On the existence of solutions of generalized equilibrium problems with $\alpha$-$\beta$-$\eta$-monotone mappings

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Abstract

The present paper is concerned with the new concept of relaxed $\alpha$-$\beta$-$\eta$-monotonicity and relaxed $\alpha$-$\beta$-$\eta$-pseudomonotonicity in Banach space which is applied to prove the existence of solutions of generalized equilibrium problem and classic equilibrium problem. In this regard, we use the well-known KKM-theory to obtain solutions of mentioned problems. ©2016 All rights reserved.

Keywords: KKM-mappings, hemicontinuity, $\alpha$-$\beta$-$\eta$-monotonicity, $\alpha$-$\beta$-$\eta$-pseudomonotonicity, semicontinuous mappings, Banach space.

2010 MSC: 47H05, 49J40.

1. Introduction

This work focuses on the existence of solutions of generalized equilibrium problems with the new concept of relaxed $\alpha$-$\beta$-$\eta$-monotonicity. The most important application of generalized equilibrium problems is in economics [1, 3], variational inequalities [5], optimization, fixed point theory [6] and so on. Over the last few years, the concept of generalized equilibrium problems has been studied by various authors and has developed rapidly (see [2, 13, 14, 17, 18]). Onjai-uea and his colleagues in [15] presented a relaxed hybrid
steepest method to find a common element for the set of fixed points of a nonexpansive mapping, the set of solutions of a variational inequality for an inverse-strongly monotone mapping and the set of solutions of generalized mixed equilibrium problems in Hilbert spaces. In 2013, Mahato and Nahak published a paper in which they obtained the existence results for mixed equilibrium problems in a reflexive Banach space [12]. Ding and his colleagues considered a collectively fixed point theorem and an equilibrium existence theorem for generalized games in product locally G-convex uniform spaces [8]. However, in recent years, the iterative algorithms of solutions for generalized equilibrium problems have been studied by several authors. For instance, a new class of generalized mixed implicit equilibrium-like problems has been introduced by Ding [7]. He used the auxiliary principle technique to obtain the solution of the mentioned problem. Zang and Deng in [19] studied the multi-valued general mixed implicit equilibrium-like problems and presented a new predictor corrector iterative algorithm by using the auxiliary principle technique. They also proved the convergence of the suggested algorithm in weaker conditions. One can refer to [4, 9, 11] for more details.

2. Preliminaries

This work has been done in real Banach space $X$. In this work, $K$ is considered as a nonempty convex subset of real Banach space $X$. In our study, we deal with the following generalized equilibrium problem:

Find $\bar{x} \in K$ such that

$$f(\bar{x}, y) + \varphi(\bar{x}, y) - \varphi(\bar{x}, \bar{x}) \geq 0, \quad \forall y \in K,$$

where $f : K \times K \to \mathbb{R}$ is an equilibrium function, that is, $f(x, x) = 0$, for all $x \in K$, and $\varphi : K \times K \to \mathbb{R}$ is a real valued function.

If $\varphi \equiv 0$, problem (2.1) reduces to the following equilibrium problem of finding $\bar{x} \in K$ such that

$$f(\bar{x}, y) \geq 0, \quad \forall y \in K.$$

Now, we present some fundamental definitions which will be used in the rest of this paper.

**Definition 2.1.** A function $f : K \to \mathbb{R}$ is said to be

1. weakly upper semicontinuous (u.s.c.) at $x_0 \in X$, if and only if

$$f(x_0) \geq \limsup_{n \to \infty} f(x_n)$$

for any sequence $\{x_n\}$ of $X$ which converges to $x_0$ weakly;

2. weakly lower semicontinuous (l.s.c.) at $x_0 \in X$, if

$$f(x_0) \leq \liminf_{n \to \infty} f(x_n)$$

for any sequence $\{x_n\}$ of $X$ which converges to $x_0$ weakly.

**Example 2.2.** The function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2} & (x, y) \neq (0, 0), \\ 0 & (x, y) = (0, 0), \end{cases}$$

is hemi-continuous on $\mathbb{R} \times \mathbb{R}$, but not continuous at $(0, 0)$.

**Definition 2.3.** Let $X$ be a Banach space. A single-valued mapping $f : X \to \mathbb{R}$ is called

1. weakly upper semicontinuous (u.s.c.) at $x_0 \in X$, if

$$f(x_0) \geq \limsup_{n \to \infty} f(x_n)$$

for any sequence $\{x_n\}$ of $X$ which converges to $x_0$ weakly;

2. weakly lower semicontinuous (l.s.c.) at $x_0 \in X$, if

$$f(x_0) \leq \liminf_{n \to \infty} f(x_n)$$

for any sequence $\{x_n\}$ of $X$ which converges to $x_0$ weakly.
Definition 2.4. A multi-valued mapping \( f : K \rightarrow 2^X \) is called a KKM-mapping, if for any \( \{y_1, \ldots, y_n\} \subset K \), \( \text{co}\{y_1, \ldots, y_n\} \subset \bigcup_{i=1}^{n} f(y_i) \), where \( 2^X \) denotes the family of all nonempty subsets of \( X \) and \( \text{co} \) denotes the convex hull.

Example 2.5. Let \( K = [0,1] \) and \( X = R \). In this case, the following mapping is a KKM-mapping.
\[
\begin{align*}
f &: [0,1] \rightarrow 2^R \\
f(x) &\mapsto [0, x].
\end{align*}
\]

Lemma 2.6 ([10]). Let \( K \) be a nonempty subset of a topological vector space \( X \) and let \( f : K \rightarrow 2^X \) be a KKM-mapping. If \( f(y) \) is closed in \( X \), for all \( y \in K \) and compact for at least one \( y \in K \), then
\[
\bigcap_{y \in K} f(y) \neq \emptyset.
\]

In the following, let us introduce a new definition of relaxed \( \alpha\)-\( \beta\)-\( \eta \)-monotone which is significant in our research.

Definition 2.7. The mapping \( f : K \times K \rightarrow R \) is called relaxed \( \alpha\)-\( \beta\)-\( \eta \)-monotone, if there exist mappings \( \eta : K \times K \rightarrow X \), \( \alpha : X \rightarrow R \) and \( \beta : K \times K \rightarrow R \) such that
\[
f(x, y) + f(y, x) \leq \alpha(\eta(x, y)) + \beta(x, y), \quad \forall x, y \in K,
\]
and
\[
\liminf_{t \to 0^+} \left[ \frac{\alpha(\eta(x, y))}{t} + \frac{\beta(x, ty + (1-t)x)}{t} \right] \leq 0.
\]

Remark that, if \( \alpha = 0 \) and \( \beta = 0 \), then the definition reduces to the definition of monotonicity of \( f \). Hence, Definition 2.7 is an extension of monotonicity.

Example 2.8. Let \( \alpha(x) = -1 \), \( \beta = 0 \) and \( \eta \) be an arbitrary function, hence
\[
\liminf_{t \to 0^+} \left[ \frac{\alpha(\eta(x, y))}{t} + \frac{\beta(x, ty + (1-t)x)}{t} \right] = -\infty \leq 0.
\]

If we choose \( f(x, y) = -2 \), in this case \( f \) is \( \alpha\)-\( \beta\)-\( \eta \)-monotone with respect to Definition 2.7, but \( f \) is not \( \alpha\)-\( \beta \)-monotone with respect to Definition 6 in [10].

3. Existence results for \( \alpha\)-\( \beta\)-\( \eta \)-monotone mappings

We start this section with the following theorem which is an existence result of solution of problem (2.1).

Theorem 3.1. Let \( f : K \times K \rightarrow R \) be relaxed \( \alpha\)-\( \beta\)-\( \eta \)-monotone, hemicontinuous in the first argument and convex in the second argument with \( f(x, x) = 0 \), for all \( x \in K \). Let \( \varphi : K \times K \rightarrow R \) be convex in the second argument. Then, the solution set of generalized equilibrium problem (2.1) is equal to the solution set of the following problem:

Find \( \overline{x} \in K \) such that
\[
f(y, \overline{x}) + \varphi(\overline{x}, \overline{x}) - \varphi(\overline{x}, y) \leq \alpha(\eta(\overline{x}, y)) + \beta(\overline{x}, y), \quad \forall y \in K.
\]

Proof. Let problem (2.1) have a solution, then
\[
\exists \overline{x} \in K \text{ such that } f(\overline{x}, y) + \varphi(\overline{x}, y) - \varphi(\overline{x}, \overline{x}) \geq 0, \quad \forall y \in K.
\]

It follows from the \( \alpha\)-\( \beta\)-\( \eta \)-monotonicity of \( f \) that
\[
f(\overline{x}, y) + f(y, \overline{x}) \leq \alpha(\eta(\overline{x}, y)) + \beta(\overline{x}, y), \quad \forall y \in K.
\]
According to problem (2.1) and equation (3.2), we get
\[ f(y, \overline{x}) + \varphi(\overline{x}, y) - \varphi(\overline{x}, y) \leq \alpha(\eta(y, y)) + \beta(\overline{x}, y) - [f(\overline{x}, y) + \varphi(\overline{x}, y) - \varphi(\overline{x}, \overline{x})] \leq \alpha(\eta(\overline{x}, y)) + \beta(\overline{x}, y), \quad \forall y \in K. \]

So, \( \overline{x} \in K \) is a solution of problem (3.1). Conversely, let \( \overline{x} \in K \) be a solution of problem (3.1). Therefore,
\[ f(y, \overline{x}) + \varphi(\overline{x}, y) - \varphi(\overline{x}, y) = \alpha(\eta(\overline{x}, y)) + \beta(\overline{x}, y), \quad \forall y \in K. \]

Let \( y \in K \) and \( t \) be an arbitrary element of \([0, 1]\). Obviously, \( x_t = ty + (1-t)\overline{x} \in K \). Hence, from (3.3), we obtain
\[ f(x_t, \overline{x}) + \varphi(\overline{x}, y) - \varphi(\overline{x}, y) \leq \alpha(\eta(\overline{x}, x_t)) + \beta(\overline{x}, x_t), \quad \forall t \in (0,1]. \]

Since \( f \) is convex in the second variable, we get
\[ 0 = f(x_t, x_t) \leq tf(x_t, y) + (1-t)f(x_t, \overline{x}), \]
and from the convexity \( \varphi \) in the second argument, we also have
\[ \varphi(\overline{x}, x_t) \leq t\varphi(\overline{x}, y) + (1-t)\varphi(\overline{x}, \overline{x}). \]

It follows from (3.4)-(3.6) that
\[ t[f(x_t, \overline{x}) - f(x_t, y) + \varphi(\overline{x}, y) - \varphi(\overline{x}, y)] \leq f(x_t, \overline{x}) + \varphi(\overline{x}, y) - \varphi(\overline{x}, x_t) \leq \alpha(\eta(\overline{x}, x_t)) + \beta(\overline{x}, x_t), \]
which implies that
\[ f(x_t, \overline{x}) - f(x_t, y) + \varphi(\overline{x}, y) - \varphi(\overline{x}, y) \leq \frac{\alpha(\eta(\overline{x}, x_t))}{t} + \frac{\beta(\overline{x}, x_t)}{t}. \]

According to hemicontinuity of \( f \) in the first argument and the definition of relaxed \( \alpha-\beta-\eta \)-monotone of \( f \), by taking \( t \to 0^+ \), we have
\[ f(\overline{x}, \overline{x}) - f(\overline{x}, y) + \varphi(\overline{x}, y) - \varphi(\overline{x}, y) \leq 0, \quad \forall y \in K, \]
and so, note \( f(\overline{x}, \overline{x}) = 0, \)
\[ f(\overline{x}, y) + \varphi(\overline{x}, y) - \varphi(\overline{x}, y) \geq 0, \quad \forall y \in K. \]

Hence, \( \overline{x} \in K \) is a solution of problem (2.1) which completes the proof.

In what follows, we demonstrate that problem (2.1) admits a solution. This topic stated in the next theorem is the most important issue in our work.

**Theorem 3.2.** Let \( K \) be a nonempty bounded closed convex subset of a real reflexive Banach space \( X \). Let \( f : K \times K \to R \) be relaxed \( \alpha-\beta-\eta \)-monotone, hemicontinuous in the first argument, convex in the second argument with \( f(x, x) = 0 \), \( \varphi : K \times K \to R \) be convex in the second variable, \( \alpha : K \to R \) be weakly upper semi-continuous and \( \beta : K \times K \to R \) be weakly upper semi-continuous in the second argument. Then, problem (2.1) admits a solution.

**Proof.** Let \( F : K \to 2^X \) be a multi-valued mapping defined by
\[ F(y) = \{ x \in K \mid f(x, y) + \varphi(x, y) - \varphi(x, x) \geq 0 \}. \]

Obviously, \( \overline{x} \in K \) is a solution of equation (2.1), if and only if \( \overline{x} \in \bigcap_{y \in K} F(y) \). We are going to show that \( \bigcap_{y \in K} F(y) \neq \emptyset \). We claim that \( F \) is a KKM-mapping. Suppose to the contrary that \( F \) is not a KKM-
mapping. So there exists a finite subset \( \{x_1, \ldots, x_n\} \) of \( K \) such that \( \text{co}\{x_1, \ldots, x_n\} \subset \bigcup_{i=1}^{n} F(x_i) \). Therefore, there exists \( x_0 \in \text{co}\{x_1, \ldots, x_n\} \) for all \( i \in \{1, \ldots, n\} \), \( x_0 \notin F(x_i) \). Hence, for \( i = 1, 2, \ldots, n \), we have
\[
f(x_0, x_i) + \varphi(x_0, x_i) - \varphi(x_0, x_0) < 0.
\] (3.7)

Thus, there exist \( \lambda_i \geq 0 \) \( (i = 1, 2, \ldots, n) \) with \( \sum_{i=1}^{n} \lambda_i = 1 \) such that \( x_0 = \sum_{i=1}^{n} \lambda_i x_i \). By multiplying both sides of relation (3.7) by \( \lambda_i \) and adding them, we obtain
\[
\sum_{i=1}^{n} \lambda_i[f(x_0, x_i) + \varphi(x_0, x_i) - \varphi(x_0, x_0)] < 0.
\]

This and our assumptions on \( f \) and \( \varphi \) lead us to the contradiction \( 0 < 0 \). Hence, the multi-valued mapping \( F \) is a KKM mapping.

We define the multi-valued mapping \( G : K \rightarrow 2^K \) by
\[
G(y) = \{x \in K : f(y, x) + \varphi(x, x) - \varphi(x, y) \leq \alpha(\eta(x, y)) + \beta(x, y)\}.
\]

It is clear that \( F(y) \) is a subset of \( G(y) \), for all \( y \in K \). Because, let \( y \) be an arbitrary element of \( K \) and \( \pi \in F(y) \), then
\[
f(\pi, y) + \varphi(\pi, y) - \varphi(\pi, \pi) \geq 0.
\]

The relaxed \( \alpha-\beta-\eta \)-monotonicity of \( f \) implies that
\[
f(y, \pi) + \varphi(\pi, \pi) - \varphi(\pi, y) \leq \alpha(\eta(\pi, y)) + \beta(\pi, y) - [f(\pi, y) + \varphi(\pi, y) - \varphi(\pi, \pi)]
\leq \alpha(\eta(\pi, y)) + \beta(\pi, y),
\]
and so \( \pi \in G(y) \). Then, \( F(y) \subset G(y) \). Since \( F \) is a KKM-mapping and \( F(y) \subset G(y) \), then \( G \) is a KKM-mapping. According to the conditions on the mappings, it is easy to verify that \( G(y) \) is weakly compact, for all \( y \in K \). Since \( K \) is a bounded, closed and convex subset of the reflexive Banach space \( X \), then it is weakly compact and consequently \( G(y) \) is weakly compact in \( K \), for all \( y \in K \). Consequently, it follows from Lemma 2.6 that \( \bigcap_{y \in K} G(y) \neq \emptyset \), and from Theorem 3.1 that \( \bigcap_{y \in K} F(y) = \bigcap_{y \in K} G(y) \). Thus, \( \bigcap_{y \in K} F(y) \neq \emptyset \). Hence, there exists \( \pi \in K \) such that
\[
f(\pi, y) + \varphi(\pi, y) - \varphi(\pi, \pi) \geq 0, \quad \forall y \in K.
\]

So, the solution set of problem (2.1) is nonempty. This completes the proof. \( \square \)

**Example 3.3.** Let \( K = [0, 1] \), \( \alpha(x) = -x \), \( \beta(x, y) = 0 \) and \( \eta(x, y) = (x + y)(x - y)^2 \). If we choose \( f(x, y) = x(y^2 - x^2) \) and \( \varphi(x, y) = x^2 + y^2 \), then all assumptions of Theorem 3.2 hold. Therefore, problem (2.1) is solvable. It is easy to see that \( \pi = 0 \) is the only solution of problem (2.1).

4. Existence results for \( \alpha-\beta-\eta \)-pseudomonotone mappings

In this section, we introduce the concept of relaxed \( \alpha-\beta-\eta \)-pseudomonotonicity and discuss the existence solution of equilibrium problems (2.1) and (2.2) using this concept.

**Definition 4.1.** A mapping \( f : K \times K \rightarrow R \) is called relaxed \( \alpha-\beta-\eta \)-pseudomonotone, if there exist functions \( \eta : K \times K \rightarrow X \), \( \alpha : X \rightarrow R \) and \( \beta : K \times K \rightarrow R \) such that for any \( x, y \in K \), we have
\[
f(x, y) \geq 0 \Rightarrow f(y, x) \leq \alpha(\eta(x, y)) + \beta(y, x),
\]
where
\[
\lim_{t \to 0^+} \left[ \frac{\alpha(\eta(x, y))}{t} + \frac{\beta(x, ty + (1 - t)x)}{t} \right] \leq 0.
\]
If we take \( \alpha = \beta = 0 \), then the definition of relaxed \( \alpha\beta\eta \)-pseudomonotonicity collapses to the usual definition of pseudomonotonicity. Moreover, note that each relaxed \( \alpha\beta\eta \)-monotone mapping is relaxed \( \alpha\beta\eta \)-pseudomonotone mapping. The following example shows that the inverse is not always true.

**Example 4.2.** Consider \( X = R \), \( K = [0,1] \) and \( f(x,y) = x - y \). We choose \( \alpha(x) = -x \), \( \beta(x,y) = 0 \) and \( \eta(x,y) = |x - y| \). If \( f(x,y) \geq 0 \), then \( x - y \geq 0 \). Hence, \( f(y,x) = y - x \leq -|x - y| = \eta(y,x) + \beta(y,x) \) and

\[
\lim_{t \to 0^+} \left[ \frac{\alpha(\eta(x,y))}{t} + \beta(x,ty + (1-t)x) \right] = -\infty \leq 0.
\]

Therefore, \( f \) is relaxed \( \alpha\beta\eta \)-pseudomonotone. Whereas, \( f \) is not relaxed \( \alpha\beta\eta \)-monotone.

**Theorem 4.3.** Let \( f : K \times K \to R \) be generalized relaxed \( \alpha\beta\eta \)-pseudomonotone, hemicontinuous in the first argument and convex in the second argument with \( f(x,x) = 0 \), for all \( x \in K \). Then, generalized equilibrium problem (2.2) is equivalent to the following problem:

Find \( \pi \in K \) such that

\[
f(y,\pi) \leq \alpha(\eta(y,\pi)) + \beta(y,\pi), \quad \forall y \in K.
\]

**Proof.** Let \( \pi \in K \) be a solution of problem (2.2), that is

\[
f(\pi,y) \geq 0, \quad \forall y \in K.
\]

So, by the relaxed \( \alpha\beta\eta \)-pseudomonotonicity of \( f \), we get

\[
f(y,\pi) \leq \alpha(\eta(y,\pi)) + \beta(y,\pi), \quad \forall y \in K.
\]

Hence, \( \pi \in K \) is a solution of problem defined by (4.1). Conversely, assume that \( \pi \in K \) is a solution of (4.1). Then, for any \( y \in K \), let \( x_t = ty + (1-t)\pi \), \( t \in (0,1] \). Obviously, \( x_t \in K \), and it follows that

\[
f(x_t,\pi) \leq \alpha(\eta(x_t,\pi)) + \beta(x_t,\pi).
\]

Since \( f \) is convex in the second argument, we obtain

\[
0 = f(x_t, x_t) \leq tf(x_t, y) + (1-t)f(x_t, \pi).
\]

Equations (4.2) and (4.3) imply that

\[
f(x_t, \pi) - f(x_t, y) \leq \frac{\alpha(\eta(x_t,\pi))}{t} + \frac{\beta(x_t, \pi)}{t}, \quad \forall y \in K.
\]

Hemicontinuity of \( f \) in the first argument and the definition of relaxed \( \alpha\beta\eta \)-monotone of \( f \), by taking \( t \to 0^+ \) imply that

\[
f(\pi, y) \geq 0, \quad \forall y \in K.
\]

Hence, \( \pi \in K \) is a solution of problem (2.2), and it completes the proof.

**Theorem 4.4.** Let \( K \) be a nonempty bounded closed convex subset of a real reflexive Banach space \( X \). Let \( f : K \times K \to R \) be relaxed \( \alpha\beta\eta \)-pseudomonotone, hemicontinuous in the first argument, convex in the second argument with \( f(x,x) = 0 \). Moreover, \( \alpha : K \to R \) is weakly upper semicontinuous and \( \beta : K \times K \to R \) is weakly upper semicontinuous in the second argument. Then, problem (2.2) admits a solution.
Proof. Let $F : K \to 2^X$ be defined by

$$F(y) = \{ x \in K \mid f(x, y) \geq 0 \}.$$ 

It is clear that $\bar{x} \in K$ is a solution of problem (2.2), if and only if $\bar{x} \in \bigcap_{y \in K} F(y)$. Hence, we prove that

$$\bigcap_{y \in K} F(y) \neq \emptyset.$$ 

It is easy to see that $F$ is a KKM-mapping. Because, otherwise, there exists a finite subset $\{ x_1, \ldots, x_n \}$ of $K$ such that $co\{ x_1, \ldots, x_n \} \nsubseteq \bigcup_{i=1}^n F(x_i)$. This means that there exists $x_0 \in co\{ x_1, \ldots, x_n \}$ such that $f(x_0, x_i) < 0$, for $i = 1, \ldots, n$. Thus, there exist $\lambda_i \geq 0$ ($i = 1, 2, \ldots, n$) with $\sum_{i=1}^n \lambda_i = 1$ such that $x_0 = \sum_{i=1}^n \lambda_i x_i$. Hence,

$$\sum_{i=1}^n \lambda_i f(x_0, x_i) < 0.$$ 

According to the convexity of $f$ in the second variable, we reach the contradiction $0 < 0$. Hence, $F$ is a KKM-mapping.

Define the set-valued mapping $G : K \to 2^X$ by

$$G(y) = \{ x \in K \mid f(y, x) \leq \alpha(\eta(y, x)) + \beta(y, x) \}.$$ 

The relaxed $\alpha$-$\beta$-$\eta$-pseudomonotonicity of $f$ implies that $F(y) \subseteq G(y)$, for all $y \in K$. Hence, $G$ is also a KKM-mapping.

By the hypothesis on the mappings, the values of the multi-valued mapping $G$ are weakly closed and since $K$ is a closed bounded subset of the reflexive Banach space $X$, then $G(y)$ is weakly compact, for all $y \in K$. Hence, the multi-valued mapping $G$ satisfies all assumptions of Lemma 2.6 and then $\bigcap_{y \in K} G(y)$ is nonempty and hence by Theorem 4.3, $\bigcap_{y \in K} F(y)$ is nonempty. Consequently, there exists $\bar{x} \in K$ such that $f(\bar{x}, y) \geq 0$, for all $y \in K$ which completes the proof.

Example 4.5. Let $K = [0, \frac{3}{2}]$, $\alpha(x) = -x$, $\beta = 0$ and $\eta(x, y) = |x - y|$. If we choose $f(x, y) = (x - y) \cos(y)$, then all assumptions of Theorem 4.4 hold. Therefore, problem (2.2) admits a solution. It is easy to see that $x = \frac{3}{2}$ is a solution of this problem.

5. Conclusion

To sum up, we have introduced a new concept of relaxed $\alpha$-$\beta$-$\eta$-monotonicity and have applied the well-known KKM-theory to obtain some existence results for solutions of generalized equilibrium problems. Moreover, we have proven the existence of solutions of equilibrium problems by using the new concept of relaxed $\alpha$-$\beta$-$\eta$-pseudomonotonicity and KKM-theory.

Acknowledgment

The second and third authors would like to thank the Department of Mathematics, Faculty of Science, Naresuan University, and the Center of Excellence in Nonlinear Analysis and Optimization, Faculty of Science, Naresuan University.

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