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Contact pseudo-slant submanifolds of a Kenmotsu manifold

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Abstract

We study the geometry of the contact pseudo-slant submanifolds of a Kenmotsu manifold. Necessary and sufficient conditions are given for a submanifold to be a pseudo-slant submanifold, contact pseudo-slant product, mixed geodesic and totally geodesic in Kenmotsu manifolds.

Keywords: Kenmotsu manifold, slant submanifold, contact pseudo-slant submanifold.
2010 MSC: 53C15, 53C25, 53C17, 53D15, 53D10.

1. Introduction

Kenmotsu [8] introduced a class of almost contact Riemannian manifolds known as Kenmotsu manifolds. In 1990, B. Y. Chen [5, 6] introduced the notion of slant submanifold, which is generalization of both the invariant and anti-invariant submanifolds. After that many research articles have been published by different authors on the existence of these submanifolds in different ambient spaces. The slant submanifolds of an almost contact metric manifolds were defined and studied by A. Lotta [10]. After, these submanifolds were studied by J. L. Cabrerizo et al. [4] of Sasakian manifolds.

The notion of semi-slant submanifolds of an almost Hermitian manifold was introduced by N. Papagiuc [11]. Cabrerizo et al. studied and characterized slant submanifolds of K- contact and Sasakian manifolds and gave several examples of such submanifolds. Cabrerizo et al. [4] defined and studied bi-slant immersions in almost contact metric manifolds and simultaneously gave the notion of pseudo-slant submanifolds. Pseudo-slant submanifolds also have been studied by Khan at al. in

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[9]. Later, U. C. De et al. [7] studied and characterized pseudo-slant submanifolds of trans Sasakian manifolds. Recently, in [1–3], Atçeken et al. studied slant and pseudo-slant submanifold in various manifolds.

In this paper, we study contact pseudo-slant submanifolds of a Kenmotsu manifold. In section 2, we review basic formulas and definitions for a Kenmotsu manifold and their submanifolds. In section 3, we study the geometry of the contact pseudo-slant submanifolds of a Kenmotsu manifold. Necessary and sufficient conditions are given for a submanifold to be a contact pseudo-slant submanifold, contact pseudo-slant product, mixed geodesic and totally geodesic in Kenmotsu manifolds.

2. Preliminaries

In this section, we give some terminology and notations used throughout this paper. We recall some necessary fact and formulas from the theory of Kenmotsu manifolds and their submanifolds.

Let $\widetilde{M}$ be a $(2m+1)$-dimensional almost contact metric manifold with structure (φ, ξ, η, g) where φ is a tensor field of type $(1,1)$, ξ a vector field, η is a 1-form, g is the Rieamanian metric on $\widetilde{M}$, which satisfy

$$\varphi^2 X = -X + \eta(X)\xi, \tag{2.1}$$

$$\varphi\xi = 0, \quad \eta \circ \varphi = 0, \quad \eta(\xi) = 1, \quad \eta(X) = g(X, \xi), \tag{2.2}$$

and

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \qquad g(\varphi X, Y) = -g(X, \varphi Y) \tag{2.3}$$

for any vector fields X, Y on $\widetilde{M}$. If in addition to above relations

$$(\widetilde{\nabla}_X \varphi)Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X, \tag{2.4}$$

then, $\widetilde{M}$ is called a Kenmotsu manifold, where $\widetilde{\nabla}$ is the Levi-Civita connection of g. We have also on a Kenmotsu manifold $\widetilde{M}$

$$\widetilde{\nabla}_X \xi = X - \eta(X)\xi \tag{2.5}$$

for any $X, Y \in \Gamma(T\widetilde{M})$.

Now, let M be a submanifold of a contact metric manifold $\widetilde{M}$ with the induced metric g. Also, let ∇ and $\nabla^\perp$ be the induced connections on the tangent bundle TM and the normal bundle $T^\perp M$ of M, respectively. Then the Gauss and Weingarten formulas are, respectively, given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \tag{2.6}$$

and

$$\widetilde{\nabla}_X V = -A_V X + \nabla^\perp_X V, \tag{2.7}$$

where σ and A_V are, respectively, the second fundamental form and the shape operator (corresponding to the normal vector field V) for the submanifold of M into $\widetilde{M}$. The second fundamental form σ and shape operator A_V are related by

$$g(A_V X, Y) = g(\sigma(X, Y), V) \tag{2.8}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$. If $\sigma(X, Y) = 0$, for each $X, Y \in \Gamma(TM)$ then M is said to be totally geodesic submanifold.

Now, let M be a submanifold of an almost contact metric manifold $\widetilde{M}$, then for any $X \in \Gamma(TM)$, we can write

$$\varphi X = PX + FX, \tag{2.9}$$

where PX and FX are the tangential and normal components of φX respectively.

Similarly for any $V \in \Gamma(T^{\perp}M)$, we can write

$$\varphi V = BV + CV, \tag{2.10}$$

where BV and CV are the tangential and normal components of φV, respectively.

Thus by using (2.1), (2.9) and (2.10), we obtain

$$P^2 = -I + \eta \otimes \xi - BF, \quad FP + CF = 0, \tag{2.11}$$

and

$$PB + BC = 0, \quad FB + C^2 = -I. \tag{2.12}$$

Furthermore, the covariant derivatives of the tensor field P, F, B and C are, respectively, defined by

$$(\nabla_X P)Y = \nabla_X PY - P\nabla_X Y, \tag{2.13}$$

$$(\nabla_X F)Y = \nabla_X^{\perp} FY - F\nabla_X Y, \tag{2.14}$$

$$(\nabla_X B)V = \nabla_X BV - B\nabla_X^{\perp} V, \tag{2.15}$$

and

$$(\nabla_X C)V = \nabla_X^{\perp} CV - C\nabla_X^{\perp} V \tag{2.16}$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.

Furthermore, for any $X, Y \in \Gamma(TM)$, we have $g(PX, Y) = -g(X, PY)$ and $V, U \in \Gamma(T^{\perp}M)$, we get $g(U, CV) = -g(CU, V)$. These show that P and C are also skew-symmetric tensor fields. Moreover, for any $X \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, we have

$$g(FX, V) = -g(X, BV), \tag{2.17}$$

which gives the relation between F and B.

A submanifold M is said to be *invariant* if F is identically zero, that is, $\varphi X \in \Gamma(TM)$ for all $X \in \Gamma(TM)$. On the other hand, M is said to be *anti-invariant* if P is identically zero, that is, $\varphi X \in \Gamma(T^{\perp}M)$ for all $X \in \Gamma(TM)$. By direct calculations, we obtain the following formulas;

$$(\nabla_X P)Y = A_{FY}X + B\sigma(X, Y) + g(PX, Y)\xi - \eta(Y)PX, \tag{2.18}$$

and

$$(\nabla_X F)Y = C\sigma(X, Y) - \sigma(X, PY) - \eta(Y)FX. \tag{2.19}$$

Similarly, for any $V \in \Gamma(T^{\perp}M)$ and $X \in \Gamma(TM)$, we obtain

$$(\nabla_X B)V = g(FX, V)\xi + A_{CV}X - PA_V X, \tag{2.20}$$

and

$$(\nabla_X C)V = -\sigma(BV, X) - FA_V X. \tag{2.21}$$

Since M is tangent to ξ, making use of (2.5), (2.6) and (2.8) we obtain

$$A_V\xi = \sigma(X,\xi) = 0 \tag{2.22}$$

for all $V \in \Gamma(T^{\perp}M)$ and $X \in \Gamma(TM)$.

In contact geometry, A. Lotta introduced slant submanifolds as follows:

A submanifold M of an almost contact metric manifold $\widetilde{M}$ is said to be slant submanifold if for any $x \in M$ and $X \in T_xM - \xi$, the angle between T_xM and φX is constant. The constant angle $[0, \frac{\pi}{2}]$ is then called slant angle of M. If $\theta = 0$, the submanifold is invariant submanifold, if $\theta = \frac{\pi}{2}$ then, it is anti-invariant submanifold, if $\theta \in (0, \frac{\pi}{2})$ then it is proper slant submanifold[10].

For slant submanifolds of contact manifolds J. L. Cabrerizo et al. proved the following Lemma.

Lemma 2.1 ([4])**.** *Let M be a slant submanifold of an almost contact metric manifold $\widetilde{M}$ such that $\xi \in \Gamma(TM)$. Then M is slant submanifold if and only if there exist a constant $\lambda \in [0,1]$ such that*

$$P^2 = \lambda(-I + \eta \otimes \xi), \tag{2.23}$$

moreover, if θ is the slant angle of M, then $\lambda = \cos^2\theta$.

Corollary 2.2 ([4])**.** *Let M be a slant submanifold of an almost contact metric manifold $\widetilde{M}$ with slant angle θ. Then for any $X, Y \in \Gamma(TM)$, we have*

$$g(PX, PY) = \cos^2\theta \left\{g(X,Y) - \eta(X)\eta(Y)\right\}, \tag{2.24}$$

and

$$g(FX, FY) = \sin^2\theta \left\{g(X,Y) - \eta(X)\eta(Y)\right\}. \tag{2.25}$$

3. Contact pseudo-slant submanifold of a Kenmotsu manifold

In this section, we study the geometry of the contact pseudo-slant submanifolds of a Kenmotsu manifold. Necessary and sufficient conditions are given for a submanifold to be a contact pseudo-slant submanifold, contact pseudo-slant product, mixed geodesic and totally geodesic in Kenmotsu manifolds.

Definition 3.1 ([9])**.** Let M be a submanifold of an almost contact metric manifold $\widetilde{M}$. M is said to be *contact pseudo-slant submanifold* of $\widetilde{M}$ if there exist two orthogonal distributions $D^{\perp}$ and D_θ on M such that:

(i) TM has the orthogonal direct decomposition $TM = D^{\perp} \oplus D_\theta$, $\xi \in \Gamma(D_\theta)$.

(ii) The distribution $D^{\perp}$ is an anti-invariant, i.e., $\varphi(D^{\perp}) \subset T^{\perp}M$.

(iii) The distribution D_θ is a slant, that is, the slant angle between of D_θ and $\varphi(D_\theta)$ is a constant.

If $\theta = 0$ then, the submanifold becomes a semi-invariant submanifold.

Let d_1 =dim$(D^{\perp})$ and d_2=dim(D_θ). We distinguish the following six cases:

(i) If $d_2 = 0$, then M is an anti-invariant submanifold.

(ii) If $d_1 = 0$ and $\theta = 0$, then M is invariant submanifold.

(iii) If $d_1 = 0$ and $\theta \in (0, \frac{\pi}{2})$, then M is a proper slant submanifold.

(iv) If $\theta = \frac{\pi}{2}$ then, M is an anti-invariant submanifold.

(v) If $d_2 d_1 \neq 0$ and $\theta = 0$, then M is a semi-invariant submanifold.

(vi) If $d_2 d_1 \neq 0$ and $\theta \in (0, \frac{\pi}{2})$, then M is a contact pseudo-slant submanifold.

If we denote the projections on $D^{\perp}$ and D_{θ} by ω_1 and ω_2, respectively, then for any vector field $X \in \Gamma(TM)$, we can write

$$X = \omega_1 X + \omega_2 X + \eta(X)\xi. \tag{3.1}$$

On the other hand, applying φ on both sides of equation (3.1), we have

$$\varphi X = \varphi\omega_1 X + \varphi\omega_2 X,$$

and

$$PX + FX = F\omega_1 X + P\omega_2 X + F\omega_2 X, \quad P\omega_1 X = 0.$$

From which, we can easily to see

$$PX = P\omega_2 X, \quad FX = F\omega_1 X + F\omega_2 X,$$

and

$$\varphi\omega_1 X = F\omega_1 X, \;\; P\omega_1 X = 0, \quad \varphi\omega_2 X = P\omega_2 X + F\omega_2 X, \quad P\omega_2 X \in \Gamma(D_{\theta}).$$

If we denote the orthogonal complementary of φTM in $T^{\perp}M$ by μ, then the normal bundle $T^{\perp}M$ can be decomposed as follows

$$T^{\perp}M = F(D^{\perp}) \oplus F(D_{\theta}) \oplus \mu. \tag{3.2}$$

We can easily see that the bundle μ is an invariant subbundle with respect to φ. Since $D^{\perp}$ and D_{θ} are orthogonal distribution on M, $g(\omega_1 Z, \omega_2 X) = 0$ for each $Z \in \Gamma(D^{\perp})$ and $X \in \Gamma(D_{\theta})$. Thus, by equation (2.3) and (2.9), we can write

$$g(F\omega_1 Z, F\omega_2 X) = g(\varphi\omega_1 Z, \varphi\omega_2 X) = g(\omega_1 Z, \omega_2 X) = 0,$$

that is, the distributions $F(D^{\perp})$ and $F(D_{\theta})$ are also mutually perpendicular. In fact, the decomposition (3.2) is an orthogonal direct decomposition.

Let M be a $(n+1)$-dimensional contact pseudo-slant submanifold of $(2m+1)$-dimensional Kenmotsu manifold $\widetilde{M}$ with d_2=dim$(D_{\theta}) = 2p+1$ and d_1 =dim$(D^{\perp}) = q$ and

$$\left\{ \begin{array}{l} e_1, e_2, ..., e_p, e_{p+1} = \sec\theta P e_1, e_{p+2} = \sec\theta P e_2, ..., \\ e_{2p} = \sec\theta P e_p, e_{2p+1} = \xi, e_{2p+2}, e_{2p+3}, ..., e_{2p+q+1} \end{array} \right\},$$

be an orthonormal basis of TM such that

$$\{e_1, e_2, ..., e_p, e_{p+1} = \sec\theta P e_1, e_{p+2} = \sec\theta P e_2, ..., e_{2p} = \sec\theta P e_p, e_{2p+1} = \xi\}$$

are tangent to $\Gamma(D_{\theta})$ and

$$\{e_{2p+2}, \;\; e_{2p+3}, ..., e_{2p+q+1}\}$$

are tangent to $\Gamma(D^{\perp})$.

Let M be a $(n+1)$-dimensional contact pseudo-slant submanifold of $(2m+1)$-dimensional Kenmotsu manifold $\widetilde{M}$ with $dim(FD_{\theta}) = 2p$, $dim(FD^{\perp}) = q$ and $dim(\mu) = 2k$,

$$\left\{\begin{array}{c} e_1^{'} = \cos ec\theta F e_1, e_2^{'} = \cos ec\theta F e_2, e_3^{'} = \cos ec\theta F e_3, ..., e_p^{'} = \\ \cos ec\theta F e_p, e_{p+1}^{'} = \sec\theta \cos ec\theta F P e_1, e_{p+2}^{'} = \sec\theta \cos ec\theta F P e_2, \\ ..., e_{2p}^{'} = \sec\theta \cos ec\theta F P e_p, e_{2p+1}^{'} = 0, e_{2p+2}^{'} = \cos ec\theta F e_{2p+2}, \\ e_{2p+3}^{'} = \cos ec\theta F e_{2p+3}, ..., e_{2p+q+1}^{'} = \cos ec\theta F e_{2p+q+1}, e_{2p+q+2}^{'} \\ , e_{2p+q+3}^{'}, ..., e_{2p+q+k+1}^{'}, \varphi e_{2p+q+k+2}^{'}, \varphi e_{2p+q+k+3}^{'}, ..., \varphi e_{2p+q+2k+1}^{'} \end{array}\right\},$$

be an orthonormal basis of $T^{\perp}M$ such that. Thus the orthonormal frames of the normal subbundles $F(D_\theta)$, $F(D^{\perp})$ and μ respectively are,

$$\left\{\begin{array}{c} e_1^{'} = \cos ec\theta F e_1, e_2^{'} = \cos ec\theta F e_2, e_3^{'} = \cos ec\theta F e_3, ..., e_p^{'} = \cos ec\theta F e_p, \\ e_{p+1}^{'} = \sec\theta \cos ec\theta F P e_1, e_{p+2}^{'} = \sec\theta \cos ec\theta F P e_2, ..., e_{2p}^{'} = \sec\theta \cos ec\theta F P e_p \end{array}\right\},$$

$$\left\{e_{2p+2}^{'} = \cos ec\theta F e_{2p+2}, ..., e_{2p+q+1}^{'} = \cos ec\theta F e_{2p+q+1}\right\}$$

and

$$\left\{e_{2p+q+2}^{'}, e_{2p+q+3}^{'}, ..., e_{2p+q+k+1}^{'}, \varphi e_{2p+q+k+2}^{'}, \varphi e_{2p+q+k+3}^{'}, ..., \varphi e_{2p+q+2k+1}^{'}\right\}.$$

Hence , we can easily see

$$\begin{aligned} g(e_i, e_j) &= 0, \quad for \quad 1 \leq i \leq 2p+1, \quad 2p+2 \leq j \leq 2p+q+1, \\ g(e_i, e_j^{'}) &= 0, \quad for \quad 1 \leq i \leq 2p+q+1, \quad 1 \leq j \leq 2p+q+k+1, \\ g(e_i^{'}, e_j^{'}) &= 0, \quad for \quad 1 \leq i \leq 2p, \quad e_{2p+1}^{'} = 0, \quad for \quad 2p+2 \leq j \leq 2p+q+1, \\ g(e_i^{'}, e_j^{'}) &= 0, \quad for \quad 1 \leq i \leq 2p, \quad 2p+q+2 \leq j \leq 2p+q+k+1, \\ g(e_i^{'}, e_j^{'}) &= 0, \quad for \quad 2p+2 \leq i \leq 2p+q+1, \quad 2p+q+2 \leq j \leq 2p+q+k+1, \\ g(e_i, \varphi e_j^{'}) &= 0, \quad for \quad 1 \leq i \leq 2p+1, \quad 2p+q+k+2 \leq j \leq 2p+q+2k+1 \end{aligned}$$

and

$$g(e_i^{'}, \varphi e_j^{'}) = 0, \quad for \quad 1 \leq i \leq 2p+q+1, \quad 2p+q+k+2 \leq j \leq 2p+q+2k+1.$$

Definition 3.2. A contact pseudo-slant submanifold M of Kenmotsu manifold $\widetilde{M}$ is said to be *D_θ-geodesic* (resp. *$D^{\perp}$-geodesic*) if $\sigma(X, Y) = 0$ for all $X, Y \in \Gamma(D_\theta)$ (resp. $\sigma(Z, W) = 0$ for all $Z, W \in \Gamma(D^{\perp})$. If for all $X \in \Gamma(D_\theta)$ and $Z \in \Gamma(D^{\perp})$, $\sigma(X, Z) = 0$, the M is called *mixed geodesic submanifold.*

Theorem 3.3. *Let M be a proper contact pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. If F is parallel, then either M is a mixed-geodesic or an anti-invariant submanifold.*

Proof. From (2.19), we obtain

$$C\sigma(X, Y) = 0$$

for any $X \in \Gamma(D_\theta)$ and $Y \in \Gamma(D^{\perp})$. Replacing X by Y in (2.19) and taking into account of F being parallel, we have

$$C\sigma(X, Y) = \sigma(Y, PX) - \eta(X)FY = 0.$$

Thus we have

$$\sigma(Y, PX) - \eta(X)FY = 0,$$

which is equivalent to

$$\sigma(Y, P^2X) = -\cos^2\theta\sigma(X, Y) = 0.$$

This proves our assertion. □

Theorem 3.4. *Let M be a proper contact pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. If B is parallel, then either M is a $D^{\perp}$ -geodesic or an anti-invariant submanifold of $\widetilde{M}$.*

Proof. If B is parallel, then making use of (2.20), we obtain

$$g(FZ, FY)\xi + A_{CFY}Z - PA_{FY}Z = 0$$

for any $Y, Z \in \Gamma(D^{\perp})$, which implies that

$$PA_{FY}Z = 0.$$

This tell us that M is either anti-invariant or $A_{FY}Z = 0$. So we obtain

$$g(\sigma(Z, W), FY) = 0$$

for any $W \in \Gamma(D^{\perp})$. Also by using (2.20), we conclude that

$$g(A_{CV}Z, Y) - g(PA_V Z, Y) = g(\sigma(Y, Z), CV) = 0$$

for any $V \in \Gamma(T^{\perp}M)$. This tells us that M is either $D^{\perp}$-geodesic or it is an anti-invariant submanifold. □

Definition 3.5. Given a proper contact pseudo-slant submanifold M of a Kenmotsu manifold $\widetilde{M}$, if the distributions D_θ and $D^{\perp}$ are totally geodesic in M, then M is said to be contact pseudo-slant product.

Theorem 3.6. *Let M be a contact pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. Then M is a contact pseudo-slant product if and only if the shape operator of M satisfies*

$$A_{FD^{\perp}}PD_\theta = A_{FPD_\theta}D^{\perp}. \tag{3.3}$$

Proof. By using (2.18), we have

$$\nabla_X PY - P\nabla_X Y = A_{FY}X + Bh(X, Y) + g(PX, Y)\xi - \eta(Y)PX$$

for any $X, Y \in \Gamma(D_\theta)$. This implies that

$$g(\nabla_X PY, Z) = g(A_{FY}X, Z) + g(Bh(X, Y), Z) \tag{3.4}$$

for any $Z \in \Gamma(D^{\perp})$. Replacing Y by PY in (3.4) and taking into account of (2.24), we obtain

$$\cos^2\theta g(\nabla_X Y, Z) = g(A_{FZ}PY - A_{FPY}Z, X). \tag{3.5}$$

Also, from (2.13), we have

$$-P\nabla_Z U = A_{FU}Z + Bh(Z, U)$$

for any $U, Z \in \Gamma(D^{\perp})$, from which

$$-g(P\nabla_Z U, PX) = g(A_{FU}Z, PX) + g(Bh(Z, U), PX),$$

that is,

$$-\cos^2\theta g(\nabla_Z U, X) = g(A_{FU}PX - A_{FPX}U, Z) \tag{3.6}$$

for any $X \in \Gamma(D_\theta)$. Equations (3.5) and (3.6) imply that (3.3). □

Example 3.7. Let M be a submanifold of $\mathbb{R}^7$ defined by equation

$$M = x(u,v,s,t,z) = (u, v, v\sin\theta, v\cos\theta, s\cos t, -s\cos t, z).$$

We can check that the tangent bundle of M is spanned by the tangent vectors

$$\begin{aligned}
e_1 &= \frac{\partial}{\partial x_1}, \quad e_5 = \xi = \frac{\partial}{\partial z},\\
e_2 &= \frac{\partial}{\partial y_1} + \sin\theta\frac{\partial}{\partial x_2} + \cos\theta\frac{\partial}{\partial y_2},\\
e_3 &= \cos t\frac{\partial}{\partial x_3} - \cos t\frac{\partial}{\partial y_3},\\
e_4 &= -s\sin t\frac{\partial}{\partial x_3} + s\sin t\frac{\partial}{\partial y_3}.
\end{aligned}$$

For the almost contact metric structure φ of $\mathbb{R}^7$, whose coordinate systems

$$(x_1, y_1, x_2, y_2, x_3, y_3, z),$$

choosing

$$\varphi(\frac{\partial}{\partial x_i}) = \frac{\partial}{\partial y_i}, \varphi(\frac{\partial}{\partial y_j}) = -\frac{\partial}{\partial x_j}, \quad 1 \leq i, j \leq 3$$

then we have

$$\begin{aligned}
\varphi e_1 &= \frac{\partial}{\partial y_1}, \quad \varphi e_5 = 0,\\
\varphi e_2 &= -\frac{\partial}{\partial x_1} + \sin\theta\frac{\partial}{\partial y_2} - \cos\theta\frac{\partial}{\partial x_2},\\
\varphi e_3 &= \cos t\frac{\partial}{\partial y_3} + \cos t\frac{\partial}{\partial x_3},\\
\varphi e_4 &= -s\sin t\frac{\partial}{\partial y_3} - s\sin t\frac{\partial}{\partial x_3}.
\end{aligned}$$

By direct calculations, we can infer $D_\theta = S_p\{e_1, e_2\}$ is a slant distribution with slant angle $\cos\alpha = \frac{g(e_2,\varphi e_1)}{\|e_2\|\|\varphi e_1\|} = \frac{\sqrt{2}}{2}$, $\alpha = \frac{\pi}{4}$. Since $g(\varphi e_3, e_i) = 0$, $i = 1,2,4,5$ and $g(\varphi e_4, e_j) = 0$, $j = 1,2,3,5$ orthogonal to M, $D^\perp = S_p\{e_3, e_4\}$ is an anti-invariant distribution. That is, M is a 5-dimensional proper pseudo-slant submanifold of $\mathbb{R}^7$ with its usual almost contact metric structure.

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