Some results on fixed points of nonlinear operators and solutions of equilibrium problems

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Abstract

The purpose of this paper is to investigate fixed points of an asymptotically quasi-$\phi$-nonexpansive mapping in the intermediate sense and a bifunction equilibrium problem. We obtain a strong convergence theorem of solutions in the framework of Banach spaces. ©2016 All rights reserved.

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1. Introduction and Preliminaries

Let $E$ be a real Banach space and let $C$ be a convex closed subset of $E$. Let $B : C \times C \to \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers, be a bifunction. Recall that the following equilibrium problem in the terminology of Blum and Oettli [4]. Find $\bar{x} \in C$ such that

$$B(\bar{x}, y) \geq 0, \forall y \in C. \quad (1.1)$$

In this paper, we use $\text{Sol}(B)$ to denote the solution set of equilibrium problem (1.1). That is, $\text{Sol}(B) = \{x \in C : B(x, y) \geq 0, \forall y \in C\}$.

The following restrictions on bifunction $B$ are essential in this paper.

(Q1) $B(a, a) \equiv 0, \forall a \in C$;
Equilibrium problem (1.1), which includes complementarity problems, variational inequality problems and inclusion problems as special cases, provides us a natural and unified framework to study a wide class of problems arising in physics, economics, finance, transportation, network, elasticity and optimization; see [3], [8], [10], [12], [14], [23], [28], and the references therein. Recently, equilibrium problem (1.1) has been extensively investigated based on fixed point algorithms in Banach spaces; see [9], [11], [13], [15]-[18], [24]-[27], [29]-[32] and the references therein.

Let $E^*$ be the dual space of $E$. Let $S^E$ be the unit sphere of $E$. Recall that $E$ is said to be a strictly convex space iff $\|x+y\| < 2$ for all $x, y \in S^E$ and $x \neq y$. Recall that $E$ is said to have a Gâteaux differentiable norm iff $\lim_{t \to 0} \frac{1}{t}(\|x\| - \|x + ty\|)$ exists for each $x, y \in S^E$. In this case, we also say that $E$ is smooth. $E$ is said to have a uniformly Gâteaux differentiable norm if for each $y \in B_E$, the limit is attained uniformly for all $x \in S^E$. $E$ is also said to have a uniformly Fréchet differentiable norm iff the above limit is attained uniformly for $x, y \in S^E$. In this case, we say that $E$ is uniformly smooth.

Recall that the normalized duality mapping $J$ from $E$ to $2^{E^*}$ is defined by

$$Jx = \{y \in E^* : \|y\|^2 = \langle x, y \rangle = \|x\|^2\}.$$ 

It is known

- if $E$ is uniformly smooth, then $J$ is uniformly norm-to-norm continuous on every bounded subset of $E$;
- if $E$ is a strictly convex Banach space, then $J$ is strictly monotone;
- if $E$ is a smooth Banach space, then $J$ is single-valued and demicontinuous, i.e., continuous from the strong topology of $E$ to the weak star topology of $E$;
- if $E$ is a reflexive and strictly convex Banach space with a strictly convex dual $E^*$ and $J^* : E^* \to E$ is the normalized duality mapping in $E^*$, then $J^{-1} = J^*$;
- if $E$ is a smooth, strictly convex and reflexive Banach space, then $J$ is single-valued, one-to-one and onto.

Recall that $p$ is said to be a fixed point of $T$ if and only if $p = Tp$. $p$ is said to be an asymptotic fixed point [22] of $T$ if and only if $C$ contains a sequence $\{x_n\}$, such that for some $p \in C$ such that $x_n - Tx_n \to 0$. From now on, we use $Fix(T)$ to stand for the fixed point set and $\tilde{Fix}(T)$ to stand for the asymptotic fixed point set.

Next, we assume that $E$ is a smooth Banach space which means $J$ is single-valued. Study the functional

$$\phi(x, y) := \|x\|^2 + \|y\|^2 - 2\langle x, Jy \rangle, \quad \forall x, y \in E.$$ 

Let $C$ be a closed convex subset of a real Hilbert space $H$. For any $x \in H$, there exists a unique nearest point in $C$, denoted by $P_Cx$, such that $\|x - P_Cx\| \leq \|x - y\|$, for all $y \in C$. The operator $P_C$ is called the metric projection from $H$ onto $C$. It is known that $P_C$ is firmly nonexpansive. In [2], Alber studied a new mapping $Proj_C$ in a Banach space $E$ which is an analogue of $P_C$, the metric projection, in Hilbert spaces. Recall that the generalized projection $Proj_C : E \to C$ is a mapping that assigns to an arbitrary point $x \in E$ the minimum point of $\phi(x, y)$, which implies from the definition of $\phi$ that

$$(\|y\|^2 + \|x\|^2) \geq \phi(x, y) \geq (\|x\|^2 - \|y\|^2), \quad \forall x, y \in E.$$ 

Recall that $T$ is said to be relatively nonexpansive [6], [7] iff

$$Fix(T) = \tilde{Fix}(T) \neq \emptyset, \phi(p, Tx) \leq \phi(p, x), \quad \forall x \in C, \forall p \in Fix(T).$$
$T$ is said to be relatively asymptotically nonexpansive [1] iff

$$\text{Fix}(T) = \overline{\text{Fix}}(T) \neq \emptyset, \phi(p,T^n x) \leq (\mu_n + 1) \phi(p,x), \; \forall x \in C, \forall p \in \text{Fix}(T), \forall n \geq 1,$$

where \(\{\mu_n\} \subset [0, \infty)\) is a sequence such that \(\mu_n \to 0\) as \(n \to \infty\).

$T$ is said to be relatively asymptotically nonexpansive in the intermediate sense iff $\text{Fix}(T) = \overline{\text{Fix}}(T) \neq \emptyset$ and

$$\limsup_{n \to \infty} \sup_{p \in \text{Fix}(T), x \in C} (\phi(p,T^n x) - \phi(p,x)) \leq 0.$$

Putting \(\xi_n = \max\{0, \sup_{p \in \text{Fix}(T), x \in C} (\phi(p,T^n x) - \phi(p,x))\}\), we see \(\xi_n \to 0\) as \(n \to \infty\).

$T$ is said to be quasi-$\phi$-nonexpansive [19] iff

$$\text{Fix}(T) \neq \emptyset, \phi(p,Tx) \leq \phi(p,x), \; \forall x \in C, \forall p \in \text{Fix}(T).$$

$T$ is said to be asymptotically quasi-$\phi$-nonexpansive [20] iff there exists a sequence \(\{\mu_n\} \subset [0, \infty)\) with \(\mu_n \to 0\) as \(n \to \infty\) such that

$$\text{Fix}(T) \neq \emptyset, \phi(p,T^n x) \leq (\mu_n + 1) \phi(p,x), \; \forall x \in C, \forall p \in \text{Fix}(T), \forall n \geq 1.$$

$T$ is said to be asymptotically quasi-$\phi$-nonexpansive in the intermediate sense [21] iff $\text{Fix}(T) \neq \emptyset$ and

$$\limsup_{n \to \infty} \sup_{p \in \text{Fix}(T), x \in C} (\phi(p,T^n x) - \phi(p,x)) \leq 0.$$

Putting \(\xi_n = \max\{0, \sup_{p \in \text{Fix}(T), x \in C} (\phi(p,T^n x) - \phi(p,x))\}\), we see \(\xi_n \to 0\) as \(n \to \infty\).

Remark 1.1. The class of relatively asymptotically nonexpansive mappings covers the class of relatively nonexpansive mappings. The class of (asymptotically) quasi-$\phi$-nonexpansive mappings (in the intermediate sense) is more desirable than the class of relatively (asymptotically) nonexpansive mappings (in the intermediate sense) because of restriction $\text{Fix}(T) = \overline{\text{Fix}}(T)$.

Remark 1.2. The class of asymptotically quasi-$\phi$-nonexpansive mappings in the intermediate sense is reduced to the class of asymptotically quasi-nonexpansive mappings in the intermediate sense, which was considered in [5] as a non-Lipschitz continuous mappings, in the framework of Hilbert spaces.

Lemma 1.3 ([2]). Let $E$ be a strictly convex, reflexive, and smooth Banach space and let $C$ be a closed and convex subset of $E$. Let $x \in E$. Then

\[ \phi(y,x) - \phi(\Pi_C x, x) \geq \phi(y, \Pi_C x), \; \forall y \in C, \]

\[ \langle y - x_0, Jx - Jx_0 \rangle \leq 0, \; \forall y \in C \text{ if and only if } x_0 = \Pi_C x. \]

Lemma 1.4 ([21]). Let $E$ be a strictly convex, smooth, and reflexive Banach space and let $C$ be a closed convex subset of $E$. Let $B$ be a function with restrictions $(Q1)$, $(Q2)$, $(Q3)$ and $(Q4)$. Let $x \in E$ and let $r > 0$. Then there exists $z \in C$ such that $rB(z,y) + \langle z - y, Jz - Jx \rangle \leq 0, \forall y \in C$ Define a mapping $W^{B,r}$ by

$$W^{B,r} x = \{z \in C : rB(z,y) + \langle y - z, Jz - Jx \rangle \geq 0, \; \forall y \in C\}.$$

The following conclusions hold:

1. $W^{B,r}$ is single-valued quasi-$\phi$-nonexpansive.

2. $\text{Sol}(B) = \text{Fix}(W^{B,r})$ is closed and convex.

Lemma 1.5 ([21]). Let $E$ be a strictly convex, smooth and reflexive Banach space such that both $E^*$ and $E$ have the KK property. Let $C$ be a convex and closed subset of $E$ and let $T$ be an asymptotically quasi-$\phi$-nonexpansive mapping in the intermediate sense on $C$. Then $\text{Fix}(T)$ is convex.
2. Main results

**Theorem 2.1.** Let $E$ be a smooth, strictly convex, and reflexive Banach space such that both $E$ and $E^*$ have the KK property and let $C$ be a convex and closed subset of $E$. Let $B$ be a bifunction satisfying (Q1), (Q2), (Q3) and (Q4) and let $T$ be an asymptotically quasi-$\phi$-nonexpansive mapping in the intermediate sense on $C$. Assume that $T$ is uniformly asymptotically regular and closed and $Fix(T) \cap Sol(B) \neq \emptyset$. Let $\{x_n\}$ be a sequence generated by

\[
\begin{align*}
  x_0 &\in E \text{ chosen arbitrarily,} \\
  C_1 &= C, x_1 = \text{Proj}_C x_0, \\
  r_n B(u_n, \mu) &\geq \langle u_n - \mu, J u_n - J x_n \rangle, \mu \in C, \\
  J y_n &= \alpha_n J T^n u_n + (1 - \alpha_n) J x_n, \\
  C_{n+1} &= \{z \in C : \phi(z, x_n) + \xi_n \geq \phi(z, y_n)\}, \\
  x_{n+1} &= \text{Proj}_{C_{n+1}} x_1,
\end{align*}
\]

where $\xi_n = \max\{\sup_{p \in Fix(T), x \in C} (\phi(p, T^n x) - \phi(p, x)), 0\}$. $\{\alpha_n\}$ is a real sequence in $[a, 1]$, where $a \in (0, 1]$ is a real number, and $\{r_n\} \subset [r, \infty)$ is a real sequence, where $r$ is some positive real number. Then $\{x_n\}$ converges strongly to $\text{Proj}_{Fix(T) \cap Sol(B)} x_1$.

**Proof.** The proof is split into seven steps.

**Step 1.** Prove $Sol(B) \cap Fix(T)$ is convex and closed.

Using Lemma 1.4 and Lemma 1.5, we find that $Sol(B)$ is convex and closed and $Fix(T)$ is convex. Since $T$ is closed, one has $Fix(T)$ is also closed. So, $Sol(B) \cap Fix(T)$ is convex and closed. $\text{Proj}_{Sol(B) \cap Fix(T)} x$ is well defined, for any element $x$ in $E$.

**Step 2.** Prove $C_n$ is convex and closed.

It is obvious that $C_1 = C$ is convex and closed. Assume that $C_m$ is convex and closed for some $m \geq 1$. Let $p_1, p_2 \in C_{m+1}$. It follows that $p = sp_1 + (1 - s)p_2 \in C_m$, where $s \in (0, 1)$. Notice that $\phi(p_1, y_m) - \phi(p_1, x_m) \leq \xi_m$, and $\phi(p_2, y_m) - \phi(p_2, x_m) \leq \xi_m$. Hence, one has

\[
\xi_m + \|x_m\|^2 - \|y_m\|^2 \geq 2 \langle p_1, Jx_m - Jy_m \rangle,
\]

and

\[
\xi_m + \|x_m\|^2 - \|y_m\|^2 \geq 2 \langle p_2, Jx_m - Jy_m \rangle.
\]

Using the above two inequalities, one has $\phi(p, x_m) + \xi_m \geq \phi(z, y_m)$. This shows that $C_{m+1}$ is closed and convex. Hence, $C_n$ is a convex and closed set. This proves that $\text{Proj}_{C_{n+1}} x_1$ is well defined.

**Step 3.** Prove $Sol(B) \cap Fix(T) \subset C_n$.

Note that $Sol(B) \cap Fix(T) \subset C_1 = C$ is clear. Suppose that $Sol(B) \cap Fix(T) \subset C_m$ for some positive integer $m$. For any $w \in Sol(B) \cap Fix(T) \subset C_m$, we see that

\[
\phi(w, y_m) = \| (1 - \alpha_m) Jx_m + \alpha_m J T^m u_m \|^2 + \| w \|^2 - 2 \langle w, (1 - \alpha_m) Jx_m + \alpha_m J T^m u_m \rangle \\
\leq \| w \|^2 - 2 \alpha_m \| w, J T^m u_m \| - 2 (1 - \alpha_m) \langle w, Jx_m \rangle \\
+ \alpha_m \| T^m u_m \|^2 + (1 - \alpha_m) \| x_m \|^2 \\
\leq \alpha_m \phi(w, u_m) + \alpha_m \xi_m + (1 - \alpha_m) \phi(w, x_m) \\
\leq \phi(w, x_m) + \xi_m,
\]

where $\xi_m = \max\{\sup_{p \in Fix(T), x \in C} (\phi(p, T^m x) - \phi(p, x)), 0\}$. This shows that $w \in C_{m+1}$. This implies that $Sol(B) \cap Fix(T) \subset C_n$.

**Step 4.** Prove $\{x_n\}$ is bounded.
Using Lemma 1.3, one has \( \langle z - x_n, Jx_1 - Jx_n \rangle \leq 0 \), for any \( z \in C_n \). It follows that
\[
0 \geq \langle w - x_n, Jx_1 - Jx_n \rangle, \forall w \in \text{Sol}(B) \cap \text{Fix}(T) \subset C_n.
\]

Using Lemma 1.3 yields that
\[
\phi(\Pi_{\text{Fix}(T) \cap \text{Sol}(B)} x_1, x_1) \geq \phi(x_n, x_1) \geq 0,
\]
which implies that \( \{ \phi(x_n, x_1) \} \). Hence \( \{ x_n \} \) is also a bounded sequence. Without loss of generality, we may assume \( x_n \rightarrow \tilde{x} \). Since \( C_n \) is convex and closed, we see \( \tilde{x} \in C_n \).

**Step 5.** Prove \( \tilde{x} \in \text{Fix}(T) \).

Using the fact \( \phi(x_n, x_1) \leq \phi(\tilde{x}, x_1) \), one has
\[
\phi(\tilde{x}, x_1) \geq \limsup_{n \to \infty} \phi(x_n, x_1) \geq \liminf_{n \to \infty} \phi(x_n, x_1) = \liminf_{n \to \infty}(\|x_n\|^2 + \|x_1\|^2 - 2\langle x_n, Jx_1 \rangle) \geq \phi(\tilde{x}, x_1).
\]

It follows that \( \lim_{n \to \infty} \phi(x_n, x_1) = \phi(\tilde{x}, x_1) \). Hence, we have
\[
\phi(x_{n+1}, x_1) - \phi(x_n, x_1) \geq \phi(x_{n+1}, x_n) \geq 0.
\]

Therefore, we have \( \lim_{n \to \infty} \phi(x_{n+1}, x_n) = 0 \). Since \( x_{n+1} \in C_{n+1} \), one sees that
\[
\phi(x_{n+1}, x_n) + \xi_n \geq \phi(x_{n+1}, y_n) \geq 0.
\]

It follows that \( \lim_{n \to \infty} \phi(x_{n+1}, y_n) = 0 \). Hence, one has \( \lim_{n \to \infty}(\|y_n\| - \|x_{n+1}\|) = 0 \). This implies that
\[
\|\tilde{x}\| = \|J\tilde{x}\| = \lim_{n \to \infty} \|Jy_n\| = \lim_{n \to \infty} \|y_n\|.
\]

This implies that \( \{ Jy_n \} \) is bounded. Assume that \( \{ Jy_n \} \) converges weakly to \( y^* \in E^* \). In view of the reflexivity of \( E \), we see that \( J(E) = E^* \). This shows that there exists an element \( u \in E \) such that \( Jy = y^* \).

It follows that \( \phi(x_{n+1}, y_n) + 2\langle x_{n+1}, Jy_n \rangle = \|x_{n+1}\|^2 + \|Jy_n\|^2 \). Taking \( \liminf_{n \to \infty} \), one has \( 0 \geq \|\tilde{x}\|^2 - 2\langle \tilde{x}, y^* \rangle + \|y^*\|^2 = \|\tilde{x}\|^2 + \|Jy\|^2 - 2\langle \tilde{x}, Jy \rangle = \phi(\tilde{x}, y) \geq 0 \). That is, \( \tilde{x} = y \), which in turn implies that \( J\tilde{x} = y^* \). Hence, \( Jy_n \rightharpoonup J\tilde{x} \in E^* \). Using the KK property, we obtain \( \lim_{n \to \infty} Jy_n = J\tilde{x} \). Since \( J^{-1} \) is demi-continuous and \( E \) has the KK property, one gets \( y_n \rightharpoonup \tilde{x} \), as \( n \to \infty \). Using the restriction on \( \{ \alpha_n \} \), one has \( \lim_{n \to \infty} \|Jx_n - JT^*u_n\| = 0 \). This implies that \( \lim_{n \to \infty} \|JT^*u_n - J\tilde{x}\| = 0 \). Since \( J^{-1} \) is demi-continuous, one has \( T^*u_n \rightharpoonup \tilde{x} \).

**Step 6.** Prove \( \tilde{x} \in \text{Sol}(B) \).

Since \( \alpha_n \phi(x_{n+1}, u_n) \leq \phi(x_{n+1}, x_1) + \xi_n \), one has \( \lim_{n \to \infty} \phi(x_{n+1}, u_n) = 0 \). Hence, one has \( \lim_{n \to \infty}(\|u_n\| - \|x_{n+1}\|) = 0 \). This implies that
\[
\|\tilde{x}\| = \|J\tilde{x}\| = \lim_{n \to \infty} \|Ju_n\| = \lim_{n \to \infty} \|u_n\|.
\]

This implies that \( \{ Ju_n \} \) is bounded. Assume that \( \{ Ju_n \} \) converges weakly to \( u^* \in E^* \). In view of the reflexivity of \( E \), we see that \( J(E) = E^* \). This shows that there exists an element \( u \in E \) such that \( Ju = u^* \).

It follows that
\[
\phi(x_{n+1}, u_n) + 2\langle x_{n+1}, Ju_n \rangle = \|x_{n+1}\|^2 + \|Ju_n\|^2.
\]

Taking \( \liminf_{n \to \infty} \), one has
\[
0 \geq \|\tilde{x}\|^2 - 2\langle \tilde{x}, u^* \rangle + \|u^*\|^2 = \|\tilde{x}\|^2 + \|Ju\|^2 - 2\langle \tilde{x}, Ju \rangle = \phi(\tilde{x}, u) \geq 0.
\]
That is, $\bar{x} = u$, which in turn implies that $u^* = J\bar{x}$. Hence, $Ju_n \to J\bar{x} \in E^*$. Using the KK property, we obtain $\lim_{n \to \infty} Ju_n = J\bar{x}$. Since $J^{-1}$ is demi-continuous and $E$ has the KK property, one gets $u_n \to \bar{x}$, as $n \to \infty$. Since

$$r_n B(y, u_n) + \langle u_n - y, Ju_n - Jy_n \rangle \geq 0, \forall y \in C_n,$$

we see that $B(y, \bar{x}) \leq 0$. Let $0 < t < 1$ and define $y_t = ty + (1 - t)\bar{x}$. It follows that $y_t \in C$, which yields that $B(y_t, \bar{x}) \leq 0$. It follows from the (Q1) and (Q4) that

$$0 = B(y_t, y_t) \leq tB(y_t, y) + (1 - t)B(y_t, \bar{x}) \leq tB(y_t, y).$$

That is, $B(y_t, y) \geq 0$. It follows from (Q3) that $B(\bar{x}, y) \geq 0, \forall y \in C$. This implies that $\bar{x} \in Sol(B)$. This completes the proof that $\bar{x} \in Sol(B) \cap Fix(T)$.

**Step 7.** Prove $\bar{x} = \text{Proj}_{Sol(B) \cap Fix(T)} x_1$.

Note the fact $\langle w - x_n, Jx_1 - Jx_n \rangle \leq 0, \forall w \in Sol(B) \cap Fix(T)$. It follows that

$$\langle x - w, Jx_1 - J\bar{x} \rangle \geq 0, \quad \forall w \in Fix(T) \cap Sol(B).$$

Using Lemma 1.3, we find that that $\bar{x} = \text{Proj}_{Fix(T) \cap Sol(B)} x_1$. This completes the proof. $\square$

From Theorem 2.1 the following results are not hard to derive.

**Corollary 2.2.** Let $E$ be a smooth, strictly convex, and reflexive Banach space such that both $E$ and $E^*$ have the KK property and let $C$ be a convex and closed subset of $E$. Let $B$ be a bifunction satisfying (Q1), (Q2), (Q3) and (Q4). Assume that $Sol(B) \neq \emptyset$. Let $\{x_n\}$ be a sequence generated by

$$\begin{cases} x_0 \in E \text{ chosen arbitrarily}, \\ C_1 = C, x_1 = \text{Proj}_{C_1} x_0, \\ r_n B(u_n, \mu) \geq \langle u_n - \mu, Ju_n - Jx_n \rangle, \mu \in C, \\ Jy_n = \alpha_n Ju_n + (1 - \alpha_n)Jx_n, \\ C_{n+1} = \{z \in C_n : \phi(z, x_n) \geq \phi(z, y_n)\}, \\ x_{n+1} = \text{Proj}_{C_{n+1}} x_1, \end{cases}$$

where $\{\alpha_n\}$ is a real sequence in $[a, 1], a \in (0, 1]$ is a real number and $\{r_n\} \subset [r, \infty)$ is a real sequence, where $r$ is some positive real number. Then $\{x_n\}$ converges strongly to $\text{Proj}_{Sol(B)} x_1$.

**Corollary 2.3.** Let $E$ be a Hilbert space and let $C$ be a convex and closed subset of $E$. Let $B$ be a bifunction satisfying (Q1), (Q2), (Q3) and (Q4) and let $T$ be an asymptotically quasi-nonexpansive mapping in the intermediate sense on $C$. Assume that $T$ is uniformly asymptotically regular and closed and $Fix(T) \cap Sol(B) \neq \emptyset$. Let $\{x_n\}$ be a sequence generated by

$$\begin{cases} x_0 \in E \text{ chosen arbitrarily}, \\ C_1 = C, x_1 = P_C x_0, \\ r_n B(u_n, \mu) \geq \langle u_n - \mu, u_n - x_n \rangle, \mu \in C, \\ y_n = \alpha_n Tu_n + (1 - \alpha_n)x_n, \\ C_{n+1} = \{z \in C_n : \|z - x_n\|^2 + \xi_n \geq \|z - y_n\|^2\}, \\ x_{n+1} = P_{C_{n+1}} x_1, \end{cases}$$

where $\xi_n = \max\{\sup_{p \in Fix(T), x \in C} \left(\|p - T^n x\|^2 - \|p - x\|^2\right), 0\}, \{\alpha_n\}$ is a real sequence in $[a, 1], a \in (0, 1]$ is a real number, and $\{r_n\} \subset [r, \infty)$ is a real sequence, where $r$ is some positive real number. Then $\{x_n\}$ converges strongly to $P_{Fix(T) \cap Sol(B)} x_1$. 
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