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Optical solitons for conformable space-time fractional nonlinear model

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Abstract

In search of the exact solutions of nonlinear partial differential equations in solitons form has become most popular to understand the internal features of physical phenomena. In this paper, we discovered various type of solitons solutions for the conformable space-time nonlinear Schrödinger equation (CSTNLSE) with Kerr law nonlinearity. To seek such solutions, we applied two proposed methods which are Sardar-subequation method and new extended hyperbolic function method. In this way several types of solitons obtained for example bright, dark, periodic singular, combined dark-bright, singular, and combined singular solitons. Some of the acquired solutions are interpreted graphically. These solutions are specific, novel, correct and may be beneficial for edifying precise nonlinear physical phenomena in nonlinear dynamical schemes. It is revealed that the proposed methods offer a straightforward and mathematical tool for solving nonlinear conformable space-time nonlinear Schrödinger equation. These results support in attaining nonlinear optical fibers in the future.

Keywords: Sardar-subequation method, conformable space-time nonlinear Schrödinger equation, the new extended hyperbolic function method, optical solitons.

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1. Introduction

Study of optical solitons have definitely increased momentum in the arena of the solitary waves. Analysis of numerous solutions have been made to the nonlinear Schrödinger equations with low group velocity dispersion, Kerr nonlinearities, dispersion terms, self-steepening, spatiotemporal dispersion, etc. Usually, these solutions are qualifying chirped free and chirped solitons, combo solitons, and dark combo soliton [4, 9, 10, 31, 33]. Although these results have many applications but communication by optic fibers is one of them. Also, solitons have revolutionized the communication system through the wave guides more recently. It is clear that the solitons establish the pillar of data transfer and communication at unbelievable distances. However, all the power of the optical system lies on famous effects, which at the same time establish conditions restrictions. Maximum of the time, pulse propagation in optical fibers

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can be concerned by nonlinearity, group velocity dispersion (GVD), and polarization type dispersion. In recent times, fractional calculus looks in many regions of engineering and science such as rheology, biology, control theory, physics, electrochemistry, systems identification, signal processing, viscoelasticity and so on. In order to define nonlinear (NL) physical phenomena the acquiring the analytical solutions for fractional differential equation (FDE) is one of the significant features. This physical phenomenon may rest on both the time instant and the time history, which can be effectively demonstrated by using the concept of derivatives and integrals of fractional order. Recently, numerous approaches have been utilized to seek exact solutions of FPDEs in the literature (Zhou et al. [54]; Eslami et al. [18]; Mirzazadeh et al. [40]; Vajargah et al. [51]; Sonomezoglu et al. [47, 48]; Zerrad et al. [52, 53]; Biswas and Suarez [49]; Gazizov et al. [21]; Biswas et al. [12]; Lukashchuk [38]; Hashemi [22]; Hosseini and Ansari [25]; Baleanu et al. [6–8]; Hosseini et al. [26]; Inc et al. [34]; Akgül et al. [3]; Baleanu et al. [5]; Rehman et al. [43]; Inc et al. [32]; Tchier et al.[50]; Hashemi et al. [24]). It is famous that some of the belongings of the fractional derivatives are very tough in contrast with the typical ones, so there is a massive inspiration to crack in discovering the solutions of some general equations similar to the fractional nonlinear Schrodinger equation (NLSE). The NLSE is an essential model which defines many type of phenomena, such as optics of nonlinear media, condensed matter physics, plasmas and photonics, (Eslami et al. [15, 16]; Eslami and Rezazadeh [20]; Khodadad et al. [36]; Eslami [14, 17]; Ekici et al. [13]; Biswas et al. [11]; Mirzazadeha et al. [39]; Eslami and Neirameh [19]; Neirameh and Eslami [41]). Additionally, NLSE explains the dynamics behavior of solitons using optical fibers. The dimensionless NLSE with STD is given by

$$\iota u_t + a u_{tx} + b u_{xx} + cF(|u|^2)u = 0, \tag{1.1}$$

where a, b, and c denote the coefficients of STD, GVD, and NL term, respectively and

$$F(|u|^2)u \in \bigcup_{r,s=1}^{\infty} c^k\left((-s,s)\times(-r,r);R^2\right). \tag{1.2}$$

The layouts of this paper are as follows. The governing equation is discussed in Section 2. Section 3 presents the analysis of the proposed methods SSM and new EHFM. In Section 4 SSM and the new EHFM are applied and Section 5 consists the results and discussion. In Section 6 conclusion of this paper is given.

2. Governing equation

In this section, we examine and analyze (1.1) given by Hashemi and Akgul [23] and

$$\iota\frac{\partial^{\alpha}u}{\partial t^{\alpha}} + a\frac{\partial^{\alpha+\beta}u}{\partial t^{\alpha}\partial x^{\beta}} + b\frac{\partial^{2\beta}u}{\partial x^{2\beta}} + cF(|u|^2)u = 0. \tag{2.1}$$

We apply two methods which are Sardar-subequation method (SSM) (Rezazadeh et al. [44]) and new extended hyperbolic function method (EHFM) (Shang [45, 46]; Nestor [42]) to discover optical solitons solutions for the CSTNLSE with Kerr law nonlinearity. It is appropriate that many models in engineering and science have an experiential parameters. Therefore, exact solutions provide authority to scientists to plan and perform experiments, by launching suitable or natural situations, to decide these parameters. Thus, exploration and finding solitons solutions is much effective in nonlinear sciences. The freshly familiarized definition of fractional derivative called conformable derivative (Hosseini et al. [27–30]; Korkmaz and Hosseini [37]; Khalil et al. [35]; Hammad and Khalil [2]; Abdeljawad [1]) is applied to transform fractional equations into ODEs, therefore discovering optical solitons solutions via two unlike analytical techniques.

3. Analysis of the methods

In this section, we have analyzed two methods which are applied to construct novel solitons solutions of the given model.

3.1. *Sardar-subequation method*

Let the FPDE

$$H(u, u_t^{\alpha}, u_x^{\beta}, u_x^{2\beta}, \dots) = 0, \tag{3.1}$$

to explore the travelling wave solutions, using following wave transformations in (1.1)

$$u(x,t) = u(\eta), \quad \eta = \frac{x^{\beta}}{\beta} + \lambda \frac{t^{\alpha}}{\alpha}, \tag{3.2}$$

Using (3.2) into (3.1), we obtain the following ODE

$$P(u, u', u'', \dots) = 0. \tag{3.3}$$

Consider (3.3) has a solution as follows

$$u(\eta) = \sum_{i=0}^{N} F_i \Phi^i(\eta),$$

where F_i $(0 \leqslant i \leqslant N)$ are constants and $\Phi(\eta)$ admits the ODE as given below

$$(\Phi'(\eta))^2 = \epsilon + \delta \Phi^2(\eta) + \Phi^4(\eta), \tag{3.4}$$

where ϵ and δ are constants. (3.4) gives the solution as follows.

Case 1: When $\delta > 0$ and $\epsilon = 0$, then

$$\Phi_1^{\pm}(\eta) = \pm\sqrt{-\delta pq}\ \mathrm{sech}_{pq}(\sqrt{\delta}\eta), \quad \Phi_2^{\pm}(\eta) = \pm\sqrt{\delta pq}\ \mathrm{csch}_{pq}(\sqrt{\delta}\eta),$$

where $\mathrm{sech}_{pq}(\eta) = \frac{2}{pe^{\eta}+qe^{-\eta}}$, $\mathrm{csch}_{pq}(\eta) = \frac{2}{pe^{\eta}-qe^{-\eta}}$.

Case 2: When $\delta < 0$ and $\epsilon = 0$, then

$$\Phi_3^{\pm}(\eta) = \pm\sqrt{-\delta pq}\ \sec_{pq}(\sqrt{-\delta}\eta), \quad \Phi_4^{\pm}(\eta) = \pm\sqrt{-\delta pq}\ \csc_{pq}(\sqrt{-\delta}\eta),$$

where $\sec_{pq}(\eta) = \frac{2}{pe^{\iota\eta}+qe^{-\iota\eta}}$, $\csc_{pq}(\eta) = \frac{2\iota}{pe^{\iota\eta}-qe^{-\iota\eta}}$.

Case 3: When $\delta < 0$ and $\epsilon = \frac{\delta^2}{4b}$, then

$$\Phi_5^{\pm}(\eta) = \pm\sqrt{-\frac{\delta}{2}}\tanh_{pq}(\sqrt{-\frac{\delta}{2}}\,\eta),$$
$$\Phi_6^{\pm}(\eta) = \pm\sqrt{-\frac{\delta}{2}}\coth_{pq}(\sqrt{-\frac{\delta}{2}}\,\eta),$$
$$\Phi_7^{\pm}(\eta) = \pm\sqrt{-\frac{\delta}{2}}\Big(\tanh_{pq}(\sqrt{-2\delta}\,\eta) \pm \iota\sqrt{pq}\ \mathrm{sech}_{pq}(\sqrt{-2\delta}\,\eta)\Big),$$
$$\Phi_8^{\pm}(\eta) = \pm\sqrt{-\frac{\delta}{2}}\Big(\coth_{pq}(\sqrt{-2\delta}\,\eta) \pm \sqrt{pq}\ \mathrm{csch}_{pq}(\sqrt{-2\delta}\,\eta)\Big),$$
$$\Phi_9^{\pm}(\eta) = \pm\sqrt{-\frac{\delta}{8}}\Big(\tanh_{pq}(\sqrt{-\frac{\delta}{8}}\,\eta) + \coth_{pq}(\sqrt{-\frac{\delta}{8}}\,\eta)\Big),$$

where $\tanh_{pq}(\eta) = \frac{pe^{\eta}-qe^{-\eta}}{pe^{\eta}+qe^{-\eta}}$, $\coth_{pq}(\eta) = \frac{pe^{\eta}+qe^{-\eta}}{pe^{\eta}-qe^{-\eta}}$.

Case 4: When $\delta > 0$ and $\epsilon = \frac{\delta^2}{4}$, then

$$\Phi_{10}^{\pm}(\eta) = \pm\sqrt{\frac{\delta}{2}}\tan_{pq}(\sqrt{\frac{\delta}{2}}\,\eta),$$

$$\Phi^{\pm}_{11}(\eta) = \pm\sqrt{\frac{\delta}{2}}\cot_{pq}(\sqrt{\frac{\delta}{2}}\,\eta),$$
$$\Phi^{\pm}_{12}(\eta) = \pm\sqrt{\frac{\delta}{2}}\Big(\tan_{pq}(\sqrt{2\delta}\,\eta) \pm \sqrt{pq}\,\sec_{pq}(\sqrt{2\delta}\,\eta)\Big),$$
$$\Phi^{\pm}_{13}(\eta) = \pm\sqrt{\frac{\delta}{2}}\Big(\cot_{pq}(\sqrt{2\delta}\,\eta) \pm \sqrt{pq}\,\csc_{pq}(\sqrt{2\delta}\,\eta)\Big),$$
$$\Phi^{\pm}_{14}(\eta) = \pm\sqrt{\frac{\delta}{8}}\Big(\tan_{pq}(\sqrt{\frac{\delta}{8}}\,\eta) + \cot_{pq}(\sqrt{\frac{\delta}{8}}\,\eta)\Big),$$

where $\tan_{pq}(\eta) = -\iota\frac{pe^{\iota\eta}-qe^{-\iota\eta}}{pe^{\iota\eta}+qe^{-\iota\eta}}$, $\cot_{pq}(\eta) = \iota\frac{pe^{\iota\eta}+qe^{-\iota\eta}}{pe^{\iota\eta}-qe^{-\iota\eta}}$.

3.2. *New extended hyperbolic function method*

The phases of the new EHFM are taken as follows.

Form 1: Let PDE as given in (3.1) with the wave transformation in (3.2) using wave transformation ODE is obtained as in (3.3). We assume that (3.3) has a solution in the next form

$$u(\eta) = \sum_{i=0}^{N} F_i\Phi^i(\eta), \tag{3.5}$$

where the coefficients $F_i(i = (1, 2, 3, \ldots, N))$ are constants and $\Phi(\eta)$ admits the ODE in next form, as

$$\frac{d\Phi}{d\eta} = \Phi\sqrt{\Lambda + \Theta\Phi^2},\ \Lambda, \Theta \in R. \tag{3.6}$$

By using balancing rule in (3.3) the value of N is found. Replacing (3.3) into (3.5) with (3.6), gives a set of equations for $F_i(i = (0, 1, 2, 3, \ldots, N))$. On solving this set, we yield set of solutions that admits (3.6), as follows.

Set 1: When $\Lambda > 0$ and $\Theta > 0$,

$$\Phi(\eta) = -\sqrt{\frac{\Lambda}{\Theta}}\operatorname{csch}(\sqrt{\Lambda}(\eta + \eta_0)).$$

Set 2: When $\Lambda < 0$ and $\Theta > 0$,

$$\Phi(\eta) = \sqrt{\frac{-\Lambda}{\Theta}}\sec(\sqrt{-\Lambda}(\eta + \eta_0)).$$

Set 3: When $\Lambda > 0$ and $\Theta < 0$,

$$\Phi(\eta) = \sqrt{\frac{\Lambda}{-\Theta}}\operatorname{sech}(\sqrt{\Lambda}(\eta + \eta_0)).$$

Set 4: When $\Lambda < 0$ and $\Theta > 0$,

$$\Phi(\eta) = \sqrt{\frac{-\Lambda}{\Theta}}\csc(\sqrt{-\Lambda}(\eta + \eta_0)).$$

Set 5: When $\Lambda > 0$ and $\Theta = 0$,

$$\Phi(\eta) = \exp(\sqrt{\Lambda}(\eta + \eta_0)).$$

Set 6: When $\Lambda < 0$ and $\Theta = 0$,

$$\Phi(\eta) = \cos(\sqrt{-\Lambda}(\eta + \eta_0)) + \iota\sin(\sqrt{-\Lambda}(\eta + \eta_0)).$$

Set 7: When $\Lambda = 0$ and $\Theta > 0$,

$$\Phi(\eta) = \pm \frac{1}{(\sqrt{\Theta}(\eta + \eta_0))}.$$

Set 8: When $\Lambda = 0$ and $\Theta < 0$,

$$\Phi(\eta) = \pm \frac{\iota}{(\sqrt{-\Theta}(\eta + \eta_0))}.$$

Form 2: Using the same pattern as previous, we adopt that $\Phi(\eta)$ admits the ODE as follows

$$\frac{d\Phi}{d\eta} = \Lambda + \Theta\Phi^2, \ \Lambda, \Theta \in R. \tag{3.7}$$

Substituting (3.5) into (3.3) along with (3.7) with value of N, makes a set of (equations with the values of $F_i (i = 1, 2, 3, \dots, N)$. Let the (3.7) accepts the solutions as follows

Set 1: When $\Lambda\Theta > 0$,

$$\Phi(\eta) = \text{sgn}(\Lambda)\sqrt{\frac{\Lambda}{\Theta}} \tan(\sqrt{\Lambda\Theta}(\eta + \eta_0)).$$

Set 2: When $\Lambda\Theta > 0$,

$$\Phi(\eta) = -\text{sgn}(\Lambda)\sqrt{\frac{\Lambda}{\Theta}} \cot(\sqrt{\Lambda\Theta}(\eta + \eta_0)).$$

Set 3: When $\Lambda\Theta < 0$,

$$\Phi(\eta) = \text{sgn}(\Lambda)\sqrt{\frac{\Lambda}{-\Theta}} \tanh(\sqrt{-\Lambda\Theta}(\eta + \eta_0)).$$

Set 4: When $\Lambda\Theta < 0$,

$$\Phi(\eta) = \text{sgn}(\Lambda)\sqrt{\frac{\Lambda}{-\Theta}} \coth(\sqrt{-\Lambda\Theta}(\eta + \eta_0)).$$

Set 5: When $\Lambda = 0$ and $\Theta > 0$,

$$\Phi(\eta) = -\frac{1}{\Theta(\eta + \eta_0)}.$$

Set 6: When $\Lambda \in R$ and $\Theta = 0$,

$$\Phi(\eta) = \Lambda(\eta + \eta_0).$$

Note: sgn is the famous sign function.

4. Application

In this section, we employ the under consideration methods to obtain optical solitons for the given model, for Kerr law nonlinearity, we have that $F(u) = u$. Suppose the next wave transformation

$$u(x, t) = u(\eta)e^{\iota\left(k\frac{x^\beta}{\beta} + w\frac{t^\alpha}{\alpha}\right)}, \quad \eta = \frac{x^\beta}{\beta} - V\frac{t^\alpha}{\alpha}, \tag{4.1}$$

substituting (4.1) into (2.1) and splitting into imaginary and real parts,

$$V = \frac{aw - 2bk}{1 - ak},$$

and

$$k^2(b - aV)u''(\eta) - (awk - w + bk^2)u(\eta) + cu^3 = 0. \tag{4.2}$$

4.1. Application of the SSM

Here, we employ the SSM for the solutions of NLSE. Using balancing pricipal on terms of u'' and u^3 in (4.2), we get $N = 1$, so (3.1) converts to

$$u(\eta) = F_0 + F_1\Phi(\eta), \tag{4.3}$$

where F_0 and F_1 are constants. Substituting (4.3) into (4.2) and comparing the coefficients of polynomials of $\Phi(\eta)$ to zero, we make a set of equations in F_0, F_1, c, and a. On resolving the set of equations, we get

$$F_0 = 0,\quad F_1 = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}, \qquad c \neq 0,\quad a = \frac{-bk^2 + \delta bk^2 + w}{k(w + kV\delta)}.$$

Case 1: When $\delta > 0$ and $\epsilon = 0$, then

$$z^{\pm}_{1,1}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}\Big(\pm\sqrt{-pq\delta}\ \mathrm{sech}_{pq}(\sqrt{\delta}\ (\eta))\Big)e^{\iota\left(k\frac{x^\beta}{\beta}+w\frac{t^\alpha}{\alpha}\right)}, \tag{4.4}$$

$$z^{\pm}_{1,2}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}\Big(\pm\sqrt{pq\delta}\ \mathrm{csch}_{pq}(\sqrt{\delta}\ (\eta))\Big)e^{\iota\left(k\frac{x^\beta}{\beta}+w\frac{t^\alpha}{\alpha}\right)}.$$

Case 2: When $\delta < 0$ and $\epsilon = 0$, then

$$z^{\pm}_{1,3}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}\Big(\pm\sqrt{-pq\delta}\ \sec_{pq}(\sqrt{-\delta}\ (\eta))\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)},$$

$$z^{\pm}_{1,4}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}\Big(\pm\sqrt{-pq\delta}\ \csc_{pq}(\sqrt{-\delta}\ (\eta))\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}.$$

Case 3: When $\delta < 0$ and $\epsilon = \frac{\delta^2}{4b}$, then

$$z^{\pm}_{1,5}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}\Big(\pm\sqrt{-\frac{\delta}{2}}\tanh_{pq}(\sqrt{-\frac{\delta}{2}}\ (\eta)\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}, \tag{4.5}$$

$$z^{\pm}_{1,6}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}\Big(\pm\sqrt{-\frac{\delta}{2}}\coth_{pq}(\sqrt{-\frac{\delta}{2}}\ (\eta)\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)},$$

$$\begin{aligned} z^{\pm}_{1,7}(x,t) = {}& \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}\Big(\pm\sqrt{-\frac{\delta}{2}}\Big(\tanh_{pq}(\sqrt{-2\delta}\ (\eta)) \\ & \pm\iota\sqrt{pq}\ \mathrm{sech}_{pq}(\sqrt{-2\delta}\ (\eta))\Big)\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}, \end{aligned} \tag{4.6}$$

$$\begin{aligned} z^{\pm}_{1,8}(x,t) = {}& \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}\Big(\pm\sqrt{-\frac{\delta}{2}}\Big(\coth_{pq}(\sqrt{-2\delta}\ (\eta)) \\ & \pm\sqrt{pq}\ \mathrm{csch}_{pq}(\sqrt{-2\delta}\ (\eta))\Big)\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}, \end{aligned} \tag{4.7}$$

$$\begin{aligned} z^{\pm}_{1,9}(x,t) = {}& \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}}\Big(\pm\sqrt{-\frac{\delta}{8}}\Big(\tanh_{pq}(\sqrt{-\frac{\delta}{8}}\ (\eta)) \\ & +\coth_{pq}(\sqrt{-\frac{\delta}{8}}\ (\eta))\Big)\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}. \end{aligned} \tag{4.8}$$

Case 4: When $\delta > 0$ and $\epsilon = \frac{\delta^2}{4}$, then

$$z^{\pm}_{1,10}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}} \left(\pm\sqrt{\frac{\delta}{2}}\tan_{pq}(\sqrt{\frac{\delta}{2}}\,(\eta))\right) e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)},$$

$$z^{\pm}_{1,11}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}} \left(\pm\sqrt{\frac{\delta}{2}}\cot_{pq}(\sqrt{\frac{\delta}{2}}\,(\eta))\right) e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)},$$

$$z^{\pm}_{1,12}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}} \left(\pm\sqrt{\frac{\delta}{2}}\left(\tan_{pq}(\sqrt{2\delta}\,(\eta)) \pm \sqrt{pq}\,\sec_{pq}(\sqrt{2\delta}\,(\eta))\right)\right) e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)},$$

$$z^{\pm}_{1,13}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}} \left(\pm\sqrt{\frac{\delta}{2}}\left(\cot_{pq}(\sqrt{2\delta}\,(\eta)) \pm \sqrt{pq}\,\csc_{pq}(\sqrt{2\delta}\,(\eta))\right)\right) e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)},$$

$$z^{\pm}_{1,14}(x,t) = \frac{k\sqrt{-2b + \frac{2V(-bk^2+w+bk^2\delta)}{k(w+kV\delta)}}}{\sqrt{c}} \left(\pm\sqrt{\frac{\delta}{8}}\left(\tan_{pq}(\sqrt{\frac{\delta}{8}}\,(\eta)) + \cot_{pq}(\sqrt{\frac{\delta}{8}}\,(\eta))\right)\right) e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}. \tag{4.9}$$

4.2. Application of the new EHFM

Form 1: Here, we employ the new EHFM for the solutions NLSE. Using balancing method in (4.2), we get $N = 1$, so (3.5) gives

$$u(\eta) = F_0 + F_1\Phi(\eta), \tag{4.10}$$

where F_0 and F_1 are constants. Replacing (3.7) into (4.2) and equating the coefficients polynomials of $\Phi(\eta)$ to zero, we get a set of equations in F_0, F_1, Λ and Θ. On resolving the set of equations, we attain

$$F_0 = 0, \quad F_1 = \frac{\sqrt{2}\sqrt{-b\Theta k^2 + a\Theta k^2 V}}{\sqrt{c}}, \quad c \neq 0,$$
$$V \neq 0, \quad \Lambda = \frac{bk^2 - w + akw}{\Theta k^2(b - aV)}, \quad \Theta = \Theta. \tag{4.11}$$

Set 1: When $\Lambda > 0$ and $\Theta > 0$,

$$z_1(x,t) = \frac{\sqrt{2}\sqrt{-b\Theta k^2 + a\Theta k^2 V}}{\sqrt{c}}\left(-\sqrt{\frac{bk^2 - w + akw}{\Theta^2 k^2(b - aV)}}\operatorname{csch}(\sqrt{\frac{bk^2 - w + akw}{\Theta k^2(b - aV)}}(\eta + \eta_0))\right)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}. \tag{4.12}$$

Set 2: When $\Lambda < 0$ and $\Theta > 0$,

$$z_2(x,t) = \frac{\sqrt{2}\sqrt{-b\Theta k^2 + a\Theta k^2 V}}{\sqrt{c}}\left(\sqrt{-\frac{bk^2 - w + akw}{\Theta^2 k^2(b - aV)}}\sec(\sqrt{-\frac{bk^2 - w + akw}{\Theta k^2(b - aV)}}(\eta + \eta_0))\right)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}.$$

Set 3: When $\Lambda > 0$ and $\Theta < 0$,

$$z_3(x,t) = \frac{\sqrt{2}\sqrt{-b\Theta k^2 + a\Theta k^2 V}}{\sqrt{c}}\left(\sqrt{-\frac{bk^2 - w + akw}{\Theta^2 k^2(b - aV)}}\operatorname{sech}(\sqrt{-\frac{bk^2 - w + akw}{\Theta k^2(b - aV)}}(\eta + \eta_0))\right)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}. \tag{4.13}$$

Set 4: When $\Lambda < 0$ and $\Theta < 0$,

$$z_4(x,t) = \frac{\sqrt{2}\sqrt{-b\Theta k^2 + a\Theta k^2 V}}{\sqrt{c}}\left(\sqrt{-\frac{bk^2 - w + akw}{\Theta^2 k^2(b - aV)}}\csc(\sqrt{-\frac{bk^2 - w + akw}{\Theta k^2(b - aV)}}(\eta + \eta_0))\right)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}.$$

Set 5: When $\Lambda = 0$ and $\Theta > 0$,

$$z_7(x,t) = \frac{\sqrt{2}\sqrt{-b\Theta k^2 + a\Theta k^2 V}}{\sqrt{c}}\Big(\pm\frac{1}{(\sqrt{\Theta}(\eta+\eta_0)}\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}.$$

Set 6: When $\Lambda = 0$ and $\Theta < 0$,

$$z_8(x,t) = \frac{\sqrt{2}\sqrt{-b\Theta k^2 + a\Theta k^2 V}}{\sqrt{c}}\Big(\pm\frac{\iota}{(\sqrt{-\Theta}(\eta+\eta_0)}\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)},$$

where $\eta = \frac{x^\beta}{\beta} - V\frac{t^\alpha}{\alpha}$.

Form 2: Operating balancing rule in (4.2), gives $N = 1$, so (3.5) reduces to

$$u(\eta) = F_0 + F_1\Phi(\eta), \tag{4.14}$$

where F_0 and F_1 are constants. Replacing (4.14) into (4.2) and equating the coefficients of polynomials of $\Phi(\eta)$ to zero, we get a set of equations in F_0, F_1, Λ, and Θ. On resolving the set of equations, we attain

$$\begin{aligned} &F_0 = 0, \quad F_1 = \frac{\Theta k\sqrt{-2b+2aV}}{\sqrt{c}}, \quad c \neq 0, \\ &V \neq 0, \quad \Lambda = \frac{bk^2 - w + akw}{2\Theta k^2(b-aV)}, \quad \Theta = \Theta. \end{aligned} \tag{4.15}$$

Set 1: When $\Lambda\Theta > 0$,

$$z_9(x,t) = \frac{\Theta k\sqrt{-2b+2aV}}{\sqrt{c}}\left(\chi\sqrt{\frac{bk^2-w+akw}{2\Theta^2k^2(b-aV)}}\tan\Big(\sqrt{\frac{bk^2-w+akw}{2k^2(b-aV)}}(\eta+\eta_0)\Big)\right)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}. \tag{4.16}$$

Set 2: When $\Lambda\Theta > 0$,

$$z_{10}(x,t) = \frac{\Theta k\sqrt{-2b+2aV}}{\sqrt{c}}\left(-\chi\sqrt{\frac{bk^2-w+akw}{2\Theta^2k^2(b-aV)}}\cot\Big(\sqrt{\frac{bk^2-w+akw}{2k^2(b-aV)}}(\eta+\eta_0)\Big)\right)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}. \tag{4.17}$$

Set 3: When $\Lambda\Theta < 0$,

$$z_{11}(x,t) = \frac{\Theta k\sqrt{-2b+2aV}}{\sqrt{c}}\left(\chi\sqrt{-\frac{bk^2-w+akw}{2\Theta^2k^2(b-aV)}}\tanh\Big(\sqrt{-\frac{bk^2-w+akw}{2k^2(b-aV)}}(\eta+\eta_0)\Big)\right)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}.$$

Set 4: When $\Lambda\Theta < 0$,

$$z_{12}(x,t) = \frac{\Theta k\sqrt{-2b+2aV}}{\sqrt{c}}\left(\chi\sqrt{-\frac{bk^2-w+akw}{2\Theta^2k^2(b-aV)}}\coth\Big(\sqrt{-\frac{bk^2-w+akw}{2k^2(b-aV)}}(\eta+\eta_0)\Big)\right)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}.$$

Set 5: When $\Lambda = 0$ and $\Theta > 0$,

$$z_{13}(x,t) = \frac{\Theta k\sqrt{-2b+2aV}}{\sqrt{c}}\Big(-\frac{1}{\Theta(\eta+\eta_0)}\Big)e^{\iota\left(\frac{x^\beta}{\beta}+\lambda\frac{t^\alpha}{\alpha}\right)}.$$

where $\chi = \text{sgn}(\frac{bk^2-w+akw}{2\Theta k^2(b-aV)})$, $\eta = \frac{x^\beta}{\beta} - V\frac{t^\alpha}{\alpha}$.

5. Results and discussions

In this study, we successfully construct novel solitons solutions along with hyperbolic and trigonometric function solutions for the CSTNLSE with Kerr law of nonlinearity using SSM and new EHFM. These methods are considered as most recent schemes in this arena and that are not utilized to this equation earlier. For physical analysis, 3-dim and 2-dim plots of some of these solutions are included with appropriate parameters. These acquired solutions discover their application in communication to convey information because solitons have the capability to spread over long distances without reduction and without changing their forms.

We only added particular figures to avoid overfilling the document. Moreover, we effectively construct various solitary wave and solitons solutions along with hyperbolic and trigonometric function solutions. Absolutely the developed results are fresh and different from that reported results. These acquired solutions discover their application in communication to convey information because solitons have the capability to spread over long distances without reduction and without changing their forms. The authors proposed different analytic approach in newly issued article and reported some fascinating findings. We can understand from all the graphs that the SSM and new EHFM are very effectual and more specific in assessing the equation under consideration. In this paper, we only added particular figures to avoid overfilling the document. The authors proposed different analytic approaches in newly issued article and reported some fascinating findings. We can understand from all the graphs that the SSM and new EHFM are very effectual and more specific in assessing the equation under consideration. For graphical representation for (1.1), the physical behavior of (4.4) using the proper values of parameters $F_2 = 2.45$, $a = 1.75$, $\nu = 0.65$, $p = 1$, $q = 1$, $\alpha = 0.75$, $k = 0.8$, and $t = 1$ are shown in Fig. 1, the physical behavior of (4.5) using the appropriate values of parameters $F_2 = 2.45$, $a = -1.75$, $\nu = 0.65$, $p = 1$, $q = 1$, $\alpha = 0.75$, $k = 0.8$, and $t = 1$ are shown in Fig. 2, the physical behavior of (4.9) using the proper values of parameters $F_2 = 2.45$, $a = 1.75$, $\nu = 0.65$, $p = 1$, $q = 1$, $\alpha = 0.75$, $k = 0.8$, and $t = 1$ are shown in Fig. 3, the entire behavior of (4.12) using the proper values of parameters $F_2 = 2.45$, $a = 1.75$, $\nu = 0.65$, $p = 1$, $q = 1$, $\alpha = 0.75$, $k = 0.8$, and $t = 1$ are shown in Fig. 4, the complete behavior of (4.13) using the proper values of parameters $F_2 = 1.4$, $\alpha = 0.4$, $\nu = 0.5$, $\Theta = 0.75$, $k = 0.65$, $\beta = 0.2$, $\gamma = 0.8$, and $t = 1$ are revealed in Fig. 5, the complete behavior of (4.16) with the proper values of parameters $F_2 = 1.4$, $\alpha = -0.4$, $\nu = 0.5$, $\Theta = -0.75$, $k = 0.65$, $\beta = 0.2$, $\gamma = 0.8$, and $t = 1$ are presented in Fig. 6, the absolute behavior of (4.17) using the proper values of parameters $F_2 = 2.45$, $\sigma = 0.95$, $\nu = 0.5$, $\Theta = -0.85$, $k = 0.45$, and $t = 1$ are revealed in Fig. 7.

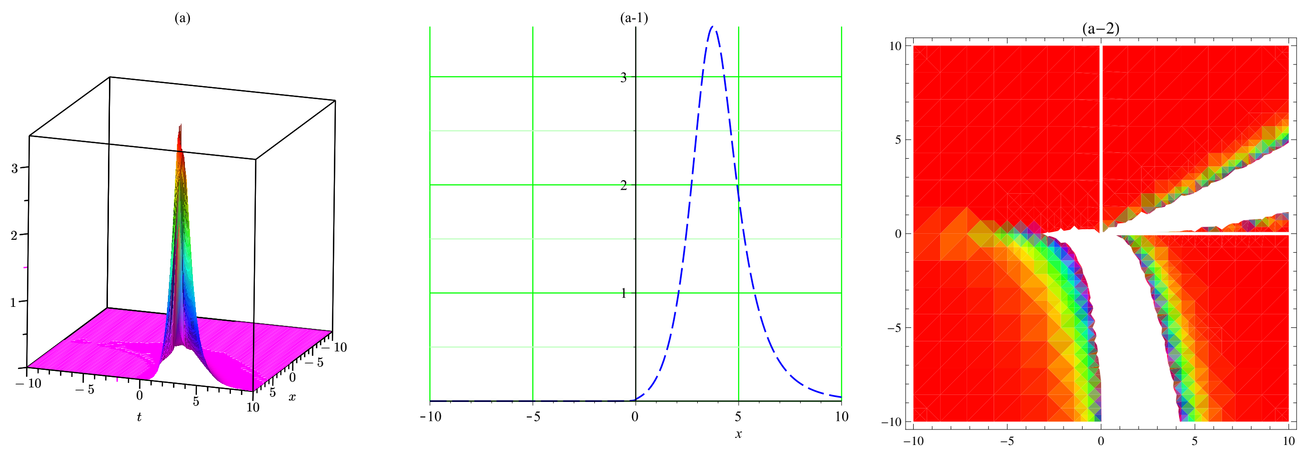

Figure 1: (a) 3D graph of (4.6) with $\alpha = 0.45$, $\beta = 0.65$, $w = 2$, $V = 2$,$p = 0.98$, $q = 0.95$, $k = 2$, $A = 3$, $b = 2$, $\delta = 2$, $c = 4$; (a-1) 2D plot of (4.4) with $t = 1$; (a-2) Contour graph of (4.4).

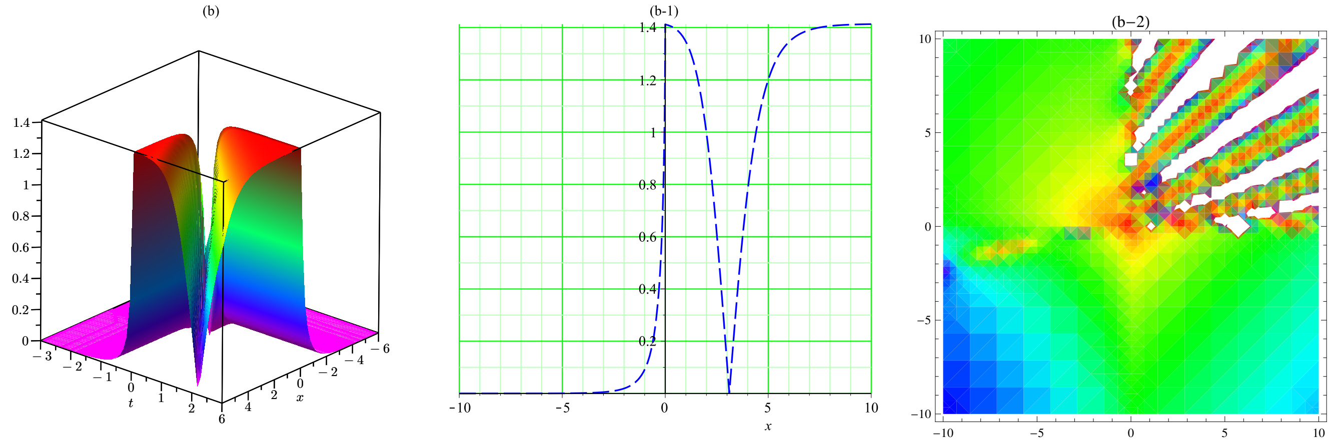

Figure 2: (b) 3D graph of (4.10) with $\alpha = 0.45$, $\beta = 0.65$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.7$, $b = 2$, $\delta = 2$, $c = 4$, $V = 2$; (b-1) 2D plot of (4.5) with $t = 1$; (b-2) Contour graph of (4.5).

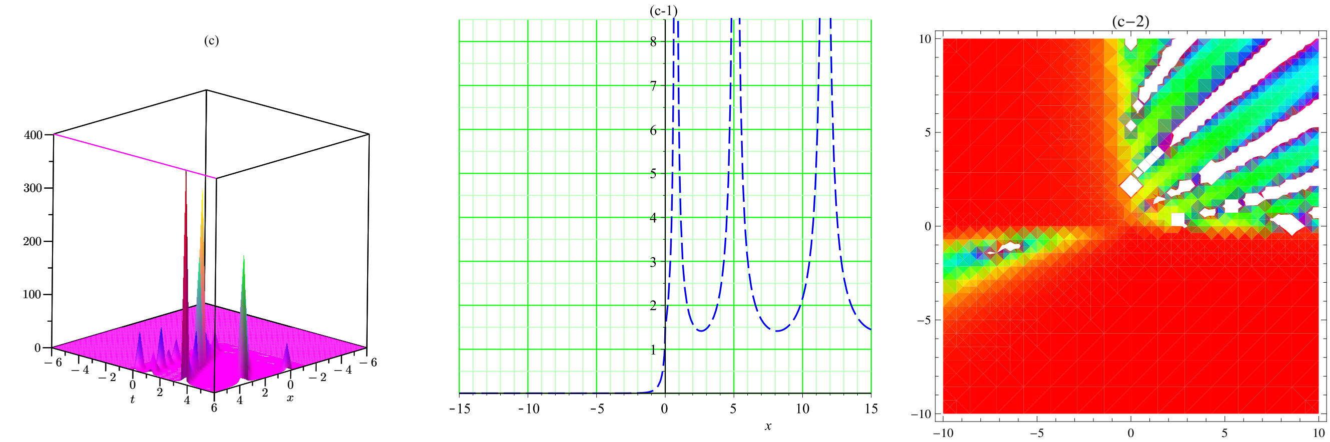

Figure 3: (c) 3D graph of (4.19) with $\alpha = 0.45$, $\beta = 0.65$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.7$, $b = 2$, $\delta = 2$, $c = 4$, $V = 2$; (c-1) 2D plot of (4.9) with $t = 1$; (c-2) Contour graph of (4.9).

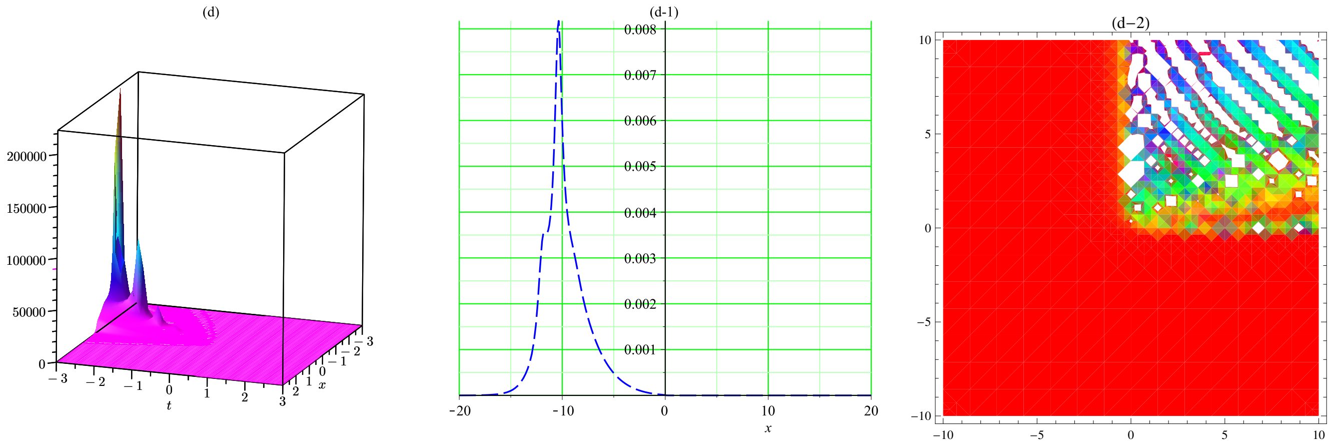

Figure 4: (d) 3D graph of (4.22) with $b = 2.5$, $a = 1.5$, $\nu = 1$, $k = 2$, $p = 0.98$, $q = 0.95$, $\alpha = 0.75$, $A = 2.7$, $\alpha = 0.45$, $\beta = 0.65$, $\theta = 4$, $V = 3$, $w = 3$; (d-1) 2D plot of (4.12) with $t = 1$; (d-2) Contour graph of (4.12).

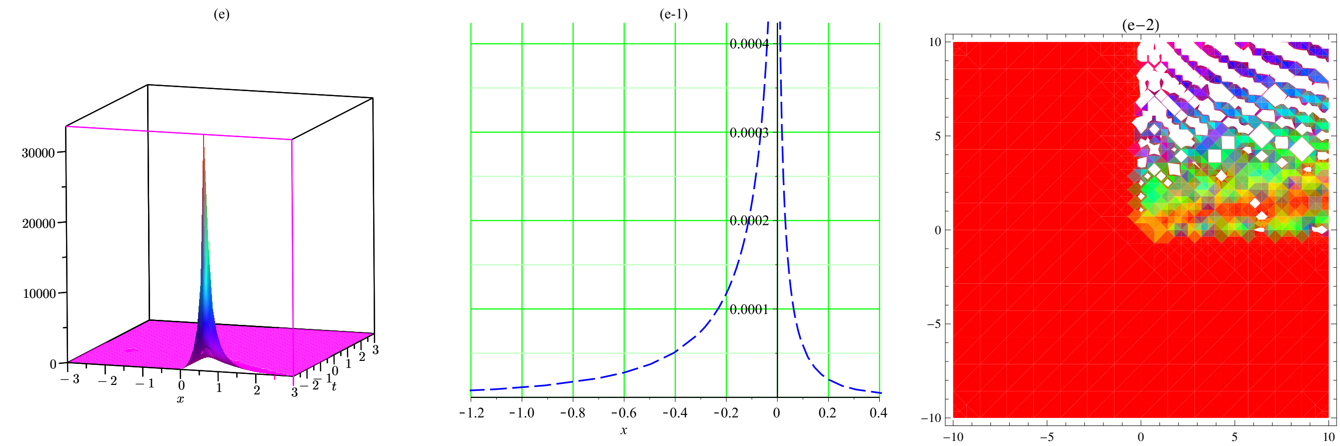

Figure 5: (e) 3D graph of (4.24) with $b = 2.5$, $a = 1.5$, $\nu = 1$, $k = 2$, $p = 0.98$, $q = 0.95$, $\alpha = 0.75$, $A = 2.7$, $\alpha = 0.45$, $\beta = 0.65$, $\theta = 4$, $V = 3$, $w = 3$; (e-1) 2D plot of (4.13) with $t = 1$; (e-2) Contour graph of (4.13).

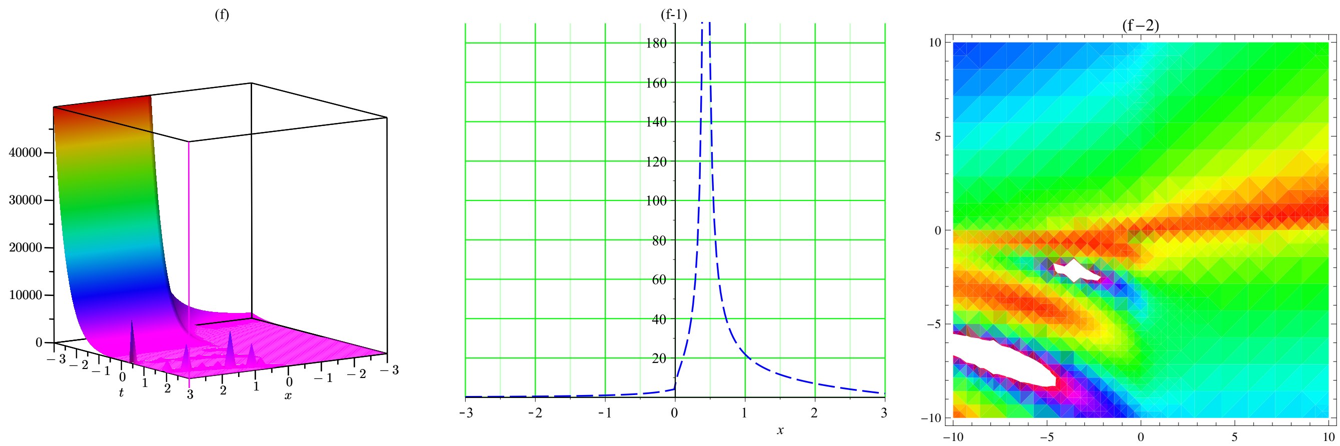

Figure 6: (f) 3D graph of (4.30) with $b = 2.5$, $a = 1.5$, $\nu = 1$, $k = 2$, $p = 0.98$, $q = 0.95$, $\alpha = 0.75$, $A = 2.7$, $\alpha = 0.45$, $\beta = 0.65$, $\theta = 4$, $V = 3$, $w = 3$; (f-1) 2D plot of (4.16) with $t = 1$; (f-2) Contour graph of (4.16).

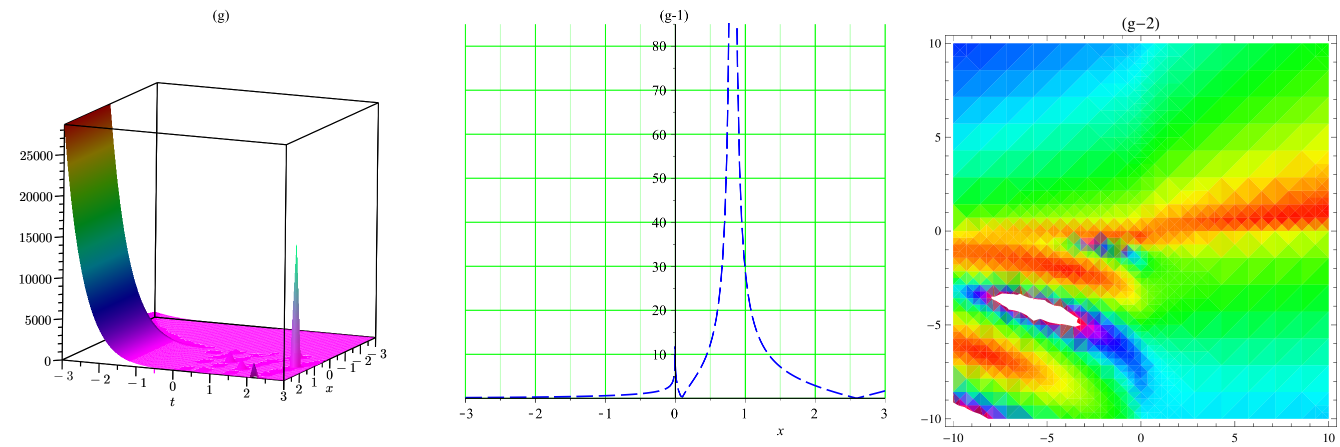

Figure 7: (g) 3D graph of (4.31) with $b = 2.5$, $a = 1.5$, $\nu = 1$, $k = 2$, $p = 0.98$, $q = 0.95$, $\alpha = 0.75$, $A = 2.7$, $\alpha = 0.45$, $\beta = 0.65$, $\theta = 4$, $V = 3$, $w = 3$; (g-1) 2D plot of (4.17) with $t = 1$; (g-2) Contour graph of (4.17).

6. Conclusion

We constructed novel optical soliton solutions for the CSTNLSE with Kerr law nonlinearity by using SSM and new EHFM. Different cases of the constraints are used to describe specific solutions. The main aim of this study is to acquire optical solitons that could satisfy the constraint conditions posed on the different parameters of the CSTNLSE. In order to describe the behavior of acquired solutions of the model, we plotted some selected solutions by giving appropriate values to the involved parameters. The obtained results may have much influence on numerous fields of nonlinear sciences. By obtained results, we can recognize that the proposed methods are proficient, consistent and beneficial for discovering the exact solutions of nonlinear FPDEs in a wide range. It has been detected that all the solutions of governing model in this study are novel and unique. In future, we will be more interested in birefringence aspect and cross-phase modulation to construct ultrashort optical pulses and soliton pulses. We need to investigate some new type optical solitons and modulation instability analysis of some fractional order NLSE type equations in the future.

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