**R$_1$-separation axioms via supra soft topological spaces**

S. Saleh$^{a,b}$, Tareq M. Al-Shami$^{c,d}$, Laith R. Flaih$^a$, Murad Arar$^{e,*}$, Radwan Abu-Gdairi$^f$

$^a$Department of Computer Science, Cihan University-Erbil, Erbil, Iraq.
$^b$Department of Mathematics, Hodeidah University, Hodeidah, Yemen.
$^c$Department of Mathematics, Sana’a University, P.O. Box 1247, Sana’a, Yemen.
$^d$Department of Engineering Mathematics & Physics, Faculty of Engineering & Technology, Future University, New Cairo, Egypt.
$^e$Department of Mathematics, College of Sciences and Humanities in Aflaj, Prince Sattam bin Abdulaziz University, Riyadh, Saudi Arabia.
$^f$Department of Mathematics, Faculty of Science, Zarqa University, P.O. Box 13110, Zarqa, Jordan.

**Abstract**

The aim of this study is to introduce and investigate two new classes of separation axioms called supra soft $R_0$ and supra soft $R_1$. They are defined in the spaces of supra soft topologies by using the notions of supra soft open sets and supra soft closure operator. We discuss the basic properties and characterizations of them. We also study the relationships between these classes and some other supra soft separation axioms with many results and explanatory examples. Moreover, the connections between the properties of these classes and those in some generated soft topologies are presented. Finally, we show that these classes are preserved under subspaces, which means they are supra soft topological properties.

**Keywords:** Soft set, soft point, supra soft open set, supra soft topology, SS-kernel, supra soft $R_0$ and supra soft $R_1$ spaces.

**2020 MSC:** 03E72, 54A05, 54A40, 54D15.

©2024 All rights reserved.

1. Introduction

In 1999, Molodtsov [38] initiated the notion of soft sets ($S$-sets) as a new approach to dealing with uncertainty data while modeling problems in the real world such as economics, engineering, data science, and health sciences. Many authors have applied the theory of $S$-sets as a mathematical framework for addressing practical problems such as [24, 43]. To found abstract structures inspired by $S$-sets, Shabir-Naz [44] introduced the idea of soft topology and studied various related concepts. Then, the intellectuals interested in abstract structures endeavored to navigate topological concepts to the spaces of topologies. For instance, it was introduced the concepts of soft compactness and Lindelöfness [6], soft separation axioms [30, 37, 42, 45], soft mappings [10, 31], soft menger [32], almost soft menger [17] and nearly soft menger spaces [18] and soft metric [25]. Furthermore, the notions of generalized open sets have been

*Corresponding author
Email address: muradshhada@gmail.com (Murad Arar)
doi: 10.22436/jmcs.032.03.07
Received: 2023-07-04 Revised: 2023-07-20 Accepted: 2023-08-05
explored in soft topologies by some authors like soft semi-open sets [23], soft somewhat open sets [7], soft Q-set [3], weakly soft α-open sets [19] and weakly soft semi-open sets [20]. Some of these principles have been examined in the frame of supra fuzzy soft topology such as [40, 41].

In 1984, Mashhour et al. [35] introduced a new extension of topology, namely, supra topology. A supra topology on \( U \) is a collection \( \mu \) of subsets of \( U \), which is closed under an arbitrary union and \( \emptyset, U \in \mu \). Then, a lot of topological researchers look at topological concepts via supra-topologies and scrutinize their properties [4, 11–13, 28]. As a natural extension, it was displayed the concept of supra soft topological spaces by Elsheikh et al. [29] as a hybridization of supra topological spaces and S-set theory. The structures of supra topology have attracted the attention of many authors. We tackle the main contributions in this framework, El-Shafei et al. [27] presented some types of operators and compactness in supra topologies. Abd El-latif examined the concepts of supra soft compact spaces [2] and supra strongly generalized soft closed sets [1]. Aras and Bayramov [21, 22] formulated supra soft separability axioms with respect to soft points. Whereas, Al-shami with his coauthors [5, 14] created four levels of supra soft separation axioms with respect to ordinary points and elucidated the interrelations among them. Also, Al-shami and El-Shafei [15] combined supra soft topology with a partial order relation. Then, they [16] established various types of supra soft separation axioms in ordered settings. We draw the reader’s attention to that supra topological spaces have been employed to handle some real-life issues as debated in [9, 33], which confirm and enhance the importance of continued contributions to this scope of work.

The framework of this manuscript is the supra soft topology. We aim to introduce and study two new classes of separation properties called supra soft \( R_0 \) and \( R_1 \) in supra soft topological spaces by using the notions of supra S-open sets and supra S-closure operator. We investigate the basic properties and characterizations of them. The relationships between these new supra soft separation axioms and their relationships with some other properties are studied. Moreover, we describe the relationships between these properties in supra soft setting and those in some generated topologies and present some results and related theorems with some illustrate examples. We show that the supra soft separation axioms SSR\(_i\), \( i = 0, 1 \) are supra soft topological properties.

2. Basic background

Through this paper, \( U \) refers to an initial universe set, \( T \) is the set of all parameters for \( U \), and \( 2^U \) is the power set of \( U \). In the following lines, we give some concepts and results about S-set theory.

**Definition 2.1** ([38]). An S-set \( G_T = (G, T) \) on \( U \) is defined as a set of ordered pairs

\[
G_T = \{ (t, G(t)) : t \in T, \ G(t) \in 2^U \},
\]

where \( G \) is a mapping from \( T \) to \( 2^U \). The class of all S-sets on \( U \) will be symbolized by SS(\( U \)).

**Definition 2.2** ([34]). For \( H_T, G_T \in SS(U) \) and \( x \in U \), we have following.

- If \( H(t) = \emptyset \) (resp. \( H(t) = U \)) for all \( t \in T \), then \( H_T \) is called a null (resp. an absolute) S-set and symbolized by \( \emptyset \) (resp. \( \hat{U} \)).

- If \( H(t) = \{ x \} \) and \( H(t') = \emptyset \) for every \( t' \in T - (t) \), then \( H_T \) is called a soft point (or S-point) in \( \hat{U} \). It is symbolized by \( x_T \). We write \( x_T \in H_T \) if for the parameter \( t \in T, x \in H(t) \). The class of all S-points in \( \hat{U} \) is denoted by Sp(\( U \)).

- The relative complement \( H_T^c \) of \( H_T \) is a mapping \( H_T^c : T \rightarrow 2^U \) given by \( H_T^c(t) = U - H(t) \) for every \( t \in T \). Clearly \( (H_T^c)^c = H_T \).

- \( H_T \) is an S-subset of \( G_T \), symbolized by \( H_T \subseteq G_T \), if \( H(t) \subseteq G(t) \) for all \( t \in T \).
• The S-union (resp. S-intersection) of $H_T$ and $G_T$ is an S-set $K_T$ (resp. $L_T$) given by $K(t) = H(t) \cup G(t)$ (resp. $L(t) = H(t) \cap G(t)$) for all $t \in T$ and it is symbolized by $H_T \sqcup G_T$ (resp. $H_T \sqcap G_T$).

**Definition 2.3** ([31]). Let $SS(U)$ and $SS(V)$ be two families of all S-sets on $U$ with a set of parameters $T$ and $V$ with a set of parameters $E$, respectively. Let $u : U \rightarrow V$ and $p : T \rightarrow E$ be two maps, then the map $f_{up} : SS(U) \rightarrow SS(V)$ is said to be a soft map (briefly, S-map) and we have:

(i) for $H_T \in SS(U)$, the image $f_{up}(H_T)$ is the S set on $V$ given by $f_{up}(H_T)(e) = \bigcup u[H(t) : t \in p^{-1}(e)]$ if $p^{-1}(e) \neq \emptyset$ and $f_{up}(H_T)(e) = \emptyset$ otherwise for any $e \in E$;

(ii) for $G_E \in SS(V)$, the preimage $f_{up}^{-1}(G_E)$ of $V_E$ is the S-set on $U$ given by $f_{up}^{-1}(G_E)(t) = u^{-1}(G(p(t)))$ for any $t \in T$.

The S-map $f_{up}$ is called one-one (resp. onto, bijective), if $u$ and $p$ are one-one (resp. onto, bijective).

**Definition 2.4** ([43]). A family $\tau \subseteq SS(U)$ under a fixed set of parameters $T$ is called a soft topology on $U$ if $\tau$ is closed under arbitrary S-union, finite S-intersections and $\emptyset, U \in \tau$. The triple $(U, \tau, T)$ is called a soft topological space (or STS). Any element in $\tau$ is an S-open set, and the complement of any S-open set is an S-closed set.

For the S-set $H_T$ in STS $(U, \tau, T)$, the S-closure $cl(H_T)$ of $H_T$ is the S-intersection of all S-closed super sets of $H_T$. The S-interior $int(H_T)$ of $H_T$ is the S-union of all S-open sets contained in $H_T$.

**Definition 2.5** ([29]). A family $\mu \subseteq SS(U)$ with a fixed set of parameters $T$ is said to be a supra soft topology (briefly, SST) on $U$ if $\emptyset, U \in \mu$ and the S-union of any number of S-sets in $\mu$ belongs to $\mu$. The triple $(X, \mu, T)$ is called a supra soft topological space (briefly, SST).

**Definition 2.6** ([14]). Let $(U, \mu, T)$ be an SST, $H_T \in SS(U)$, and $x \in U$, then:

• every element of $\mu$ is called supra soft open set (briefly, SS-open set), and the set of all SS-open sets on $U$ is denoted by $SSO(U)$;

• $H_T$ is called a supra soft closed set (briefly, SS-closed set) in $U$ if $H_T^c \in \mu$ and the set of all SS-closed sets is denoted by $SSC(U)$;

• if $(U, \tau, T)$ is an STS, we say that, $\mu$ is an SST associated with $\tau$ if $\tau \subseteq \mu$;

• an S-set $H_T$ is called a supra soft neighborhood of an S-point $x_t$ if there is an SS-open set $G_T$ such that $x_t \in G_T \subseteq H_T$.

**Definition 2.7** ([15, 26, 44]). For $H_T \in SS(U)$, $Y \subseteq U$, and $x \in U$, we have:

1. $x \in H_T$ if $x \in H(t)$ for all $t \in T$, and $x \notin H_T$ if $x \notin H(t)$ for some $t \in T$;

2. if $H(t) = \{x\}$ for all $t \in T$, then $H_T$ is called an S-singleton point, denoted by $x_T$, we write $x_T \in H_T$ if $x \in H_T$ for all $t \in T$;

3. for $x_t, y_t \in SP(U)$, we write $x_t \neq y_t$ if $x \neq y$ or $t \neq r$;

4. $Y = (Y, T)$ refers to the S-set on $U$ for which $Y(t) = Y$ for all $t \in T$, is called stable.

**Definition 2.8** ([29]). Let $(U, \mu, T)$ be an SST and $F_T \in SS(U)$. The supra soft closure (briefly, SS-closure) of $F_T$, denoted by $cl_s(F_T)$ is the intersection of all SS-closed super sets of $F_T$, and the supra soft interior (briefly, SS-interior) of $F_T$, denoted by $int_s(F_T)$ is the union of all SS-open subsets of $F_T$.

**Theorem 2.9** ([14]). For two SS-sets $G_T$ and $F_T$ in SST $(U, \mu, T)$, we have:

1. $G_T \subseteq SSC(U) \iff cl_s(G_T) = G_T$;

2. $G_T \subseteq F_T \Rightarrow cl_s(G_T) \subseteq cl_s(F_T)$;

3. $x_t \in cl_s(G_T) \iff F_T \cap G_T \neq \emptyset$ for all $F_T \in \mu$ and $x_t \notin F_T$. 
Definition 2.10 ([14]). For SSTs \((U, \mu, T)\) and \(Y \subseteq U\). The family \(\mu_Y = \{\overline{\cap} F_T : F_T \in \mu\}\) is an SST on \(Y\), and \((Y, \mu_Y, T)\) is called an SS-subspace of \((U, \mu, T)\).

For the SST-subspace \((Y, \mu_Y, T)\) of \((U, \mu, T)\) and \(H_T \in SS(Y)\) we have, \(H_T\) is an SS-open set in \((Y, \mu_Y, T)\) if and only if \(H_T = \overline{\cap} G_T\) for some \(G_T \in \mu\).

Definition 2.11 ([14]). An S-map \(f_{up} : (U, \mu, T) \to (V, \theta, E)\) is called:

(i) SS-continuous if \(f_{up}^{-1}(H_E) \in \mu\) for any SS-open set \(H_E \in \theta;\)

(ii) SS-open if \(f_{up}(G_T) \in \theta\) for any SS-open set \(G_T \in \mu;\)

(iii) SS-homeomorphism if it is bijective, SS-continuous, and SS-open.

Definition 2.12 ([22]). An SSTs \((U, \mu, T)\) is said to be:

(i) supra soft \(T_0\) (briefly, \(ST_0\)) iff for any \(x_t \neq y_t \in SP(U)\) there are SS-open sets \(H_T\) and \(F_T\) such that \(x_t \in \overline{\cap} F_T\) and \(y_t \notin \overline{\cap} H_T\) or \(y_t \in \overline{\cap} H_T\) and \(x_t \notin \overline{\cap} F_T;\)

(ii) supra soft \(T_1\) (briefly, \(ST_1\)) iff for any \(x_t \neq y_t \in SP(U)\) there are SS-open sets \(H_T\) and \(F_T\) such that \(x_t \in \overline{\cap} H_T, y_t \notin \overline{\cap} H_T\) and \(y_t \in \overline{\cap} F_T;\)

(iii) supra soft \(T_2\) (briefly, \(ST_2\)) iff for any \(x_t \neq y_t \in SP(U)\) there are SS-open sets \(H_T\) and \(F_T\) such that \(x_t \in \overline{\cap} H_T, y_t \notin \overline{\cap} H_T\) and \(H_T \cap \overline{\cap} F_T = \emptyset.\)

Remark 2.13 ([22]). \(ST_2 \implies ST_1 \implies ST_0.\)

Definition 2.14 ([39]). An SSTs \((U, \tau, T)\) is called:

(i) soft \(R_0\) (briefly, \(SR_0\)) if for any \(x_t \neq y_t \in SP(U)\) with \(cl(x_t) \neq cl(y_t)\) implies \(cl(x_t) \cap cl(y_t) = \emptyset;\)

(ii) soft \(R_1\) (briefly, \(SR_1\)) if for any \(x_t \neq y_t \in SP(U)\) with \(cl(x_t) \neq cl(y_t)\), there are disjoint S-open subsets \(H_T, G_T\) of \(U\) such that \(x_t \in H_T\) and \(y_t \in G_T.\)

Remark 2.15 ([39]). Every \(SR_1\) space is \(SR_0.\)

Definition 2.16 ([36]). A supra topological space \((U, \tau, T)\) is called:

(i) supra \(R_0\) (or \(sup-R_0\)) if for any \(x \neq y \in U\) with \(cl(x) \neq cl(y)\), \(cl(x) \cap cl(y) = \emptyset;\)

(ii) supra \(R_1\) (or \(sup-R_1\)) if for any \(x \neq y \in U\) with \(cl(x) \neq cl(y)\), there are disjoint supra open subsets \(A, B\) of \(U\) such that \(x \in A\) and \(y \in B.\)

3. On supra soft \(R_0\) and \(R_1\) spaces

In the following, we introduce and discuss a new class of supra soft separation properties called \(SSR_i, i = 0, 1\) and investigate some characterizations for them.

First, we give some definitions and lemmas which are used in the sequel.

Definition 3.1. Let \((U, \mu, T)\) be an SSTs, \(F_T \in SS(U),\) and \(x_t \in SP(U)\). The supra soft kernel (briefly, \(SSK\)) of \(F_T\) is denoted by \(SSK(F_T) = \overline{\cap}(G_T \subseteq \mu : F_T \subseteq G_T).\) In particular, the supra soft kernel of \(x_t\) is defined by \(SSK(x_t) = \overline{\cap}(G_T \subseteq \mu : x_t \subseteq G_T).\)

Lemma 3.2. Let \((U, \mu, T)\) be an SSTs and \(F_T \in SS(U),\) then \(SSK(F_T) = \overline{U}(x_t \in SP(U) : cl_s(x_t) \cap F_T \neq \emptyset).\)

Proof. Let \(x_t \in SSK(F_T).\) Assume that \(cl_s(x_t) \cap F_T = \emptyset,\) we get \(F_T \subseteq cl_s(x_t)\) and so, \(x_t \notin cl_s(x_t)\) which is an SS-open set contains \(F_T.\) This contradicts with \(x_t \in SSK(F_T).\) So \(cl_s(x_t) \cap F_T \neq \emptyset\) and \(SSK(F_T) \subseteq \overline{U}(x_t \in SP(U) : cl_s(x_t) \cap F_T \neq \emptyset).\)

Conversely, let \(cl_s(x_t) \cap F_T \neq \emptyset.\) Suppose \(x_t \notin SSK(F_T),\) there is \(G_T \subseteq \mu\) such that \(F_T \subseteq G_T\) and \(x_t \notin G_T.\) Let \(y_t \in cl_s(x_t)\) we have \(y_t \in cl_s(x_t)\) and \(G_T\) is an SS-open set containing \(y_t\) imply \(x_t \in G_T,\) a contradiction. So that \(x_t \in SSK(F_T).\) This completes the proof. \(\square\)
Lemma 3.3. Let $(U, \mu, T)$ be an SSTS and $x_t \in \text{SP}(U)$, then $x_t \in \overline{\text{cl}}_s(y_r)$ if and only if $y_r \in \overline{\text{SSK}}(x_t)$. 

Proof. It is clear. □

Definition 3.4. An SSTS $(U, \mu, T)$ is said to be:

(i) supra soft $R_0$ (briefly, SSR$_0$) iff $x_t \neq y_r \in \text{SP}(U)$ with $x_t \in \overline{\text{cl}}_s(y_r)$ implies $y_r \in \overline{\text{cl}}_s(x_t)$;

(ii) supra soft $R_1$ (briefly, SSR$_1$) iff $x_t \neq y_r \in \text{SP}(U)$ with $\overline{\text{cl}}_s(x_t) \neq \overline{\text{cl}}_s(y_r)$, there are SS-open sets $F_T, G_T$ such that $x_t \in F_T$ and $y_r \in G_T$ with $F_T \cap G_T = \emptyset$.

Theorem 3.5. Every SSR$_1$ space is SSR$_0$.

Proof. Let $x_t \neq y_r \in \text{SP}(U)$ with $x_t \in \overline{\text{cl}}_s(y_r)$, then $\overline{\text{cl}}_s(x_t) \neq \overline{\text{cl}}_s(y_r)$. Since $(U, \mu, T)$ is SSR$_1$, there is $G_T \in \mu$ such that $y_r \in G_T$ and $x_t \notin G_T$. Thus $y_r \notin \overline{\text{cl}}_s(x_t)$, and this completes the proof. □

The converse of the above theorem is not necessary true, the next example shows it.

Example 3.6. Let $U$ be an infinite set. The class $\mu = \{\emptyset\} \cup \{H_T : (H_t) \in \mathcal{C}\}$ is a finite subset of $U$ for all $t \in T$ is SST on $U$ and $(U, \mu, T)$ is called an SS-cofinite space. Now one can verify $\mu$ is SSR$_0$. But it is not SSR$_1$. Indeed, suppose that $(U, \mu, T)$ is SSR$_1$ and $x_t \neq y_r \in \text{SP}(U)$ with $\overline{\text{cl}}_s(x_t) \neq \overline{\text{cl}}_s(y_r)$, there are $F_T, G_T \in \mu$ such that $x_t \in F_T$ and $y_r \in G_T$ with $F_T \cap G_T = \emptyset$ implies $(F(t)) \in \mathcal{C} \cup (G(t)) \in \mathcal{C} = U$. Since $(F(t)) \in \mathcal{C}$ and $(G(t)) \in \mathcal{C}$ are finite subsets of $U$, this means that $U$ is finite. This is a contradiction. Thus $(U, \mu, T)$ is not SSR$_1$.

Theorem 3.7. Let $(U, \tau, T)$ be an SSTS and $\mu$ be an SST associated with $\tau$, then $(U, \mu, T)$ is SSR$_0$ if and only if $\overline{\text{cl}}_s(x_t) \subseteq F_T$ for any $F_T \in \mu$ and $x_t \notin F_T$.

Proof. Let $(U, \mu, T)$ be SSR$_0$. Suppose $\overline{\text{cl}}_s(x_t) \subseteq F_T$ for some $F_T \in \mu$ and $x_t \notin F_T$, there exists an S-point $y_r$ such that $y_r \in \overline{\text{cl}}_s(x_t)$, $y_r \notin F_T$. So that $y_r \in \overline{\text{cl}}_s(y_r)$ for some $F_T \in \mu$, $x_t \in F_T$ where $x_t, y_r$ are different S-points in $U$. Thus $x_t \notin \overline{\text{cl}}_s(y_r)$. This is a contradiction. Hence, the necessary part holds. Conversely, let $x_t \notin \overline{\text{cl}}_s(y_r)$, there is an SS-open set $G_T$ contains $x_t$ such that $y_r \notin G_T$ imply $y_r \notin G_T$. Clearly $\overline{\text{cl}}_s(x_t) \subseteq G_T$, we have $y_r \notin \overline{\text{cl}}_s(x_t)$. We obtain the desired result. □

Theorem 3.8. Let $(U, \tau, T)$ be an SSTS and $\mu$ be an SST associated with $\tau$, then $(U, \mu, T)$ is SSR$_0$ if and only if for any $F_T \in \text{SSC}(U)$ with $x_t \notin F_T$ implies $F_T \subseteq X_T$ and $x_t \notin X_T$ for some $X_T \in \mu$.

Proof:

''$\Longrightarrow$'' Let $F_T \in \text{SSC}(U)$ and $x_t \notin F_T$, then $x_t \notin F_T \supseteq F_T$ which is an SS-open set contains $x_t$. Since $(U, \mu, T)$ is SSR$_0$, we have $\overline{\text{cl}}_s(x_t) \subseteq F_T \supseteq F_T$. Take $G_T = (\overline{\text{cl}}_s(x_t)) \in \mathcal{C}$, then $G_T \in \mu$ and $x_t \notin G_T$.

''$\Longleftarrow$'' Let $x_t \neq y_r \in \text{SP}(U)$. Suppose that $x_t \notin \overline{\text{cl}}_s(y_r) \in \text{SSC}(U)$. By hypothesis, there is an SS-open set $G_T$ such that $\overline{\text{cl}}_s(y_r) \subseteq G_T$, this implies $x_t \notin G_T$. So, $\overline{\text{cl}}_s(x_t) \subseteq G_T$ but $y_r \notin G_T$. Thus $y_r \notin \overline{\text{cl}}_s(x_t)$ and the result holds. □

Theorem 3.9. For SSTS $(U, \mu, T)$, the next statements are equivalent:

(1) $(U, \mu, T)$ is SSR$_0$;

(2) $F_T \in \text{SSC}(U) \implies F_T = \text{SSK}(F_T)$;

(3) $H_T \in \text{SSC}(U)$ and $x_t \notin H_T \implies \text{SSK}(x_t) \subseteq H_T$;

(4) $x_t \in \text{SP}(U) \implies \text{SSK}(x_t) \subseteq \overline{\text{cl}}_s(x_t)$.

Proof:

(1) $\implies$ (2) Let $F_T \in \text{SSC}(U)$. Suppose that $x_t \notin F_T$, we have $x_t \notin F_T \supseteq F_T$ which is an SS-open set containing $x_t$. Since $(U, \mu, T)$ is SSR$_0$, we get $\overline{\text{cl}}_s(x_t) \subseteq F_T$ this implies that $\overline{\text{cl}}_s(x_t) \cap F_T = \emptyset$. From Lemma 3.2, we get $x_t \notin \text{SSK}(F_T)$. So that $F_T = \text{SSK}(F_T)$.
(2) \implies (3) It follows from that K_{T} \subseteq G_{T} implies SSK(K_{T}) \subseteq SSK(G_{T}).

(3) \implies (4) It is clear.

(4) \implies (1) Let x_{t} \neq y_{r} \in SP(U) with x_{t} \notin cl_{s}(y_{r}). From Lemma 3.3, we have y_{r} \notin SSK(x_{t}). Since x_{t} \notin cl_{s}(x_{t}) which is an SS-closed set and from (4), we get y_{r} \notin SSK(x_{t}) \subseteq cl_{s}(x_{t}), that is y_{r} \notin cl_{s}(x_{t}) and this completes the proof.

Proposition 3.10. An SSTS (U, \mu, T) is SSR_{0} if and only if cl_{s}(x_{t}) \subseteq SSK(x_{t}) for all x_{t} \in SP(U).

Proof. It follows from Lemma 3.3 and Theorem 3.7.

Theorem 3.11. For SSTS (U, \mu, T), the next statements are equivalent:

(i) (U, \mu, T) is SSR_{0};

(ii) for any F_{T} \in SSC(U) with x_{t} \notin cl_{s}(y_{r}), there is z_{e} \notin cl_{s}(x_{t}) and z_{e} \notin cl_{s}(y_{r}) or z_{e} \notin cl_{s}(y_{r}) and z_{e} \notin cl_{s}(x_{t}). So there is F_{T} \in \mu such that y_{r} \notin F_{T}, z_{t} \notin F_{T} and so, x_{t} \notin F_{T}. Therefore x_{t} \notin cl_{s}(y_{r}). From (ii) we obtain cl_{s}(x_{t}) \subseteq cl_{s}(y_{r}). The proof of the remaining case is similar.

(iii) for any x_{t} \neq y_{r} \in SP(U), either cl_{s}(x_{t}) \subseteq cl_{s}(y_{r}) or cl_{s}(x_{t}) \subseteq cl_{s}(y_{r}) = \emptyset.

Proof. (i) \implies (ii) Let F_{T} \in SSC(U) and x_{t} \notin F_{T}, then x_{t} \notin cl_{s}(y_{r}) \subseteq cl_{s}(y_{r}) = \emptyset.

(ii) \implies (iii) Let x_{t} \neq y_{r} \in SP(U) with x_{t} \notin cl_{s}(y_{r}), then cl_{s}(x_{t}) \subseteq cl_{s}(y_{r}) = \emptyset. From (iii) we get cl_{s}(x_{t}) \subseteq cl_{s}(y_{r}) = \emptyset.

Corollary 3.12. Let (U, \tau, T) be an STS and \mu be an SST associated with \tau, then (U, \mu, T) is SSR_{0} if and only if for any two different S-points x_{t}, y_{r} in U with cl_{s}(x_{t}) \neq cl_{s}(y_{r}) \implies \emptyset.

Proof. It follows from above theorem.

Lemma 3.13. Let (U, \mu, T) be an SSTS, x_{t}, y_{r} \in SP(U), then SSK(x_{t}) \neq SSK(y_{r}) if and only if cl_{s}(x_{t}) \neq cl_{s}(y_{r}).

Proof. Suppose that SSK(x_{t}) \neq SSK(y_{r}), there is z_{e} \in SP(U) with z_{e} \notin SSK(x_{t}) and z_{e} \notin SSK(y_{r}). Since z_{e} \notin SSK(x_{t}) from Lemma 3.2, we get x_{t} \notin cl_{s}(z_{e}) implies y_{r} \notin cl_{s}(z_{e}), that is cl_{s}(x_{t}) \subseteq cl_{s}(z_{e}). Similarly, since z_{e} \notin SSK(y_{r}) we get y_{r} \notin cl_{s}(z_{e}). Since cl_{s}(z_{e}) \subseteq cl_{s}(z_{e}) and y_{r} \notin cl_{s}(z_{e}), we have y_{r} \notin cl_{s}(z_{e}). Conversely, let cl_{s}(x_{t}) \neq cl_{s}(y_{r}) there is z_{e} \in SP(U) with z_{e} \notin cl_{s}(x_{t}) and z_{e} \notin cl_{s}(y_{r}). Thus, there is an SS-open set containing z_{e} and so x_{t} but not y_{r}. Hence y_{r} \notin SSK(x_{t}). The proof is complete.

Theorem 3.14. An SSTS (U, \mu, T) is SSR_{0} if and only if x_{t} \neq y_{r} \in SP(U) with SSK(x_{t}) \neq SSK(y_{r}) implies SSK(x_{t}) \subseteq SSK(y_{r}) = \emptyset.

Proof. Suppose (i) \implies (ii) (U, \mu, T) is SSR_{0} and x_{t} \neq y_{r} \in SP(U) with SSK(x_{t}) \neq SSK(y_{r}). By Lemma 3.13, we get cl_{s}(x_{t}) \neq cl_{s}(y_{r}). Suppose SSK(x_{t}) \subseteq SSK(y_{r}) = \emptyset, there is z_{e} \notin SSK(x_{t}) \subseteq SSK(y_{r}). Since z_{e} \notin SSK(x_{t}), from Lemma 3.3, we have x_{t} \notin cl_{s}(z_{e}) implies cl_{s}(x_{t}) \subseteq cl_{s}(z_{e}). Since x_{t} \notin cl_{s}(x_{t}) and from Corollary 3.12, we get cl_{s}(x_{t}) = cl_{s}(z_{e}). Similarly, since z_{e} \notin SSK(y_{r}), we have cl_{s}(y_{r}) = cl_{s}(z_{e}) = cl_{s}(x_{t}). This is contradiction therefore, SSK(x_{t}) \subseteq SSK(y_{r}) = \emptyset.
"⇐" Let \( x_t \neq y_r \in \text{SP(}U\text{)} \) with \( \text{cl}_s(x_t) \neq \text{cl}_s(y_r) \). From Lemma 3.13, we have \( \text{SSK}(x_t) \neq \text{SSK}(y_r) \). By hypothesis, we get \( \text{SSK}(x_t) \cap \text{SSK}(y_r) = \emptyset \). Suppose that \( \text{cl}_s(x_t) \cap \text{cl}_s(y_r) \neq \emptyset \), there is \( z_e \in U \) such that \( z_e \in \text{cl}_s(x_t) \) and \( z_e \in \text{cl}_s(y_r) \). Form Lemma 3.3, we have \( x_t \in \text{SSK}(z_e) \) and \( y_r \in \text{SSK}(z_e) \) and by Lemma 3.2, we obtain, \( \text{SSK}(x_t) \cap \text{SSK}(z_e) \neq \emptyset \) and \( \text{SSK}(y_r) \cap \text{SSK}(z_e) \neq \emptyset \). By hypothesis we get, \( \text{SSK}(x_t) = \text{SSK}(z_e) \) and \( \text{SSK}(y_r) = \text{SSK}(z_e) \). Hence \( \text{SSK}(x_t) = \text{SSK}(y_r) \). So, \( \text{SSK}(x_t) \cap \text{SSK}(y_r) \neq \emptyset \). This is a contradiction. Hence \( \text{cl}_s(x_t) \cap \text{cl}_s(y_r) = \emptyset \). From Corollary 3.12, we obtain the result.

**Proposition 3.15.** An SSTSS \((U, \mu, T)\) is SSR\(_0\) if and only if \( \text{cl}_s(x_t) \subseteq \text{SSK}(x_t) \) for all \( x_t \in \text{SP}(U) \).

**Proof.** It follows from Definition 3.1 and Theorem 3.7.

From Lemma 3.3 and the above proposition one can verify the next corollary.

**Corollary 3.16.** An SSTSS \((U, \mu, T)\) is SSR\(_0\) for any \( x_t \in \text{SP}(U) \), \( \text{SSK}(x_t) = \text{cl}_s(x_t) \).

**Theorem 3.17.** An SSTSS \((U, \mu, T)\) is SSR\(_1\) if and only if for any \( x_t \neq y_r \in \text{SP}(U) \) with \( \text{SSK}(x_t) \neq \text{SSK}(y_r) \), there are \( F_T, G_T \in \mu \) such that \( \text{cl}_s(x_t) \subseteq F_T \), \( \text{cl}_s(y_r) \subseteq G_T \) and \( F_T \cap G_T = \emptyset \).

**Proof.** It follows from Lemma 3.13.

**Proposition 3.18.** For an SSTSS \((U, \mu, T)\), the next statements are equivalence:

1. \((U, \mu, T)\) is SSR\(_1\);
2. for any \( x_t \neq y_r \in \text{SP}(U) \) with \( x_t \in \text{cl}_s(y_r) \), there are \( H_T, G_T \in \mu \) such that \( x_t \in H_T, y_r \subseteq G_T \) and \( H_T \cap G_T = \emptyset \);
3. for any \( x_t \neq y_r \in \text{SP}(U) \) with \( \text{cl}_s(x_t) \neq \text{cl}_s(y_r) \), there are \( F_T, G_T \in \mu \) such that \( \text{cl}_s(x_t) \subseteq F_T \) and \( \text{cl}_s(y_r) \subseteq G_T \) with \( F_T \cap G_T = \emptyset \).

**Proof.** It follows from the above theorem and Lemma 3.13.

**Theorem 3.19.** Every SS-subspace \((Y, \mu_Y, T)\) of an SSR\(_i\) space \((U, \mu, T)\) is SSR\(_i\), \( i = 0, 1 \).

**Proof.** We will show the case \( i = 1 \). The proof of the rest case is similar. To prove that \((Y, \mu_Y, T)\) is SSR\(_1\), let \( x_t \neq y_r \in \text{SP}(Y) \) with \( \text{cl}_s(x_t) \neq \text{cl}_s(y_r) \), then also \( x_t, y_r \) are different S-points in \( U \) with \( \text{cl}_s(x_t) \neq \text{cl}_s(y_r) \). Since \((U, \mu, T)\) is SSR\(_1\), there are \( F_T, G_T \in \mu \) such that \( x_t \subseteq F_T, y_r \subseteq G_T \), and \( F_T \cap G_T = \emptyset \). So there are SS-open sets in \( Y \) say, \( H^Y_T = \text{Y} \cap F_T \in \mu_Y \) and \( V^Y_T = \text{Y} \cap G_T \in \mu_Y \), which are containing \( x_t, y_r \), respectively, with \( H^Y_T \cap V^Y_T = \emptyset \). The result holds.

**Corollary 3.20.** An open SS-subspace of SSR\(_i\) space need not be SSR\(_i\) space, \( i = 0, 1 \).

**Proof.** Clearly, it follows from that, the S-intersection of an S-open set and an SS-open set need not to be SS-open soft.

**Remark 3.21.** Clearly, every SSR\(_i\) space is SSR\(_i\), for \( i = 0, 1 \).

## 4. More characterizations and relations

This section is devoted to investigating how the concepts of supra soft \( R_0 \) and \( R_1 \) spaces navigate between supra soft topological spaces and their parametric supra topological spaces. To demonstrate the obtained relationships we provide some elucidative examples.

**Definition 4.1.** Let \((U, \sigma)\) be a supra topological space and \( T \) be a fixed set of parameters. The family \( \mu_\sigma = \{ H_T : H(t) = A \text{ for all } t \in T \text{ and } A \in \sigma \} \) defines an SST called stable SST on \( U \) derived from \( \sigma \). In general, an SSTSS \((U, \mu, T)\) is called stable if any SS-open set in \((U, \mu, T)\) is stable.
Definition 4.2. An SSTS \((U, \mu, T)\) is called a strong stable if \(\mu = \{H_T : H(t) = B \text{ for all } t \in T \text{ and } B \subset U\}\). In this case any SS-singleton point \(x_T\) is SS-open set.

Definition 4.3. Let \((U, \mu, T)\) be an SSTS, then the class \(\mu_t = \{F(t) : F_t \in \mu\}\) for each \(t \in T\) defines a supra topology on \(U\) which is called a parametric supra topology [14].

Remark 4.4.

1. If \((U, \mu, T)\) is a strong stable SSTS, we have:
   (i) any element in \((U, \mu, T)\) is both SS-open and SS-closed set;
   (ii) \((U, \mu_t)\) is a discrete space for all \(t \in T\);
   (iii) every stable SSTS \((U, \mu_\sigma, T)\) is a subspace of a strong stable SSTS \((U, \mu, T)\);
   (iv) every strong stable SSTS \((U, \mu, T)\) is a subspace of soft discrete space \(\tau = SS(U)\).

2. If \((U, \sigma)\) is a discrete topological space, then the SST \(\mu_\sigma\), which is defined in Definition 3.1, is a strong stable SST on \(U\).

Definition 4.5. For an SST \((U, \mu, T)\), if \(F^c_t \in \mu\) for every \(F_t \in \mu\), then \((U, \mu, T)\) is called a complemenatal SST.

Example 4.6.

(1) Let \(U = \{a, b, c\}, T = \{t_1, t_2\}\), and the family \(\emptyset = \{\emptyset, U, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}\). Then \(\emptyset\) is an supra topology on \(U\) and from Definition 4.1, we have the family \(\mu_{\emptyset} = \{\emptyset, \tilde{U}, \{a, b\}, \{b, c\}, \{a, c\}\}\) is a stable SST on \(U\).

(2) Let \(U = \{x, y, z\}, T = \{t_1, t_2\}\) and \(\tau = SS(U)\) be a soft discrete topology on \(U\). The family \(\mu = \emptyset, \tilde{U}, x_T, y_T, z_T, \{x, y\}, \{x, z\}, \{y, z\}\) is a strong stable SST on \(U\) and any element in \(\mu\) is both SS-open and SS-closed set. Moreover, \(\mu\) is a complemenatal SST and it is a subspace of \(\tau\). On other hand, \(\mu_{t_1} = \mu_{t_2} = \emptyset, U, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}\) is a discrete topology on \(U\).

Theorem 4.7. Every strong stable SST \((U, \mu, T)\) is \(SSR_i, i = 0, 1\).

Proof. We will prove only the case \(i = 1\). The proof of the other cases is similar. Let \((U, \mu, T)\) be a strong stable SSTS and \(x_t \neq y_t \in SP(U)\) such that \(\text{cl}_s(x_t) \neq \text{cl}_s(y_t)\). From Definition 4.2, there are SS-open sets \(x_T, y_T\) such that \(x_t \in x_T\) and \(y_t \in y_T\) with \(x_T \cap y_T = \emptyset\). Hence \((U, \mu, T)\) is \(SSR_1\).

Corollary 4.8. Every stable SST \((U, \mu, T)\) is \(SSR_i, i = 0, 1\).

Theorem 4.9. Every complemenatal SST \((U, \mu, T)\) is \(SSR_i, i = 0, 1\).

Proof. We will prove only the case \(i = 1\). The proof of other case is similar. Let \(x_t \neq y_t \in SP(U)\) and \(x_t \notin \text{cl}_s(y_t)\), then \(x_t \in \text{cl}_s(x_t) \subset F_T \in \mu\). Since \((U, \mu, T)\) is a complemenatal SST, we have \(y_t \in \text{cl}_s(y_t) = G_T \in \mu\). Clearly, \(F_T \cap G_T = \emptyset\) and so, from Proposition 3.18 (2), we obtain \((U, \mu, T)\) is \(SSR_1\).

Theorem 4.10. If \((U, \mu, T)\) is \(SSR_i\), then \((U, \mu_t)\) is \(sup-R_i\) for all \(t \in T\) and \(i = 0, 1\).

Proof. We will prove the case \(i = 1\). The proof of the case \(i = 0\) is similar. To prove that \((U, \mu_t)\) is \(sup-R_i\), let \(x \neq y \in U\) with \(\text{cl}(x) \neq \text{cl}(y)\), then either \(x \notin \text{cl}(y)\) or \(y \notin \text{cl}(x)\). Thus \(x_t \notin \text{cl}_s(y_t)\) or \(y_t \notin \text{cl}_s(x_t)\) imply \(\text{cl}_s(x_t) \neq \text{cl}_s(y_t)\). Since \((U, \mu, T)\) is \(SSR_i\), there are \(F_T, G_T \in \mu\) such that \(x_t \in F_T\) and \(y_t \in G_T\) with \(F_T \cap G_T = \emptyset\) and so, there are \(F(t)\) and \(G(t) \in \mu\) such that \(x \in F(t)\) and \(y \in G(t)\) with \(F(t) \cap G(t) = \emptyset\) for all \(t \in T\). Therefore \((U, \mu_t)\) is \(sup-R_i\) for all \(t \in T\).

The next example shows that the converse of the above theorem may not be true.
Example 4.11. Let \( U = \{u_1, u_2\} \) and \( T = \{t_1, t_2\} \). Consider the family \( \mu = (\emptyset, \hat{U}, F_{1T}, F_{2T}, F_{3T}, F_{4T}) \), where \( F_{1T} = \{(t_1, \{u_1\})\}, F_{2T} = \{(t_1, \{u_1\}), (t_2, \{u_1\})\}, F_{3T} = \{(t_1, \{u_1\}), (t_2, \{u_2\})\}, \) and \( F_{4T} = \{(t_1, \{u_1\}), (t_2, \{u_2\})\} \). Then \( \mu \) is an SST on \( U \) and the class \( \mu \) is \( \{\emptyset, \{u_1\}, \{u_2\}\} \) is a supra topology on \( U \). It is clear that \((U, \mu, T)\) is sup-R\(_1\) and sup-R\(_0\). But \((U, \mu, T)\) is not SSR\(_0\). Indeed, for \((u_1)\) is not SSR\(_0\). Hence \((U, \mu, T)\) is not SSR\(_1\).

Proposition 4.12. If \((U, \mu, T)\) is a strong stable SSTS, then \((U, \mu, T)\) is SSR\(_1\) if and only if \((U, \mu, T)\) is sup-R\(_1\) for all \( t \in T \) and \( i = 0, 1 \).

Proof. We will give the proof for \( i = 1 \). The proof for the case \( i = 0 \) is similar.

\[ \implies \] The proof follows from that of Theorem 4.10.

\[ \Leftarrow \] Let \( x_1 \neq y_r \in SP(U) \) with \( cl_s(x_1) \neq cl_s(y_r) \), either \( x \notin cl_s(y_r) \) or \( y \notin cl_s(x_1) \) implies \( x \notin cl_s(y_r) \) or \( y \notin cl_s(x_1) \), then \( cl_s(x_1) \neq cl_s(y_r) \). Since \((U, \mu, T)\) is sup-R\(_1\), there are supra open subsets \( F, K \) of \( U \) such that \( x \in F \) and \( y \in K \) with \( F \cap K = \emptyset \) imply there are SS-open sets \( H_T, V_T \) such that \( F = H(t) \) and \( K = V(t) \) for all \( t \in T \) with \( x \notin H_T \) and \( y \notin V_T \) with \( H_T \cap V_T = \emptyset \). This completes the proof.

Theorem 4.13. A supra topological space \((U, \emptyset)\) is sup-R\(_1\) if and only if \((U, \emptyset, T)\) is SSR\(_i\), \( i = 0, 1 \).

Proof. We will give the proof for \( i = 1 \). The proof for the case \( i = 0 \) is similar.

\[ \implies \] It is similar to that of the converse part in the above proposition.

\[ \Leftarrow \] Let \( x_1 \neq y_r \in SP(U) \) with \( cl_s(x_1) \neq cl_s(y_r) \), either \( x \notin cl_s(y_r) \) or \( y \notin cl_s(x_1) \) implies \( x \notin cl_s(y_r) \) or \( y \notin cl_s(x_1) \), then \( cl_s(x_1) \neq cl_s(y_r) \). Since \((U, \mu, T)\) is SSR\(_1\), there are \( F_T, G_T \) such that \( x_1 \notin F_T \) and \( y_r \notin G_T \), and \( F_T \cap G_T = \emptyset \). Thus, there are disjoint supra open sets \( A, B \) such that \( x \notin F(t) \) and \( y \notin G(t) \) for all \( t \in T \). Hence \((U, \emptyset)\) is sup-R\(_1\).

Theorem 4.14. If \((U, \mu, T)\) is SSTS\(_i\) then it is SSR\(_{i-1}\), \( i = 1, 2 \).

Proof. We will show the case \( i = 1 \). The proof for the case \( i = 2 \) is obvious.

Let \((U, \mu, T)\) be SSTS\(_1\) and \( F_T \) be an SS-open set contains \( x_1 \). We want to show that \( cl_s(x_1) \subseteq F_T \). So let \( y_r \notin F_T \), then \( x_1 \notin cl_s(y_r) \). Since \((U, \mu, T)\) is SSTS\(_1\), there is \( H_T \notin \emptyset \) such that \( y \notin H_T \) and \( x_1 \notin H_T \). Therefore \( cl_s(x_1) \subseteq F_T \). This completes the proof.

The converse of the above theorem may not be true. The next example shows it.

Example 4.15. Let \( U = \{x, y\}\) and \( T = \{t_1, t_2\} \). The family \( \mu = (\emptyset, \hat{U}, G_{1T}, G_{2T}) \), where, \( G_{1T} = \{t_1, U\} \) and \( G_{2T} = \{t_2, U\} \) is an SST on \( U \). One can verify \((U, \mu, T)\) is SSR\(_0\) and SSR\(_1\) but not SSTS\(_1\). Indeed, for two S-points \( x_1, y_{t_1} \), the SS-open sets which are containing \( x_1 \) are \( U \) and \( G_{1T} \) but also, they are containing \( y_{t_1} \). Thus \((U, \mu, T)\) is not SSTS\(_1\). Moreover, one can check that \((U, \mu, T)\) is not SSTS\(_2\).

Theorem 4.16. For SSTS \((U, \mu, T)\), the next items hold:

1. \((U, \mu, T)\) is SSTS\(_1\) \iff\((U, \mu, T)\) is both SSR\(_0\) and SSR\(_0\);
2. \((U, \mu, T)\) is SSTS\(_2\) \iff\((U, \mu, T)\) is both SSR\(_1\) and SSR\(_0\).

Proof. Necessity for two cases follows from Theorem 4.14 and Remark 2.13. The proof of converse part is obvious, for two cases.

Corollary 4.17. \((U, \mu, T)\) is SSTS\(_2\) \iff\((U, \mu, T)\) is both SSR\(_1\) and SSTS\(_1\).

Definition 4.18. A property is called an SS-topological property if the property is preserved by SS-homeomorphism.

Theorem 4.19. For a bijective SS-continuous map \( f_{UW} : (U, \mu, T) \rightarrow (V, \emptyset, E) \), if \((V, \emptyset, E)\) is SSR\(_i\), then \((U, \mu, T)\) is also SSR\(_i\), \( i = 0, 1 \).
Proof. We will prove only the case $i = 1$. The proof of the rest case is similar. To show that $(U, \mu, T)$ is SSR$_i$, let $x_t \neq y_{t'} \in SP(U)$. Since $f_{up}$ is one-one, there are two different S-points $a_e, b_e'$ in $V$ such that $f_{up}(x_t) = a_e$ and $f_{up}(y_{t'}) = b_e'$. Since $(V, \emptyset, E)$ is SSR$_i$, then there are two SS-open sets $G_{1E}, G_{2E} \in \emptyset$ such that $a_e \in G_{1E}$ and $b_e' \in G_{2E}$ and so, $x_t = f_{up}^{-1}(G_{1E})$ and $y_{t'} = f_{up}^{-1}(G_{2E})$. Since $f_{up}$ is SS-continuous, we have $f_{up}^{-1}(G_{1E}), f_{up}^{-1}(G_{2E})$ are SS-open sets in $(U, \mu, T)$ with $f_{up}^{-1}(G_{1E}) \cap f_{up}^{-1}(G_{2E}) = \emptyset$. This complete the proof. \hfill $\square$

**Theorem 4.20.** For a bijective SS-open map $f_{up} : (U, \mu, T) \to (V, \emptyset, E)$, if $(U, \mu, T)$ is SSR$_i$, then $(V, \emptyset, E)$ is also SSR$_i$, $i = 0, 1$.

**Proof.** We will prove only the case $i = 1$. The proof of the rest case is similar. To show that $(V, \emptyset, E)$ is SSR$_i$. Let $a_e \neq b_e' \in SP(V)$. Since $f_{up}$ is onto, there are two different S-points $x_t, y_{t'}$ in $U$ such that $f_{up}(x_t) = a_e$ and $f_{up}(y_{t'}) = b_e'$. By hypothesis, there are two disjoint SS-open sets $F_{1T}, F_{2T} \in \mu$ such that $x_t \in F_{1T}, y_{t'} \in F_{2T}$ and so, $a_e \in f_{up}(F_{1T})$ and $b_e' \in f_{up}(F_{2T})$. Since $f_{up}$ is SS-open, we have $f_{up}(F_{1T}), f_{up}(F_{2T})$ are SS-open sets in $(V, \emptyset, E)$ with $f_{up}(F_{1T}) \cap f_{up}(F_{2T}) = \emptyset$. The result holds. \hfill $\square$

From the two above theorems, we have the next theorem.

**Theorem 4.21.** Let $f_{up} : (U, \mu, T) \to (V, \emptyset, E)$ be an SS-homeomorphism map, then $(U, \mu, T)$ is SSR$_i$ if and only if $(V, \emptyset, E)$ is SSR$_i$, $i = 0, 1$.

**Corollary 4.22.** The supra soft properties SSR$_i$ are SS-topological property, for $i = 0, 1$.

**Corollary 4.23.** From Remarks 2.13, 2.15, and Corollary 4.17, the following implications hold and describe the relationships between SSR$_i$ and other soft separation properties.

\[
\begin{align*}
\text{SST}_2 & \implies \text{SST}_1 \implies \text{SST}_0 \\
\Downarrow & \quad \Downarrow \\
\text{SSR}_1 & \implies \text{SSR}_0 \\
\Uparrow & \quad \Uparrow \\
\text{SR}_1 & \implies \text{SR}_0
\end{align*}
\]

5. Conclusion

The theory of soft sets proves its efficiency as a fruitful mathematical approach to tackling the incompleteness and vagueness of knowledge, which is the utmost importance to cognitive analysis and artificial intelligence. This theory has been exploited to initiate the concept of soft topologies and some of their generalizations such as supra soft topologies and infra soft topologies \cite{8}.

In this work, we have defined and studied a new class of supra soft properties called supra soft R$_i$ and supra soft R$_i$ axioms in supra soft topological spaces, and have obtained basic properties of these axioms. We have investigated more characterizations and relationships between these classes and other soft separability properties. Several results and supported examples have been presented. Also, we have showed that the properties of begin SSR$_i$ and $i = 0, 1$ are supra soft topological properties.

In future works, we will continue study some generalizations of some topological notions via supra soft topological spaces such as supra soft $\delta$-topology. It is stated that the results obtained in this paper may be useful for further research on supra soft topology and its applications.

**Acknowledgment**

This study is supported via funding from Prince Sattam bin Abdulaziz University project number (PSAU/2023/R/1444).
References

[14] T. M. Al-shami, M. E. El-Shafei, Two types of separation axioms on supra soft separation spaces, Demonstr. Math., 52 (2019), 147–165. 1, 2.6, 2.9, 2.10, 2.11, 4.3