}$ is computed as below:

$$\mathcal{V}_1 = (0.7489, 0.2838, -0.4694, -0.8541, 0.7831, 0.1320, -0.3757, -0.4301),$$
$$\mathcal{V}_2 = (0.5451, 0.2290, -0.4139, -0.6698, 0.7488, 0.1600, -0.2962, -0.5561),$$
$$\mathcal{V}_3 = (0.5662, 0.2987, -0.4190, -0.5111, 0.6502, 0.1837, -0.3446, -0.3401).$$

Step 4. The distance between the alternative and $\mathcal{BAR}$ is evaluated as $d_{11} = 0.4849$, $d_{12} = 0.3518$, $d_{13} = -0.6113$, $d_{14} = -0.2819$, $d_{21} = 0.7874$, $d_{22} = 0.5017$, $d_{23} = 0.9084$, $d_{24} = 0.4214$, $d_{31} = -0.5273$, $d_{32} = -0.3715$, $d_{33} = 0.7466$, $d_{34} = -0.4857$, $d_{41} = -0.7629$, $d_{42} = 0.6539$, $d_{43} = 0.6654$, $d_{44} = -0.5683$, $d_{51} = -0.6089$, $d_{52} = -0.5655$, $d_{53} = -0.9108$, $d_{54} = 0.761$.

Step 5. The value sum is derived as $\mathcal{S}_1 = -0.007$, $\mathcal{S}_2 = 0.3273$, $\mathcal{S}_3 = -0.0797$, $\mathcal{S}_4 = -0.0014$, $\mathcal{S}_5 = -0.1655$.

Step 6. Hence, $\mathcal{E}_2 \succ \mathcal{E}_4 \succ \mathcal{E}_1 \succ \mathcal{E}_3 \succ \mathcal{E}_5$ (Figure 5).

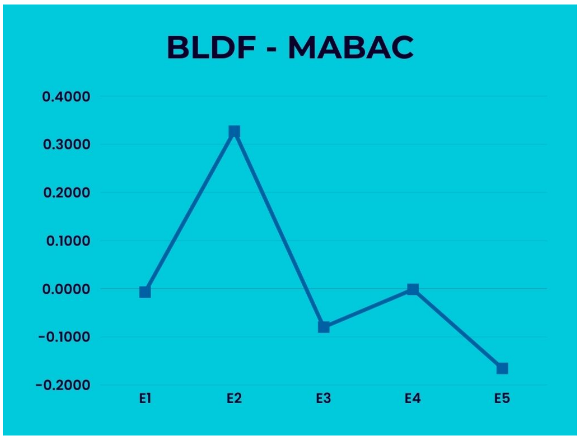

Figure 5: Graphical representation of the obtained result through $\mathcal{BLDF}MABAC$.

7. Discussion and conclusion

Regarding resolving multi-attribute group decision-making, the $\mathcal{EDAS}$ method is one of the numerous traditional decision-making methods. It excels in selecting the best options, unlike other ways like $\mathcal{VIKOR}, \mathcal{TOPSIS}, \mathcal{AHP}$, and $\mathcal{ELECTRE}$ into various extensions. It is quite successful when there are conflicting criteria in $\mathcal{MAGDM}$ issues. In contrast to those other methods, the $\mathcal{EDAS}$ relies on average solutions derived from positive and negative distances (PDA & NDA) rather than ideal ones. It provides opportunities to use a variety of strategies to expand this strategy. These make it possible for "$\mathcal{EDAS}$" to function as an appropriate auxiliary and produce credible decisions. The new novel method $\mathcal{BLDFEDAS}$ is a useful tool in handling uncertainty to resolve a $\mathcal{MAGDM}$ problem in a malleable manner. Also, it manipulates each circumstance in both positive and negative ways with control parameters, which gives more flexibility to our proposed method. The proposed approach is more ubiquitous than the other existing techniques. Uncertainty modeling has rapidly advanced in recent decades to tackle ambiguous inputs. And so, it paves a path to hybrid structures of fuzzy sets. On comparing the proposed $\mathcal{BLDFEDAS}$ to other models, there are two main advantages. First off, the computational results of $\mathcal{EDAS}$ for various attribute weights are quite accurate and consistent. Next, the calculations used for the outcomes are much simpler. It can foresee the zero and negative results in the average solution. Also, while comparing the results of our method to another valuable technique, $\mathcal{BLDF}MABAC$, the same outcome is procured, which validates the model's accuracy even if there are inconsistent and unpredictable data. The $\mathcal{MAGDM}$ has a great deal of potential and a rigorous strategy to evaluate numerous competing factors in different domains of experts to offer more gratifying & pragmatic results. A few essential concepts are implemented for a new notion of $\mathcal{BLDFS}$ with some basic operators. The incorporation of control parameters gives an individual a wide range of preferences for the right choices. The score and accuracy function of $\mathcal{BLDFS}$ play a major role in the flow of the manuscript. A $\mathcal{BLDFWAO}$ is originated for the development of $\mathcal{BLDFEDAS}$ method for a $\mathcal{MAGDM}$ problem. The proposed technique is applied to obtain an optimal $\mathcal{FDI}$ system, to be integrated into an application in regards to solving serious issues as a consequence of the rapid increase of criminal activities. Similar to Bipolar Linear Diophantine Fuzzy $\mathcal{EDAS}$, the outcome of Bipolar Linear Diophantine Fuzzy $\mathcal{MABAC}$ is obtained. Thus, the result strongly supports our innovative approach. The recommended approach can be widely used in any critical situation of our day-to-day lives to make the correct choice based on our priorities. The results of the proposed technique demonstrate flexibility and high accuracy especially when managing critical data and handling positive and negative influences. The control parameters have a significant impact on multi-attribute decision-making. The main drawback of our proposed model is that it is unable to handle parameters and sub-parameters. Also, sometimes it may not be able to deal with a wide range of attributes, which could lead to incorrect choices because of the unpredictable data. The limitation dealing with sub-parameters suggests potential possibilities for future enhancement via bipolar linear Diophantine fuzzy soft set ($\mathcal{BLDFSS}$), bipolar linear Diophantine fuzzy hypersoft set ($\mathcal{BLDFHSS}$), and extensions into multi-fuzzy and lattice ordered frameworks. A few future directions in various application fields include supply chain management by optimizing logistics, healthcare decision systems in optimizing plans for treatments, and smart city infrastructure.

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