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Fractional order system of integro-differential equations: theoretical and numerical simulations using Appell-type Changhee polynomials

M. Adel

Department of Mathematics, Faculty of Science, Islamic University of Madinah, Medina 42210, KSA.
Department of Mathematics, Faculty of Science, Cairo University, Giza 12613, Egypt.

Abstract

We examine the behavior of the solutions for a fractional-order integro-differential equation (FIDE) system using an efficient simulation presented in this paper. The FIDEs are treated by a presumably novel approximation method based on Changhee polynomials of Appell type (ACPs). In this work, we use the method of spectral collocation (SCM), which is based on the advantages of ACPs. Using this technique on the given model, it generates an algebraic equation system. Through the evaluation of the residual error function (REF), we verify the efficiency of the approach that has been offered. To verify the effectiveness and originality of the suggested algorithm, the results are compared with the precise solutions. Our results show that the method employed provides a straightforward and efficient tool to simulate the solution for such models. The suggested method's primary benefits are that it only requires a few easy steps and doesn't generate secular terms.

Keywords: Fractional integro-differential equations, Liouville-Caputo fractional derivative, ACPs, SCM.

2020 MSC: 34A12, 41A30, 47H10, 65N20.

1. Introduction

The FIDEs play an important role in different physical phenomena in science and engineering ([6, 9, 27]). In [25], Volterra integro-differential equations were solved numerically by using Genocchi polynomials. Haar collocation scheme is devoted to obtaining the solution for linear IDEs with fractional order [8]. The system of FIDEs was solved numerically by using different techniques ([12, 13]). Shah et al. [29] presented Haar wavelet technique for solving non-linear variable order integro-differential equations. Also, there are many numerical techniques that depend on famous polynomials like ([3, 10, 16, 20]) can be used.

Fractional analysis is considered an essential tool of mathematical analysis, which is capable of dealing with modeling and analysis in many real-life situations ([1, 4, 31]). Fractional-order analysis has gained more interest due to the study of biological models [2]. One of these mathematical equations is used in the model under study. This model was studied by many researchers ([5, 14, 26, 30, 33]).

This article presents an application of the SCM based on ACPs for solving the fractional-order system of FIDEs with a few recent innovations using the collocation method. The system of FIDEs is transformed

Email address: adel@sci.cu.edu.eg (M. Adel)

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into an algebraic system of equations when the approach is used. The system is then solved to get the unknowns coefficients, which leads to the approximate solution for the original problem.

For this class of problems, the SCM offers certain advantages, since the ACPs's coefficients for the solution can exist rather simply once the numerical algorithms are used. This makes that procedure far faster than the others. Their favorable characteristics in the approximation of functions have led to their widespread usage. This approach is also a fast-converging, highly accurate numerical methodology that may be applied to a variety of problems in both finite and infinite domains. An additional benefit of this approach is that it does not require the domain discretization or the nonlinear terms to be approximated.

In this manuscript, the ACPs will be used to approximate the fractional derivative formula, which will then be applied to solve a system of the FIDEs by using the SCM.

The remainder of the manuscript is structured as follows. Definitions and concepts relating to fractional derivatives, the ACPs, and other related notions are presented in Section 2. Section 3 will examine the error analysis pertaining to a function that is approximated by the ACPs. Using the ACPs, we provide an approximation formula for the fractional derivatives in Section 4. We will provide the suggested method's implementation through Section 5. A numerical simulation of the suggested problem is shown in Section 6. Lastly, Section 7 contains the conclusions.

2. Preliminaries and definitions

2.1. Fractional integration and fractional derivative

Fractional-order integration and differentiation have been defined in a number of ways in the literature [14]. The two most significant fractional derivatives that were used to advance the theory of fractional calculus are the Riemann-Liouville and Caputo derivatives, which have the following definitions.

Definition 2.1. For a given function $\psi(t)$, the Riemann-Liouville fractional integral of fractional order ν is defined as [14]:

$$I^{\nu}\psi(t) = \frac{1}{\Gamma(\nu)}\int_0^t (t-\tau)^{\nu-1}\psi(\tau)\,d\tau, \quad t > 0,\ \nu \in \mathbb{R}^+,$$

where $\Gamma(\cdot)$ is the gamma function. The operator I^{ν} possesses the following property:

$$I^{\nu}\, I^{\rho}\psi(t) = I^{\nu+\rho}\psi(t), \quad \nu, \rho > 0.$$

Definition 2.2. For a given function $\psi(t)$, the Riemann-Liouville fractional derivative of order ν is defined as follows [14]:

$${}^{RL}D^{\nu}\psi(t) = \frac{d^m}{dt^m}\left(I^{m-\nu}\psi(t)\right), \quad m-1 < \nu \leqq m, \ \ m \in \mathbb{N},$$

where, as usual, $\mathbb{N}$ denotes the set of natural numbers.

Several restrictions are applied to the Riemann-Liouville definition for simulating certain real-world issues [14]. On the other hand, and to address these problems, the Liouville-Caputo (LC) formulation was developed. We use it, as the definition.

Definition 2.3. In the Liouville-Caputo sense, the fractional derivative ${}^{LC}D^{\nu}$ of a function $\psi(t)$ has the following definition:

$${}^{LC}D^{\nu}\psi(t) = \frac{1}{\Gamma(n-\nu)}\int_0^t \frac{\psi^{(n)}(\tau)}{(t-\tau)^{\nu-n+1}}d\tau, \quad n-1 < \nu < n,\ n \in \mathbb{N}.$$

The Liouville-Caputo fractional derivative ${}^{LC}D^{\nu}$ possesses the following properties:

$${}^{LC}D^{\nu}C = 0, \qquad C \text{ is a constant},$$

and

$$ {}^{LC}D^{\nu}t^{\theta} = \frac{\Gamma(\theta+1)}{\Gamma(\theta+1-\nu)}t^{\theta-\nu}, \quad \theta \in \mathbb{N}\cup\{0\},\ \theta \geqq \lceil \nu \rceil. \tag{2.1} $$

Furthermore, we have

$$ {}^{LC}D^{\nu}(c_1\psi_1(t)+c_2\psi_2(t)) = c_1\ {}^{LC}D^{\nu}\psi_1(t) + c_2\ {}^{LC}D^{\nu}\psi_2(t), $$

for some constants c_1 and c_2.

2.2. A few ideas about Changhee polynomials

Numerous researchers have examined Changhee polynomials in relation to numerous special numbers and polynomials in recent years, there are many interesting results that can be found in [17]. DEs derived from λ-Changhee polynomials were examined, and several novel and precise identities for the λ-Changhee polynomials related to DEs were provided in [21]. The Changhee polynomials $Ch_m(t)$ and the Changhee number Ch_m are given by ([15, 18, 19, 21, 24])

$$ \frac{2}{z+2}(1-z)^t = \sum_{m=0}^{\infty} Ch_m(t)\frac{z^m}{m!}, $$

where $Ch_m = Ch_m(0)$ are the Changhee numbers, see [20]. The ACPs $Ch_m^*(t)$ are ([23])

$$ \frac{2}{z+2}e^{tz} = \sum_{m=0}^{\infty} Ch_m^*(t)\,\frac{t^m}{m!}. $$

The ACPs of degree m are defined by

$$ Ch_m^*(t) = \sum_{j=0}^{m} \binom{m}{j} Ch_{m-j}^*\, t^j. \tag{2.2} $$

From (2.2), one can get:

$$ \frac{d}{dt}Ch_m^*(t) = m\, Ch_{m-1}^*(t), \tag{2.3} $$

therefore from (2.3), we get

$$ Ch_m^*(t) = \int_0^t m\, Ch_{m-1}^*(y)dy + Ch_m^*. $$

Also, note that $Ch_0^* = 1$ and $2Ch_m^* + m\, Ch_{m-1}^* = 0,\ \forall\, m \geqslant 1$. We can prove that the ACPs satisfy

$$ \int_0^1 Ch_n^*(t)Ch_m^*(t)dt = \sum_{i=0}^{m}\sum_{k=0}^{m-i} \binom{m}{i} \frac{(-1)^{m-i-1}(m-i)\binom{m-i}{k}Ch_k^*(1)Ch_i^*}{(2(m-i)-k+1)\binom{2(m-i)-k}{m-i}}. $$

Let $\left\{Ch_i^*(t)\right\}_{i=1}^{m} \subset L^2[0,1]$ be the set of ACPs and let

$$ \Omega = \mathrm{Span}\{Ch_i^*(t)\}_{i=1}^{m} $$

be a finite-dimensional subspace of $L^2[0,1]$ ([23]). For a function $u(t)$ of $L^2[0,1]$ having a unique approximation in Ω, say $u^*(t)$, then the subsequent error estimate can be held as

$$ \|u(t)-u^*(t)\|_2 \leqslant \|u(t)-v(t)\|_2, \quad \forall\, v(t) \in \Omega. $$

But since Ω is a closed subspace of $L^2[0,1]$, then according to [28], we can find that

$$L^2[0,1] = \Omega \oplus \Omega^{\perp},$$

where $\Omega^{\perp}$ is the orthogonal complement of Ω, and so $u(t) = v(t) + g(t)$, and then $g(t) = u(t) - v(t)$, which means that $u(t) - u^*(t) \in \Omega^{\perp}$. Therefore

$$\langle u(t) - u^*(t), v(t) \rangle = 0, \quad \forall\, v(t) \in \Omega. \tag{2.4}$$

Since $u^*(t) \in \Omega$, then

$$u(t) \approx u^*(t) = \sum_{i=1}^{m} c_i \, Ch_i^*(t) = C^T \, \mathbf{Ch}^*(t), \tag{2.5}$$

where

$$\mathbf{C} = [c_1, c_2, \ldots, c_m]^T, \quad \mathbf{Ch}^*(t) = [Ch_1^*(t), Ch_2^*(t), \ldots, Ch_m^*(t)]^T.$$

Let $v(t) = Ch_i^*(t)$ and using (2.5) in (2.4), we get

$$\left\langle u(t) - C^T \, \mathbf{Ch}^*(t), Ch_i^*(t) \right\rangle = 0.$$

Furthermore, from (2.5), we get

$$\langle u(t), \mathbf{Ch}^*(t) \rangle = C^T \, \langle \mathbf{Ch}^*(t), \mathbf{Ch}^*(t) \rangle .$$

3. Error analysis

Theorem 3.1. *Let the function $u(t) \in C[0,1]$ has continuous derivatives up to $(m+1)^{th}$ times and $u_m(t)$ be the best approximation of the function $u(t)$ defined in Eq. (2.5), then we have*

$$\|u(t) - u_m(t)\| \leqslant \frac{\alpha \beta}{(m+1)!},$$

where

$$\alpha = \max_{0 \leqslant t \leqslant 1} u^{(m+1)}(t), \quad \beta = \max\{t_0, 1 - t_0\}.$$

Proof. The Taylor series approximation of $u(t)$ in the neighborhood of a point $t = t_0$ is given by

$$u(t) = u(t_0) + \frac{(t-t_0)}{1!}u^{(1)}(t_0) + \frac{(t-t_0)^2}{2!}u^{(2)}(t_0) + \cdots + \frac{(t-t_0)^m}{m!}u^{(m)}(t_0) + \frac{(t-t_0)^{m+1}}{(m+1)!}u^{(m+1)}(\xi),$$

where $t_0 \in [0,1], \xi \in (t_0, t)$. Assume

$$\tilde{u}(t) = u(t_0) + \frac{(t-t_0)}{1!}u^{(1)}(t_0) + \frac{(t-t_0)^2}{2!}u^{(2)}(t_0) + \cdots + \frac{(t-t_0)^m}{m!}u^{(m)}(t_0),$$

then

$$|u(t) - \tilde{u}_m(t)| = \left| \frac{(t-t_0)^{m+1}}{(m+1)!}u^{(m+1)}(\xi) \right|.$$

Since $u_m(t)$ is the best approximation of $u(t)$, then

$$\|u(t) - u_m(t)\|^2 \leqslant \|u(t) - \tilde{u}_m(t)\|^2 = \int_0^1 |u(t) - \tilde{u}_m(t)|^2 \, dt \leqslant \int_0^1 \left| \frac{(t-t_0)^{m+1}}{(m+1)!}u^{(m+1)}(\xi) \right|^2 dt.$$

It is assumed that $u(t)$ has continuous derivatives up to $(m+1)^{th}$ times, therefore there exists a constant α such that

$$\alpha = \max_{0 \leqslant t \leqslant 1} u^{(m+1)}(t).$$

Now we have

$$\|u(t)-u_m(t)\|^2 \leqslant \int_0^1 \left|\frac{\alpha}{(m+1)!}(t-t_0)^{m+1}\right|^2 dt. \tag{3.1}$$

Considering $\beta = \max\{t_0, 1-t_0\}$, then Eq. (3.1) becomes

$$\|u(t)-u_m(t)\|^2 \leqslant \frac{\alpha^2\,\beta^2}{[(m+1)!]^2}\int_0^1 dt.$$

Hence,

$$\|u(t)-u_m(t)\| \leqslant \frac{\alpha\,\beta}{(m+1)!}.$$

So the proof is completed. □

4. Fractional derivative approximation with ACPs

In this section, we will show that the ${}^{LC}D^{\beta}$ for $u_m(t)$ that was introduced in (2.5) can be approximated using the following result.

Theorem 4.1. *Assume that* $\nu > 0$ *and* $u_m(t)$ *as was introduced in* (2.5)*, then we have*

$${}^{LC}D^{\beta}\,u_m(t) = \sum_{i=\lceil\nu\rceil}^{m}\sum_{j=\lceil\nu\rceil}^{i} c_i\,\kappa_{i,j,\nu}\,t^{j-\nu}, \quad \kappa_{i,j,\nu} = \frac{(i)!\,Ch^*_{i-j}}{(i-j)!\,\Gamma(j+1-\nu)}, \tag{4.1}$$

where Ch^*_{i-j} *is the Changhee number.*

Proof. Consider the ACP, $Ch^*_i(t)$ of degree i, with $i = 0, 1, \ldots, m$, and using (2.1) and (2.2) we can get

$$\begin{aligned}{}^{LC}D^{\beta}\,u_m(t) = \sum_{i=0}^{m} c_i\,D^{\nu}\,Ch^*_i(t) &= \sum_{i=\lceil\nu\rceil}^{m}\sum_{j=\lceil\nu\rceil}^{i} c_i\,\frac{(i!)Ch^*_{i-j}}{(j!)(i-j)!}\;{}^{LC}D^{\beta}\,t^j\\ &= \sum_{i=\lceil\nu\rceil}^{m}\sum_{j=\lceil\nu\rceil}^{i} c_i\,\frac{(i!)\,Ch^*_{i-j}}{(i-j)!\,\Gamma(j+1-\nu)}\,t^{j-\nu} = \sum_{i=\lceil\nu\rceil}^{m}\sum_{j=\lceil\nu\rceil}^{i} c_i\,\kappa_{i,j,\nu}\,t^{j-\nu},\end{aligned}$$

where $\kappa_{i,j,\nu}$ is defined in (4.1), then the proof is completed. □

5. Numerical implementation

Now, we give an outline of the implementation of the presented method to simulate the solution for the system in its Liouville-Caputo fractional-order form [7]:

$$\begin{aligned}{}^{LC}D^{\beta}\psi_1(t) &= h_1(t) + NF_1(\psi_1(t),\psi_2(t)) - \int_0^t (NF_2(\psi_1(\tau),\psi_2(\tau))\,d\tau,\\ {}^{LC}D^{\beta}\psi_2(t) &= h_2(t) + NG_1(\psi_1(t),\psi_2(t)) - \int_0^t (NG_2(\psi_1(\tau),\psi_2(\tau))\,d\tau,\end{aligned} \tag{5.1}$$

where $\psi_1(t)$ and $\psi_2(t)$ are unknown functions, $h_1(t)$ and $h_2(t)$ are nonhomogeneous terms, and $NF_i(\psi_1(t), \psi_2(t))$ and $NG_i(\psi_1(t),\psi_2(t))$, $(i = 1, 2)$ are continuous linear or nonlinear functions. Consider the ICs

$$\psi_k(0) = \varepsilon_k, \quad k = 1, 2. \tag{5.2}$$

Let $\psi_{\gamma,m}(t)$, $\gamma = 1,2$ be the approximate of the unknown functions $\psi_\gamma(t)$, in terms of ACPs as

$$\psi_{\gamma,m}(t) = \sum_{i=0}^{m} c_{\gamma,i}\, Ch_i^*(t), \quad \gamma = 1,2. \tag{5.3}$$

Upon substituting from (5.3) and (4.1) into the system (5.1), we get

$$\sum_{i=\lceil\beta\rceil}^{m} \sum_{j=\lceil\beta\rceil}^{i} c_{1,i}\, \kappa_{i,j,\beta}\, t^{j-\beta} = h_1(t) + NF_1(\psi_{1,m}(t), \psi_{2,m}(t)) - \int_0^t NF_2(\psi_{1,m}(\tau), \psi_{2,m}(\tau))\, d\tau, \tag{5.4}$$

$$\sum_{i=\lceil\beta\rceil}^{m} \sum_{j=\lceil\beta\rceil}^{i} c_{2,i}\, \kappa_{i,j,\beta}\, t^{j-\beta} = h_2(t) + NG_1(\psi_{1,m}(t), \psi_{2,m}(t)) - \int_0^t NG_2(\psi_{1,m}(\tau), \psi_{2,m}(\tau))\, d\tau. \tag{5.5}$$

By collocating (5.4)-(5.5) at $m-1$ points $t_r = \frac{r}{m-1} + 1$, $r = 1,2,\ldots,m-1$, it will be reduced to the following system of equations in $c_{\gamma,i}$, $\gamma = 1,2$, $i = 0,2,\ldots,m$:

$$\sum_{i=\lceil\beta\rceil}^{m} \sum_{j=\lceil\beta\rceil}^{i} c_{1,i}\, \kappa_{i,j,\beta}\, t_r^{j-\beta} = h_1(t_r) + NF_1(\psi_{1,m}(t_r), \psi_{2,m}(t_r)) - \int_0^{t_r} NF_2(\psi_{1,m}(\tau), \psi_{2,m}(\tau))\, d\tau, \tag{5.6}$$

$$\sum_{i=\lceil\beta\rceil}^{m} \sum_{j=\lceil\beta\rceil}^{i} c_{2,i}\, \kappa_{i,j,\beta}\, t_r^{j-\beta} = h_2(t_r) + NG_1(\psi_{1,m}(t_r), \psi_{2,m}(t_r)) - \int_0^{t_r} NG_2(\psi_{1,m}(\tau), \psi_{2,m}(\tau))\, d\tau. \tag{5.7}$$

Furthermore, upon substituting (5.3) into (5.2), the initial conditions (5.2) will be converted to the following algebraic equations:

$$\sum_{i=0}^{m} c_{\gamma,i}\, Ch_i^*(0) = \varepsilon_\gamma, \quad \gamma = 1,2. \tag{5.8}$$

Using the Newton iteration method for solving the system consisting of the equations (5.6)-(5.8) for $c_{\gamma,i}$, $\gamma = 1,2$, $i = 0,1,\ldots,m-1$. This then leads us to use substitution to generate the approximate solution in the form (5.3).

6. Numerical simulation

In this part, we go on to provide a numerical simulation on three test examples, where we address the system (5.1) with various values of β, for various nonlinear functions [7], in order to check the precision and quality of the provided scheme. We will compare the outcomes of the suggested method with the exact solution in each example, as well as the behavior of the approximate solution for various values of β. We assess the REF [11] to gauge the precision and value of the suggested strategy.

Example 6.1. Consider the following linear system of Volterra FIDEs [22]:

$$^{LC}D^{\beta}\psi_1(t) = 1 + t + t^2 - \psi_2(t) - \int_0^t (\psi_1(\tau) + \psi_2(\tau))d\tau,$$

$$^{LC}D^{\beta}\psi_2(t) = -1 - t + 2e^t - \psi_1(t) - \int_0^t (\psi_1(\tau) - \psi_2(\tau))d\tau,$$

where $t \in [0,1]$ and $\beta \in (0,1]$, with $\psi_1(0) = 1$, $\psi_2(0) = -1$. The exact solution for $\beta = 1$ is $\psi_1(t) = t + e^t$ and $\psi_2(t) = t - e^t$. The numerical results that obtained for this case using the introduced method are illustrated in Figures 1-3. In Figure 1, we compare the numerical results with the exact solution at ($\beta = 1$) with $m = 7$. In Figure 2, the solution for $\beta = 1.0, 0.98, 0.96, 0.94$, with $m = 6$ is presented. In Figure 3, the REF of the solution at $m = 7$, $\beta = 0.98$ is given.

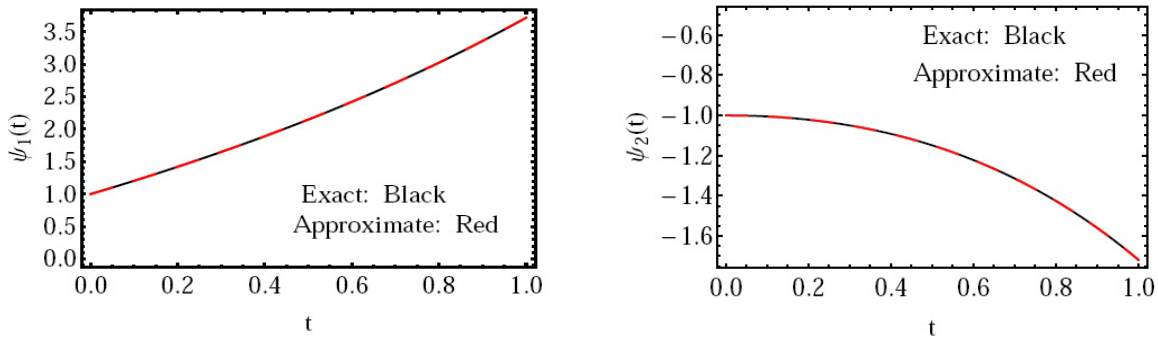

Figure 1: The approximate and exact solutions for Example 6.1, $\psi_i(t)$, $i = 1, 2$ with $\beta = 1$.

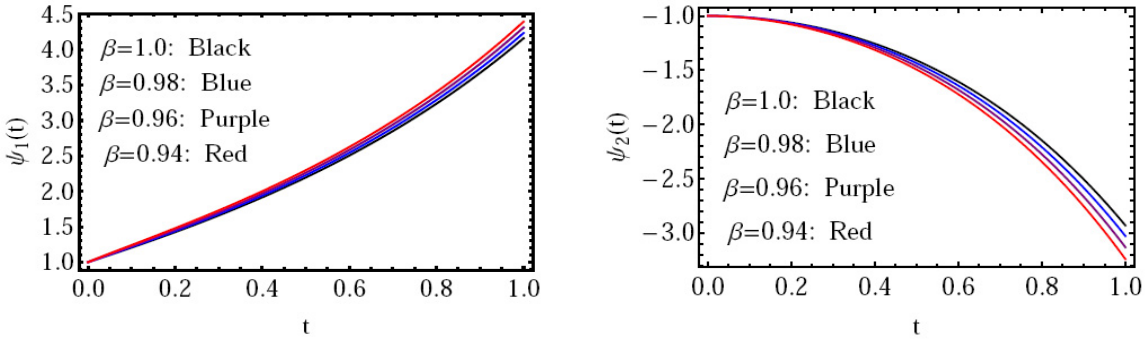

Figure 2: The approximate solution for Example 6.1, $\psi_i(t)$, $i = 1, 2$ with distinct β.

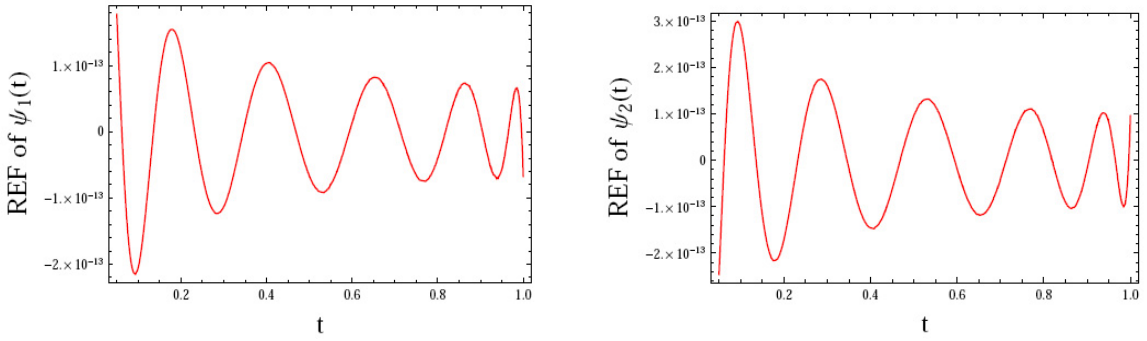

Figure 3: The REF of $\psi_i(t)$, $i = 1, 2$, for Example 6.1.

Example 6.2. Consider the system of nonlinear Volterra FIDEs [32]:

$$
\begin{aligned}
{}^{LC}D^{\beta}\psi_1(t) &= \frac{1}{3}\psi_1(t)\psi_2(t) - \frac{1}{2}\psi_2^2(t) + 2\psi_2(t) - \int_0^t (\psi_1(\tau) + \psi_2(\tau))\, d\tau, \\
{}^{LC}D^{\beta}\psi_2(t) &= \frac{1}{3}\psi_1(t)\psi_2(t) - \psi_1(t) - \int_0^t (\psi_1(\tau) - 2\psi_2(\tau))\, d\tau,
\end{aligned}
$$

where $t \in [0, 1]$ and $\beta \in (0, 1]$, with $\psi_1(0) = \psi_2(0) = 0$. The exact solution for $\beta = 1$ is $\psi_1(t) = t^2$ and $\psi_2(t) = t$. The numerical results that obtained for this case using the introduced method are illustrated in Figures 4-6. In Figure 4, we compare the numerical results with the exact solution at ($\beta = 1$) with $m = 4$. In Figure 5, the approximate solution for $\beta = 1.0, 0.9, 0.8, 0.7$, with $m = 5$ is presented. In Figure 6, the REF of the solution at $m = 6$, $\beta = 0.98$ is given.

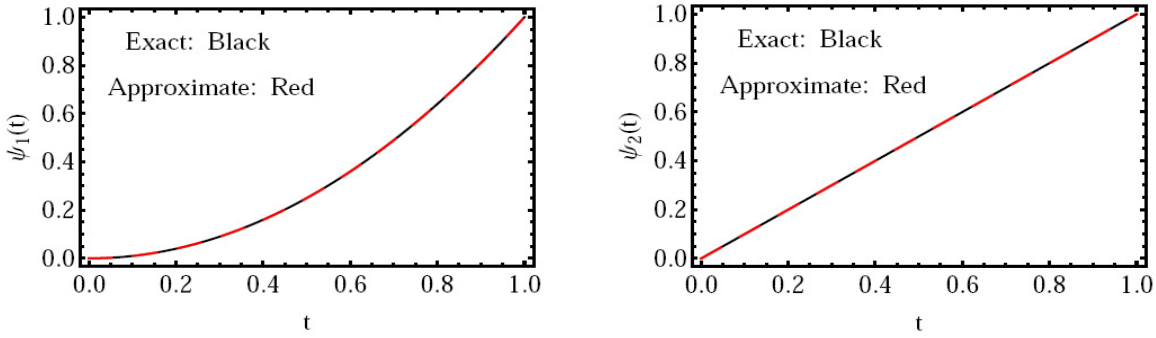

Figure 4: The approximate and exact solutions for Example 6.2, $\psi_i(t)$, $i = 1, 2$ with $\beta = 1$.

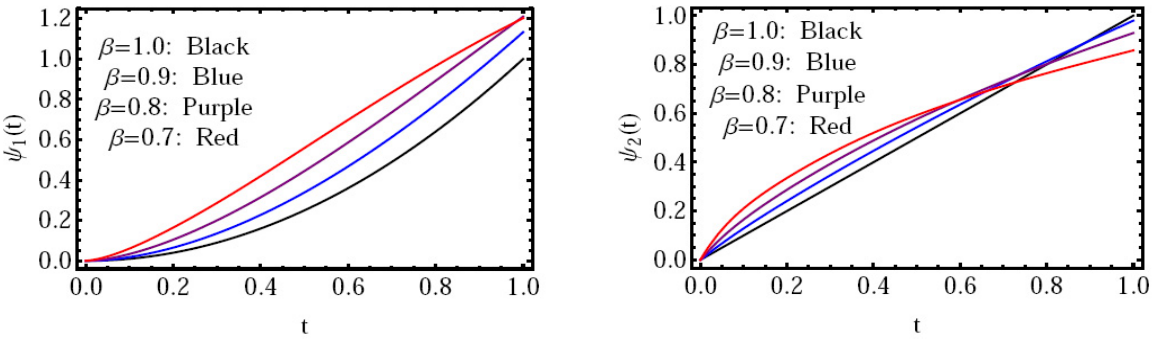

Figure 5: The approximate solution for Example 6.2, $\psi_i(t)$, $i = 1, 2$ with distinct β.

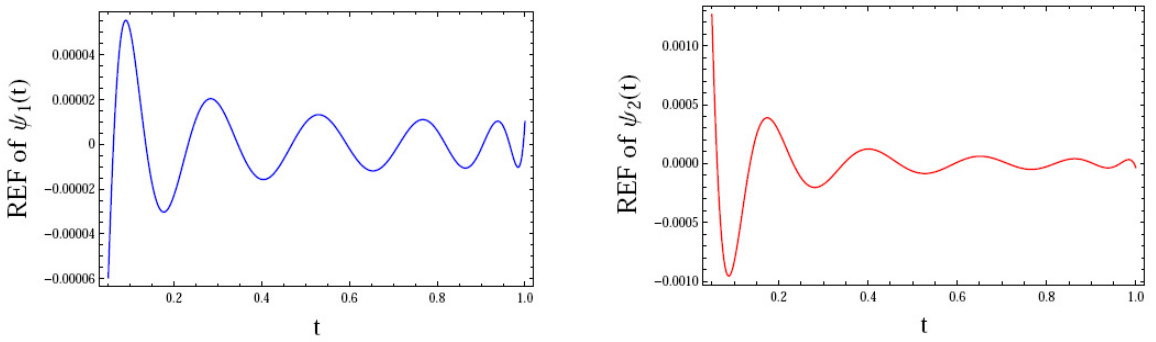

Figure 6: The REF of $\psi_i(t)$, $i = 1, 2$, for Example 6.2.

Example 6.3. Consider the nonlinear system of Volterra FIDEs [32]:

$$
\begin{aligned}
{}^{LC}D^{\beta}\psi_1(t) &= \psi_1^2(t) + \psi_2^2(t) - \int_0^t \psi_1(\tau)\, d\tau, \\
{}^{LC}D^{\beta}\psi_2(t) &= -\frac{1}{2}(\psi_2^2(t) + 2\psi_1(t) - 1) - \int_0^t (\psi_1(\tau)\psi_2(\tau))\, d\tau,
\end{aligned}
$$

where $t \in [0, 1]$ and $\beta \in (0, 1]$, with $\psi_1(0) = 0$, $\psi_2(0) = 1$. The exact solution for $\beta = 1$ is $\psi_1(t) = \sin(t)$ and $\psi_2(t) = \cos(t)$. The numerical results that obtained for this case using the introduced method are illustrated in Figures 7-9. In Figure 7, we compare the numerical results with the exact solution at ($\beta = 1$) with $m = 6$. In Figure 8, the approximate solution for $\beta = 1.0, 0.9, 0.8, 0.7$, with $m = 7$ is presented. In Figure 9, the REF of the solution at $m = 7$, $\beta = 0.95$ is given.

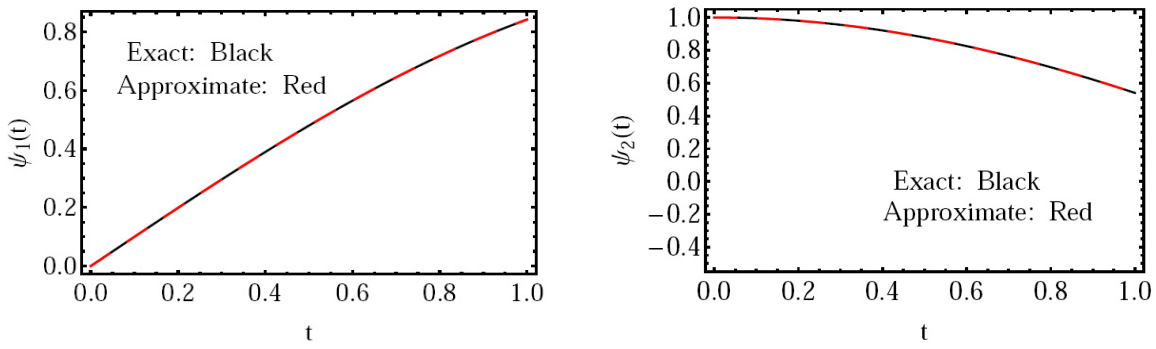

Figure 7: The approximate and exact solutions for Example 6.3, $\psi_i(t)$, $i = 1, 2$ with $\beta = 1$.

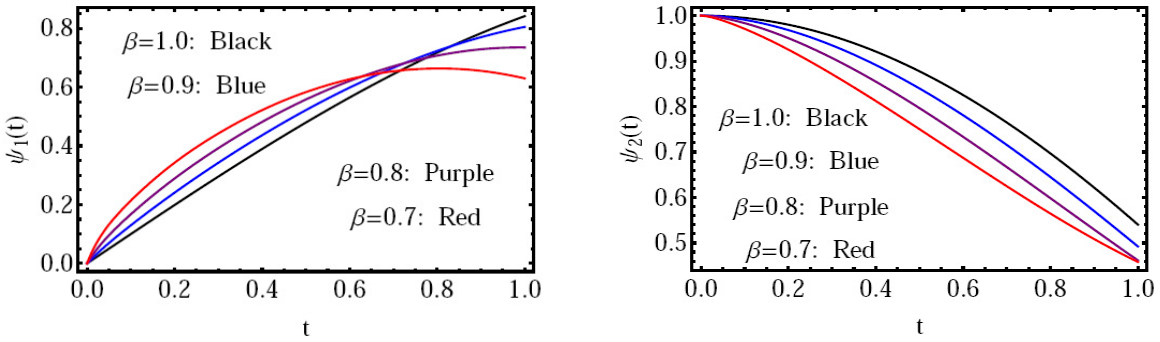

Figure 8: The approximate solution for Example 6.3, $\psi_i(t)$, $i = 1, 2$ with distinct β.

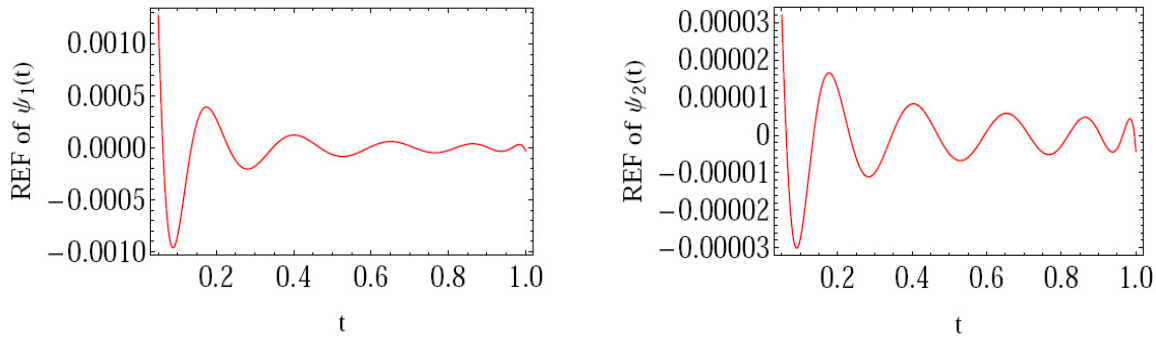

Figure 9: The REF of $\psi_i(t)$, $i = 1, 2$, for Example 6.3.

These results demonstrate that the fractional derivative of the Liouville-Caputo sense can be used to solve the suggested model in an efficient manner. The way in which the suggested method behaves in the numerical solution depends on β and m. Additionally, the efficiency and outcomes of the strategy are markedly enhanced by the proposed approach.

7. Conclusions and future work

This work's primary objective is to use fractional calculus tools and methodologies to examine the dynamical behavior of the system of FIDEs using the LC-fractional derivative operator. The numerical solutions were found in this work using various values of the approximation order m, and the fractional order β. Lastly, we have shown that learning this mathematical model is remarkably effective with the proposed strategy. Moreover, by raising m or adding more terms to the approximation solution series,

we are able to control and reduce the error's precision. We also concluded that numerical simulations with the LC-fractional derivative operator are more appropriate for the mathematical model studied in this work. Our results also show the accuracy and computational efficiency of the suggested strategy. We plan to handle the same situation in a future analysis utilizing a new form of polynomials or a different kind of fractional derivative as a generalization of our current work.

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