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Dynamic behaviors of two species amensalism model with a cover for the first species

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Abstract

In this paper, a two species amensalism model with a cover for the first species takes the form

$$\frac{dx}{dt} = a_1x(t) - b_1x^2(t) - c_1(1-k)x(t)y(t),$$

$$\frac{dy}{dt} = a_2y(t) - b_2y^2(t),$$

is investigated, where $a_i, b_i, i = 1, 2$ and c_1 are all positive constants, k is a cover provided for the species x, and $0 < k < 1$. Our study shows that if $0 \le k < 1 - \dfrac{a_1b_2}{a_2c_1}$, then $E_2(0, \frac{a_2}{b_2})$ is globally stable, and if $1 > k > 1 - \dfrac{a_1b_2}{a_2c_1}$, then $E_3(x^*, y^*)$ is the unique globally stable positive equilibrium. More precisely, the conditions which ensure the local stability of $E_2(0, \frac{a_2}{b_2})$ is enough to ensure its global stability, and once the positive equilibrium exists, it is globally stable. Some numerical simulations are carried out to illustrate the feasibility of our findings. ©2016 All rights reserved.

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1. Introduction

During the last two decades, the study of dynamic behaviors of population system incorporating

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a refuge for some species become one of the most important research topic, (see [5–7, 9, 10, 13, 15–21, 24–28] and the references cited therein). Also, there are several papers on the dynamic behaviors of amensalism model, see [1–4, 8, 12, 14, 22, 23, 29, 30]. However, only [22] considered the influence of refuge on the amensalism model.

Sita Rambabu, Narayan and Bathul[22] considered a two species amensalism model with a partial cover for the first species to protect it from the second species, the model is as follows:

$$\begin{aligned} \frac{dx}{dt} &= a_1x(t) - b_1x^2(t) - c_1(1-k)x(t)y(t), \\ \frac{dy}{dt} &= a_2y(t) - b_2y^2(t), \end{aligned} \tag{1.1}$$

where $a_i, b_i, i = 1, 2$ and c_1 are all positive constants, k is a cover provided for the species x, and $0 < k < 1$. The series solution of above system was approximated by the Homotopy analysis method (HAM). However, the author did not give any analysis about the cover parameter k, to show how the cover influence the dynamic behaviors of the system (1.1). To find out the influence of refuge (which is represent by k), one should give a thoroughly analysis of the global dynamic behaviors of system (1.1).

The aim of this paper is to investigate the local and global stability property of the possible equilibria of system (1.1) and to find out the influence of refuge. We arrange the paper as follows: In the next section, we will investigate the existence and local stability property of the equilibria of system (1.1). In Section 3, by constructing a suitable Lyapunov function and applying the Dulac Theorem, we will investigate the global stability property of the system; In Section 4, an example together with its numeric simulations is presented to show the feasibility of our main results.

2. The existence and stability of the equilibria

The equilibria of system (1.1) is determined by the system

$$\begin{aligned} &a_1x - b_1x^2 - c_1(1-k)xy = 0, \\ &a_2y - b_2y^2 = 0. \end{aligned}$$

Hence, system (1.1) admits four possible equilibria, $E_0(0, 0)$, $E_1\big(\frac{a_1}{b_1}, 0\big)$, $E_2\big(0, \frac{a_2}{b_2}\big)$ and $E_3\big(x^*, y^*\big)$, where $x^* = \dfrac{a_1b_2 - a_2c_1(1-k)}{b_1b_2}, y^* = \dfrac{a_2}{b_2}$. Obviously, E_3 is a positive equilibrium if and only if $k > 1 - \dfrac{a_1b_2}{a_2c_1}$.

Concerned with the local stability property of the above four equilibria, we have,

Theorem 2.1. *$E_0(0, 0)$ and $E_1(\frac{a_1}{b_1}, 0)$ are unstable; If $k < 1 - \dfrac{a_1b_2}{a_2c_1}$, then $E_2(0, \frac{a_2}{b_2})$ is stable and if $k > 1 - \dfrac{a_1b_2}{a_2c_1}$, then $E_2(0, \frac{a_2}{b_2})$ is unstable; If $k > 1 - \dfrac{a_1b_2}{a_2c_1}$ holds, $E_3\Big(\dfrac{a_1b_2 - a_2c_1(1-k)}{b_1b_2}, \dfrac{a_2}{b_2}\Big)$ is stable.*

Proof. The Jacobian matrix of the system (1.1) is calculated as

$$J(x, y) = \begin{pmatrix} a_1 - 2b_1x - c_1(1-k)y & -c_1(1-k)x \\ 0 & -2b_2y + a_2 \end{pmatrix}.$$

Hence the Jacobian matrix of the system (1.1) about the equilibrium $E_1(0,0)$ is given by

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}.$$

Clearly $E_0(0,0)$ is unstable.
For $E_1(\frac{a_1}{b_1},0)$, its Jacobian matrix is given by

$$\begin{pmatrix} -a_1 & -\dfrac{c_1(1-k)a_1}{b_1} \\ 0 & a_2 \end{pmatrix}.$$

Clearly $E_1(\frac{a_1}{b_1},0)$ is unstable.
For $E_2(0,\frac{a_2}{b_2})$, its Jacobian matrix is given by

$$\begin{pmatrix} a_1 - \dfrac{c_1 a_2(1-k)}{b_2} & 0 \\ 0 & -a_2 \end{pmatrix}.$$

Hence, if $a_1 > \dfrac{c_1 a_2(1-k)}{b_2}$ holds, $E_2(0,\frac{a_2}{b_2})$ is unstable, and if $a_1 < \dfrac{c_1 a_2(1-k)}{b_2}$ holds, $E_2(0,\frac{a_2}{b_2})$ is stable. That is to say, if $k < 1 - \dfrac{a_1 b_2}{a_2 c_1}$ (in this case, system (1.1) has no positive equilibrium), then $E_2(0,\frac{a_2}{b_2})$ is stable and if $k > 1 - \dfrac{a_1 b_2}{a_2 c_1}$ (in this case, system (1.1) has positive equilibrium E_3), then $E_2(0,\frac{a_2}{b_2})$ is unstable. The Jacobian matrix about the equilibrium E_3 is given by

$$\begin{pmatrix} \dfrac{a_2 c_1(1-k) - a_1 b_2}{b_2} & \dfrac{c_1(k-1)(a_2 c_1(k-1) + a_1 b_2)}{b_1 b_2} \\ 0 & -a_2 \end{pmatrix},$$

if $k > 1 - \dfrac{a_1 b_2}{a_2 c_1}$, then $a_2 c_1(1-k) - a_1 b_2 < 0$, hence, the eigenvalues of the above matrix are negative and E_3 is stable. This ends the proof of Theorem 2.1. □

Remark 2.2. Theorem 2.1 shows that if the positive equilibrium is exist, then it is locally stable. Theorem 2.1 also shows that if the cover for the species is large enough, such that $k > 1 - \dfrac{a_1 b_2}{a_2 c_1}$, then two species could be coexisted in a stable state.

3. Global stability of the equilibria

Following we will further investigate the global stability of the boundary equilibrium E_2 and the positive equilibrium E_3. As a direct corollary of Lemma 1.1.4 of [11], we have,

Lemma 3.1. *System*

$$\frac{dy}{dt} = a_2 y(t) - b_2 y^2(t), \tag{3.1}$$

has a unique globally attractive positive equilibrium $y^* = \frac{a_2}{b_2}$.

Theorem 3.2. *If* $k < 1 - \dfrac{a_1 b_2}{a_2 c_1}$, *then* $E_2(0, \frac{a_2}{b_2})$ *is globally stable.*

Proof. $k < 1 - \dfrac{a_1 b_2}{a_2 c_1}$ is equivalent to $a_1 - c_1(1-k)\frac{a_2}{b_2} < 0$, hence, one could choose small enough $\varepsilon > 0$ such that

$$a_1 - c_1(1-k)(\frac{a_2}{b_2} - \varepsilon) < 0 \tag{3.2}$$

holds. For this ε, it follows from Lemma 3.1 that there exists a $T > 0$, such that every positive solution $y(t)$ of (3.1) satisfies

$$\frac{a_2}{b_2} - \varepsilon < y(t) < \frac{a_2}{b_2} + \varepsilon. \tag{3.3}$$

Now let's consider the Lyapunov function

$$V_1(x, y) = x + y - y^* - y^* \ln \frac{y}{y^*},$$

where $y^* = \frac{a_2}{b_2}$. Calculating the derivative of V along the solution of the system (1.1), by using equalities (3.3) and (3.2), we have

$$\begin{aligned} \dot{V}_1 &= a_1 x - b_1 x^2 - c_1(1-k)xy + (a_2 y - b_2 y^2)(1 - \frac{y^*}{y}) \\ &= -b_1 x^2 + (a_1 - c_1(1-k)y)x + (a_2 - b_2 y)(y - y^*) \\ &\leq -b_1 x^2 + \big(a_1 - c_1(1-k)(\frac{a_2}{b_2} - \varepsilon)\big)x - \frac{1}{b_2}(a_2 - b_2 y)^2. \end{aligned}$$

Obviously, $\frac{dV_1}{dt} < 0$ strictly for all $x, y > 0$ except the positive equilibrium $E_2(0, \frac{a_2}{b_2})$, where $\frac{dV_1}{dt} = 0$. Thus, $V_1(x, y)$ satisfies Lyapunov's asymptotic stability theorem, and the boundary equilibrium $E_2(0, \frac{a_2}{b_2})$ of system (1.1) is globally stable. This ends the proof of Theorem 3.2. □

Remark 3.3. Theorem 3.2 shows that if the boundary equilibrium E_2 is locally stable, then it is globally stable.

Theorem 3.4. *If* $k > 1 - \dfrac{a_1 b_2}{a_2 c_1}$ *holds, then* $E_3\big(x^*, y^*\big)$ *is globally stable.*

Proof. Firstly we proof that every solution of system (1.1) that starts in R^2_+ is uniformly bounded. From the first equation of (1.1) one has

$$\frac{dx}{dt} \leq a_1 x - b_1 x^2.$$

By using the differential inequality, we obtain

$$\limsup_{t \to +\infty} x(t) \leq \frac{a_1}{b_1}. \tag{3.4}$$

From (3.3) and (3.4), there exists a $\varepsilon > 0$ such that for all $t > T$

$$x(t) < \frac{a_1}{b_2} + \varepsilon, \;\; y(t) < \frac{a_2}{b_2} + \varepsilon.$$

Let $B = \{(x, y)| \in R^2_+ : x < \frac{a_1}{b_2} + \varepsilon, \;\; y < \frac{a_2}{b_2} + \varepsilon\}$. Then every solution of system (1.1) starts in R^2_+ is uniformly bounded on B. Also, from Theorem 2.1 there is a unique local stable positive equilibrium $E_3(x^*, y^*)$. To ensure $E_3(x^*, y^*)$ is globally stable in above area, we consider the Dulac function $u(x, y) = x^{-1}y^{-1}$, then

$$\frac{\partial(uP)}{\partial x} + \frac{\partial(uQ)}{\partial y} = -b_1y^{-1} - b_2x^{-1} < 0,$$

where $P(x, y) = a_1x - b_1x^2 - c_1xy, Q(x, y) = a_2y - b_2y^2$. By Dulac Theorem, there is no closed orbit in area B. So $E_3(x^*, y^*)$ is globally asymptotically stable. This completes the proof of Theorem 3.4. □

Remark 3.5. Theorem 3.4 shows that if the positive equilibrium E_3 exists, then it is globally stable.

Remark 3.6. Noting that $x^*(k) = \dfrac{a_1b_2 - a_2c_1(1-k)}{b_1b_2}$, and $\frac{dx^*(k)}{dk} = \dfrac{a_2c_1}{b_1b_2} > 0$, which means that the cover can increase the densities of the first species and thus can reduce the chance of the extinction of the first species.

4. Numeric simulations

Now let us consider the following example.

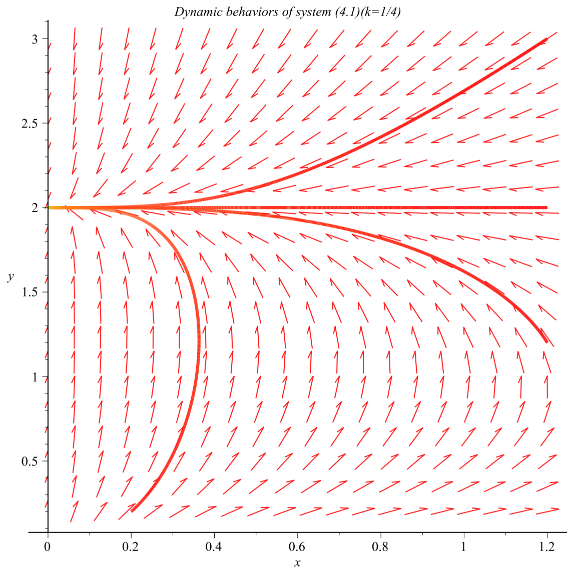

Figure 1: Numeric simulations of system (4.1) with $k = \frac{1}{4}$, the initial conditions $(x(0), y(0)) = (1.2, 1.2), (0.2, 2), (1.2, 3), (1.2, 0.2)$ and $(0.2, 0.2)$, respectively.

Example 4.1. Consider the following system

$$\begin{aligned} \frac{dx}{dt} &= x(t) - \frac{1}{4}x^2(t) - (1-k)x(t)y(t), \\ \frac{dy}{dt} &= 2y(t) - y^2(t). \end{aligned} \tag{4.1}$$

In this system, corresponding to system (1.1), we take $a_1 = b_2 = c_1 = 1, a_2 = 2, b_1 = \frac{1}{4}$. First, let us consider the case $k = \frac{1}{4}$, obviously, in this case $k < 1 - \frac{a_1b_2}{a_2c_1}$, and so, from Theorem 3.2, the prey species go extinct while predator species reaches its maximum environment carrying capacity. Figure 1 shows the dynamics behavior of species x and y, where $k = \frac{1}{4}$. It follows from Theorem 3.4 that for all $k > \frac{1}{2}$, system (4.1) admits a unique positive equilibrium, which is globally asymptotically stable. Figure 2 shows the dynamics behavior of species x and y, where $k = \frac{3}{4}$. The figures confirm the effect of refuge for the first species.

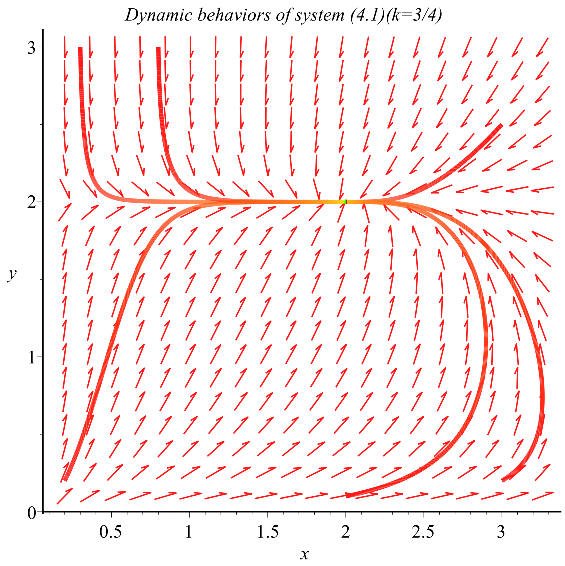

Figure 2: Numeric simulations of system (4.1) with $k = \frac{3}{4}$, the initial conditions $(x(0), y(0)) = (0.3, 3), (0.2, 0.2), (3, 2.5), (0.8, 3), (3, 0.2)$ and $(2, 0.1)$, respectively.

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