COMMON FIXED POINTS FOR D-MAPS SATISFYING INTEGRAL TYPE CONDITION

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Abstract. In this paper, we obtain two common fixed point theorems one for two pairs of single and set-valued mappings and another for four set-valued mappings satisfying integral type conditions.

1. Introduction and preliminaries

Recently Ali and Imdad \cite{8} obtained some common fixed point theorems for four self maps using implicit relations in a metric space. Branciari \cite{4} introduced integral type contractive conditions and proved a fixed point theorem for a self map on a metric space. Based on this concept, Bouhadjera and Djoudi \cite{3} proved common fixed point theorems for pairs of single and set-valued D-maps satisfying an integral type condition. In this paper, we obtain a theorem different from that of \cite{3} and obtain a generalization of a theorem of \cite{8}. We also obtain common fixed point theorems for four set-valued mappings and obtain a generalization of theorems of \cite{8} and \cite{2}.

In the sequel, we need the following

Let \((X, d)\) be a metric space and \(B(X)\), the set of all nonempty bounded subsets of \(X\). For \(A, B \in B(X)\), define \(\delta(A, B) = \sup\{d(a, b) : a \in A, b \in B\}\).

If \(A = \{a\}\), then we write \(\delta(A, B) = \delta(a, B)\) and also if \(B = \{b\}\) then we write \(\delta(A, B) = d(a, b)\).

From the definition of \(\delta(A, B)\), we have \(\delta(A, B) = \delta(B, A) \geq 0\),
\( \delta(A, B) = 0 \) iff \( A = B = \{a\} \), \( \delta(A, B) \leq \delta(A, C) + \delta(C, B) \),
\( \delta(A, A) = \text{diam}A \) for all \( A, B, C \in B(X) \).

**Definition 1.1.** ([6]): A sequence \( \{A_n\} \) of nonempty subsets of \( X \) is said to be convergent to a subset \( A \) of \( X \) if
(i) each point \( a \) in \( A \) is the limit of a convergent sequence \( \{a_n\} \), where \( a_n \) is in \( A_n \) for \( n \in \mathbb{N} \),
(ii) for arbitrary \( \epsilon > 0 \), there exists an integer \( m \) such that \( A_n \subseteq A \) for \( n > m \), where \( A_n \)
denotes the set of all points \( x \in X \) for which there exists a point \( a \in A \), depending on \( x \),
such that \( d(x, a) < \epsilon \). \( A \) is then said to be the limit of the sequence \( \{A_n\} \).

**Lemma 1.2.** ([6]): If \( \{A_n\} \) and \( \{B_n\} \) are sequences in \( B(X) \) converging to \( A \) and \( B \) in \( B(X) \), respectively, then the sequence \( \{\delta(A_n, B_n)\} \) converges to \( \delta(A, B) \).

**Lemma 1.3.** ([7]): Let \( \{A_n\} \) be a sequence in \( B(X) \) and \( y \) be a point in \( X \) such that \( \delta(A_n, y) \to 0 \). Then the sequence \( \{A_n\} \) converges to the set \( \{y\} \) in \( B(X) \).

**Definition 1.4.** ([9]): The maps \( f : X \to X \) and \( F : X \to B(X) \) are weakly compatible or coincidentally commuting (some authors call it as subcompatible) if \( \{t \in X/\text{Ft} = \{ft\}\} \subseteq \{t \in X/\text{Ft} = \text{Ft}\} \).

The following definition is an extension of (E.A.) property due to Aamri and Moutawakil [1].

**Definition 1.5.** ([5]): The maps \( f : X \to X \) and \( F : X \to B(X) \) are said to be D-maps if there exists a sequence \( \{x_n\} \) in \( X \) such that \( \lim f x_n = t \) and \( \lim F x_n = \{t\} \) for some \( t \in X \).

Recently in 2008, Bouhadjera and Djoudi [3] proved the following:

**Theorem 1.6.** (Theorem 2.1 of [3]): Let \( f, g \) be self maps of a metric space \( (X, \phi) \) and let \( F, G : X \to B(X) \) be two set-valued maps such that
(1.6.1) \( FX \subseteq gX \) and \( GX \subseteq fX \),
(1.6.2) \[ \int_0^\phi \left( \delta(Fx, Gy), d(fx, gy), \delta(fx, Fx), \delta(gy, Gy), \delta(fx, Gy), \delta(gy, Fx) \right) \varphi(t) dt \leq 0 \]
for all \( x, y \in X \), where \( \phi : R_+ \to R \) is a continuous function satisfying
(i) \( \int_0^\phi \varphi(t) dt \leq 0 \) implies \( u = 0 \),
(ii) \( \int_0^\phi \varphi(t) dt \leq 0 \) implies \( u = 0 \),
(iii) \( \int_0^\phi \varphi(t) dt > 0 \) for all \( u > 0 \) and
\( \varphi : R_+ \to R \) is a Lebesgue-integrable map which is summable.
(1.6.3)(a) \( f \) and \( F \) are subcompatible D-maps; \( g \) and \( G \) are subcompatible and \( FX \) is closed,
(or)
(1.6.3)(b) \( g \) and \( G \) are subcompatible D-maps; \( f \) and \( F \) are subcompatible and \( GX \) is closed.
Then \( f, g, F \) and \( G \) have a unique common fixed point \( t \in X \) such that \( Ft = Gt = \{ft\} = \{gt\} = \{t\} \).

In this paper we prove a slight variation theorem of the above theorem using more general contractive condition.
2. Main results

First implicit relation:
Let \( \phi : R^4_+ \rightarrow R \) be a lower semi continuous function satisfying
\[
\int_0^{\phi(u,u,u,u)} \varphi(t) dt \leq 0 \text{ implies } u = 0 \text{, where } \varphi : R_+ \rightarrow R \text{ is a Lebesgue-integrable map which is summable.}
\]
Now we give some examples.
(i) Let \( \phi(t_1,t_2,t_3,t_4) = t_1 - k \max\{t_2,t_3,t_4\} \), where \( k \in [0,1] \) and \( \varphi(t) = t \) or \( \varphi(t) = \frac{3\pi}{4(1+t^2)} \cos(\frac{3\pi t}{4(1+t^2)}) \) for all \( t \in R_+ \).
Then \( \phi(u,u,u,u) = (1-k)u \).
Case: Suppose \( \varphi(t) = t \).
Then \( \int_0^{\phi(u,u,u,u)} \varphi(t) dt \leq 0 \) implies \( \frac{1}{2}(1-k)^2u^2 \leq 0 \) so that \( u \leq 0 \). But \( u \geq 0 \). Hence \( u = 0 \).
Case: Suppose \( \varphi(t) = \frac{3\pi}{4(1+t^2)} \cos(\frac{3\pi t}{4(1+t^2)}) \).
Then \( \int_0^{\phi(u,u,u,u)} \varphi(t) dt \leq 0 \) implies \( \sin(\frac{3\pi(1-k)u}{4(1+(1-k)u)}) \leq 0 \) so that \( u = 0 \) since
\[
0 \leq \frac{3\pi(1-k)u}{4(1+(1-k)u)} < \pi.
\]
The following \( \phi \) functions satisfy the first implicit relation with \( \varphi(t) = t \) for all \( t \in R_+ \) or \( \varphi(t) = \frac{3\pi}{4(1+t^2)} \cos(\frac{3\pi t}{4(1+t^2)}) \).
(ii) \( \phi(t_1,t_2,t_3,t_4) = t_1 - k (\max\{t_2,t_3,t_4\})^\frac{1}{2} \), where \( k \in [0,1] \).
(iii) \( \phi(t_1,t_2,t_3,t_4) = t_1^3 - \alpha \max\{t_2^2,t_3^2,t_4^2\} - \beta \max\{t_2t_3,t_3t_4\} \), where \( \alpha, \beta \geq 0 \) such that \( \alpha + \beta < 1 \).
(iv) \( \phi(t_1,t_2,t_3,t_4) = t_1^3 - \alpha \max\{t_it_jt_k/i,j,k \in \{2,3,4\}\} \), where \( \alpha \in [0,1] \).

Theorem 2.1. Let \( f, g \) be self maps of a metric space \( (X,d) \) and let \( F,G : X \rightarrow B(X) \) be two set-valued maps such that
\[
(2.1.1) \quad \int_0^{\phi(u,u,u,u)} \varphi(t) dt \leq 0 \text{ for all } x, y \in X, \text{ where } \phi : R^4_+ \rightarrow R \text{ is a lower semi continuous function satisfying}
\]
\( \varphi : R_+ \rightarrow R \text{ is a Lebesgue-integrable map which is summable,} \)
\[
(2.1.2) \quad (f,F) \text{ and } (g,G) \text{ are subcompatible pairs,}
\]
\[
(2.1.3)(a) \quad (f,F) \text{ is a pair of } D\text{-maps} \Rightarrow Fx \subseteq g(X) \:\forall x \in X \text{ and } f(X) \text{ is closed (or)}
\]
\[
(2.1.3)(b) \quad (g,G) \text{ is a pair of } D\text{-maps} \Rightarrow Gx \subseteq f(X) \:\forall x \in X \text{ and } g(X) \text{ is closed.}
\]
Then \( f,g,F \) and \( G \) have a unique common fixed point in \( X \).

Proof. Suppose (2.1.3)(a) holds.
Since \( (f,F) \) is a pair of \( D\)-maps, there exists a sequence \( \{x_n\} \) in \( X \) such that \( \lim f x_n = t \) and \( \lim F x_n = \{t\} \) for some \( t \in X \).
Since \( Fx \subseteq g(X) \:\forall x \in X \), there exists \( \alpha_n \in F x_n \) and \( y_n \in X \) such that \( \alpha_n = gy_n \:\forall n \).
Also \( d(gy_n,t) = d(\alpha_n,t) \leq d(F x_n,t) \rightarrow 0 \) as \( n \rightarrow \infty \).
Suppose \( \lim G y_n = A \). Now
\[
\int_0^\phi \left( \delta(Fx_n, Gy_n), d(fx_n, gy_n) + \delta(fx_n,Fx_n) + \delta(gy_n, Gy_n), \right) \varphi(t) dt \leq 0
\]
Letting \( n \to \infty \), we get
\[
\int_0^\phi \left( \delta(t, A), \delta(t, A), \delta(t, A) \right) \varphi(t) dt \leq 0
\]
Hence \( \delta(t, A) = 0 \) so that \( A = \{t\} \). Thus \( \lim G y_n = \{t\} \).

Since \( f(X) \) is closed, there exists \( u \in X \) such that \( t = fu \). Now,
\[
\int_0^\phi \left( \delta(Fu, Gy_n), d(fu, gy_n) + \delta(fu,Fu) + \delta(gy_n, Gy_n), \right) \varphi(t) dt \leq 0
\]
Letting \( n \to \infty \), we get
\[
\int_0^\phi \left( \delta(Fu, t), \delta(Fu, t), \delta(Fu, t) \right) \varphi(t) dt \leq 0
\]
Hence \( \delta(Fu, t) = 0 \) so that \( Fu = \{t\} \). Thus \( Fu = \{t\} = \{fu\} \).

Since \( \{t\} = Fu \subseteq g(X) \), there exists \( w \in X \) such that \( t = gw \). Now,
\[
\int_0^\phi \left( \delta(Fx_n, Gw), d(fx_n, gw) + \delta(fx_n,Fx_n) + \delta(gw, Gw), \right) \varphi(t) dt \leq 0
\]
Letting \( n \to \infty \), we get
\[
\int_0^\phi \left( \delta(t, Gw), \delta(t, Gw), \delta(t, Gw) \right) \varphi(t) dt \leq 0
\]
Hence \( \delta(t, Gw) = 0 \) so that \( Gw = \{t\} \). Thus \( Gw = \{t\} = \{gw\} \).

Since \( (f, F) \) is subcompatible, we have \( Ft = Ffu = fFu = \{ft\} \). Now,
\[
\int_0^\phi \left( \delta(Ft, Gw), d(ft, gw) + \delta(ft, Ft) + \delta(gw, Gw), \right) \varphi(t) dt \leq 0
\]
which implies
\[
\int_0^\phi \left( \delta(Ft, t), \delta(Ft, t), \delta(Ft, t) \right) \varphi(t) dt \leq 0
\]
Hence \( \delta(Ft, t) = 0 \) so that \( Ft = \{t\} \). Thus \( Ft = \{t\} = \{ft\} \).

Since \( (g, G) \) is subcompatible, we have \( Gt = Ggw = gGw = \{gt\} \). Now,
\[
\int_0^\phi \left( \delta(Fu, Gt), d(fu, gt) + \delta(fu, Fu) + \delta(gt, Gt), \right) \varphi(t) dt \leq 0
\]
which implies

\[ \int_0^\phi \left( \delta(t, Gt), \delta(t, Gt), \delta(t, Gt), \delta(t, Gt) \right) \varphi(t) dt \leq 0 \]

Hence \( \delta(t, Gt) = 0 \) so that \( Gt = \{t\} \). Thus \( Gt = \{t\} = \{gt\} \).

Thus \( t \) is a common fixed point of \( F, G, f \) and \( g \). Uniqueness of common fixed point follows easily from (2.1.1). Similarly, we can prove the theorem if (2.1.3)(b) holds. \( \square \)

Let \( \Psi_6 \) denote the set of all lower semicontinuous functions \( \psi : R^6_+ \rightarrow R \) satisfying

(i) \( \psi(t, 0, t, 0, t) > 0 \ \forall t > 0 \),
(ii) \( \psi(t, 0, t, 0, t) > 0 \ \forall t > 0 \),
(iii) \( \psi(t, t, 0, t, t) > 0 \ \forall t > 0 \).

Clearly the conditions (i),(ii) and (iii) imply \( \phi(t, t, t, t, t, t) \leq 0 \) if we define \( \phi(t_1, t_2, t_3, t_4, t_5, t_6) = \phi(t_1, t_2 + t_3 + t_4, t_3 + t_5, t_4 + t_6) \).

We observe that \( \phi(t, t, t, t, 0) \leq 0 \Rightarrow t = 0 \) need not imply(i),(ii),(iii) if we take \( \phi(t_1, t_2, t_3, t_4) = t_1 - k \max\{t_2, t_3, t_4\} \), where \( k \in [0,1) \) and \( \psi(t_1, t_2, t_3, t_4, t_5, t_6) = \phi(t_1t_2, t_2t_3, t_3t_4, t_4t_5) \).

Clearly \( \psi(t, 0, t, 0, 0, t) = \phi(0, 0, 0, 0, 0) = 0 \).

Theorem 2.1 is a generalization of the following

**Theorem 2.2.** (Theorem 3.3,[8]): Let \( A, B, S \) and \( T \) be self mappings of a metric space \((X, d)\) satisfying

\[
\psi(d(Ax, By), d(Sx, Ty), d(Sx, Ax), d(Ty, By), d(Sx, By), d(Ty, Ax)) \leq 0
\]

for all \( x, y \in X \), where \( \psi \in \Psi_6 \).

Suppose that (2.2.2) the pair \((A, S) \) ( or \((B, T) \) ) has Property(E.A.),

(2.2.3) \( A(X) \subseteq T(X) \) ( or \( B(X) \subseteq S(X) ) \),

(2.2.4) \( S(X) \) ( or \( T(X) \) ) is a closed subset of \( X \) and

(2.2.5) the pairs \((A, S) \) and \((B, T) \) are weakly compatible.

Then \( A, B, S \) and \( T \) have a unique common fixed point.

**Proof.** Let \( F = \{A\}, G = \{B\}, f = S, g = T \) be single valued mappings and \( \varphi(t) = 1 \) for all \( t > 0 \) in Theorem 2.1. Define \( \psi(t_1, t_2, t_3, t_4, t_5, t_6) = \phi(t_1 + t_2 + t_3 + t_4, t_3 + t_5, t_4 + t_6) \).

Clearly the conditions (i),(ii),(iii) on \( \psi \) imply that \( \phi(t, t, t, t) \leq 0 \) implies that \( t = 0 \). The rest follows from Theorem 2.1. \( \square \)

Now we prove a common fixed point theorem for four set-valued mappings.

**Theorem 2.3.** Let \( F, G, f \) and \( g \) : \( X \rightarrow B(X) \) be set- valued mappings satisfying

\[
\int_0^\phi \left( \delta(Fx, Gy), \delta(fx, gy) + \delta(fx, Fx), \delta(gy, Gy) \right) \varphi(t) dt \leq 0
\]

for all \( x, y \in X \), where \( \phi \) and \( \varphi \) are as in Theorem 2.1,

(2.3.2)(a) Suppose that there exists a sequence \( \{x_n\} \) in \( X \) such that \( \{Fx_n\} \) and \( \{fx_n\} \) converge to the same limit \( \{z\} \) for some \( z \in X \). ( or )
Hence \( \delta \) implies \( \delta \).

Letting \( n \to \infty \), we get

\[
\int_0^\phi \left( \delta(Fx_n, Gv), \delta(fx_n, gv) + \delta(fx_n, Fx_n) + \delta(gv, Gv), 
\delta(fx_n, Fx_n) + \delta(fx_n, Fx_n) + \delta(gv, Gv) \right) \varphi(t)dt \leq 0
\]

Hence \( \delta(z, Gv) = 0 \) so that \( Gv = \{z\} \). Thus \( Gv = \{z\} = gv \).

Since \((g, G)\) is coincidentally commuting, we have \( Gz = Gv = gGv = gz = \text{singleton} \) from (2.3.5). Now,

\[
\int_0^\phi \left( \delta(Fx_n, Gz), \delta(fx_n, gz) + \delta(fx_n, Fx_n) + \delta(gz, Gz), 
\delta(fx_n, Fx_n) + \delta(fx_n, Fx_n) + \delta(gz, Gz) \right) \varphi(t)dt \leq 0
\]

Hence \( \delta(z, Gz) = 0 \) so that \( Gz = \{z\} \). Thus \( Gz = \{z\} = gz \).

which implies

\[
\int_0^\phi \left( \delta(Fu, z), \delta(Fu, z), \delta(Fu, z), \delta(Fu, z) \right) \varphi(t)dt \leq 0
\]

Hence \( \delta(Fu, z) = 0 \) so that \( Fu = \{z\} \). Thus \( Fu = \{z\} = fu \).

Since \((f, F)\) is coincidentally commuting, we have \( Fz = Fu = fFu = fz = \text{singleton} \) from (2.3.5). Now,

\[
\int_0^\phi \left( \delta(Fz, Gz), \delta(fz, gz) + \delta(fz, Fz) + \delta(gz, Gz), 
\delta(fz, Fz) + \delta(fz, Gz), \delta(gz, Gz) + \delta(gz, Fz) \right) \varphi(t)dt \leq 0
\]
Similarly we can show that

\[ \text{Case : Suppose } \phi \]

\[ \text{Let } \delta \]

\[ \text{Proof.} \]

\[ \text{Theorem 2.4.} \]

Hence \( \delta(Fz,z) = 0 \) so that \( Fz = \{ z \} \). Thus \( Fz = \{ z \} = fz \). Thus \( z \) is a common fixed point of \( F, G, f \) and \( g \). Uniqueness of common fixed point follows easily from (2.3.1).

Suppose \( f = \{ w \} = \{ w \} \) for some \( w \in X \).

\[ \int_0^\infty \left( \delta(Fw,Gz), \delta(fw,gz) + \delta(fw,Fw) + \delta(gz,Gz), \right) \varphi(t)dt \leq 0 \]

which implies

\[ \int_0^\infty \left( d(w,z), d(w,z) \right) \varphi(t)dt \leq 0 \]

Hence \( d(w,z) = 0 \) so that \( w = z \). Thus \( z \) is the unique common fixed point of \( f \) and \( F \).

Similarly we can show that \( z \) is the unique common fixed point of \( g \) and \( G \). Similarly, we can prove the theorem when (2.3.2)(b) holds.

\[ \square \]

Theorem 2.3 is a generalization of the following

**Theorem 2.4.** (Theorem 3.1,[8]) Let \( A, B, S \) and \( T \) be self mappings of a metric space \( (X,d) \) satisfying (2.2.1) of Corollary (2.2). Suppose that

(2.4.1) the pairs \( \{ A, S \} \) and \( \{ B, T \} \) enjoy the common property (E.A.),

(2.4.2) \( S(X) \) and \( T(X) \) are closed subsets of \( X \),

(2.4.3) the pairs \( \{ A, S \} \) and \( \{ B, T \} \) are weakly compatible.

Then \( A, B, S \) and \( T \) have a unique common fixed point in \( X \).

**Proof.** Let \( F = \{ A \} \), \( G = \{ B \} \), \( f = \{ S \} \), \( g = \{ T \} \) be single valued mappings and \( \varphi(t) = 1 \) for all \( t > 0 \) in Theorem 2.3. Define \( \psi(t) = \delta(t_1,t_2+t_3+t_4,t_5+t_6) \). From (2.4.1), there exist sequences \( \{ x_n \} \) and \( \{ y_n \} \) in \( X \) such that

\[ \lim Ax_n = \lim Sx_n = \lim By_n = \lim T y_n = z \]

for some \( z \in X \).

From (2.4.2), there exist \( u, v \in X \) such that \( z = Su = Tv \). The rest follows from Theorem 2.3. \( \square \)

**Second implicit relation :**

Let \( \phi : R_+^5 \to R \) be an upper semi continuous function satisfying

\[ \int_0^{\phi(0,u,u,u,u)} \varphi(t)dt \geq 0 \text{ or } \int_0^{\phi(u,u,u,u,u)} \varphi(t)dt \geq 0 \implies u = 0 \]

where \( \varphi : R_+ \to R \) is a Lebesgue-integrable map which is summable.

Now, we give some examples .

(i) Let \( \phi(t_1,t_2,t_3,t_4,t_5) = t_1 - k \min\{ t_2,t_3,t_4,t_5 \} \), where \( k > 1 \) and \( \varphi(t) = t^2 \) or \( \varphi(t) = \frac{3\pi}{4(1-t)} \cos \left( \frac{3\pi t}{4(1-t)} \right) \) for all \( t \in R_+ \).

Case : Suppose \( \varphi(t) = t^2 \).

Then \( \int_0^{\phi(0,u,u,u,u)} \varphi(t)dt \geq 0 \implies -\frac{1}{3}k^3u^3 \geq 0 \implies u \leq 0 \).

But \( u \geq 0 \). Hence \( u = 0 \).

Case : \( \varphi(t) = \frac{3\pi}{4(1-t)} \cos \left( \frac{3\pi t}{4(1-t)} \right) \).
Then \( \int_0^{\phi(0,u,u,u,u)} \varphi(t)dt \geq 0 \Rightarrow \sin(\frac{3\pi ku}{4(1+ku)}) \geq 0 \Rightarrow \sin(\frac{3\pi ku}{4(1+ku)}) \leq 0 \Rightarrow u = 0 \) since \( 0 \leq \frac{3\pi ku}{4(1+ku)} < \pi \).

The following \( \phi \) functions satisfy the second implicit relation with \( \varphi(t) = t^2 \) or \( \varphi(t) = \frac{3\pi}{4(1-t^2)} \cos(\frac{3\pi t}{4(1-t)}) \) for all \( t \in R_+ \).

(ii) \( \varphi(t_1, t_2, t_3, t_4, t_5) = t_1 - at_2 - b \frac{(t_2 t_3 + t_3 t_4)}{(t_3 + t_4)} \), where \( a \geq 0, b \geq 0 \) with \( a + b > 1 \).

(iii) \( \varphi(t_1, t_2, t_3, t_4, t_5) = t_1 - \alpha t_2 - \beta \min\{t_3, t_4\} - \gamma \min\{t_2 + t_3, t_4 + t_5\} \), where \( \alpha, \beta, \gamma > 0 \) with \( \alpha + \beta + 2\gamma > 1 \).

Finally, we state the following theorem with expansive condition for four set - valued mappings.

**Theorem 2.5.** Theorem 2.3 holds if the inequality (2.3.1) is replaced by (2.5.1)

\[
\int_0^{\phi(0,u,u,u,u)} \varphi(t)dt \geq 0 \Rightarrow \sin(\frac{3\pi ku}{4(1+ku)}) \geq 0 \Rightarrow \sin(\frac{3\pi(k-1)u}{4(1+(k-1)u)}) \leq 0 \Rightarrow u = 0.
\]

for all \( x, y \in X \), where \( \phi : R_+^5 \longrightarrow R \) is an upper semi continuous function satisfying \( \int_0^{\phi(0,u,u,u,u)} \varphi(t)dt \geq 0 \) or \( \int_0^{\phi(0,u,u,u,u)} \varphi(t)dt \geq 0 \) implies \( u = 0 \) and \( \varphi \) is as in Theorem 2.1.

**Remark 2.6:** Theorem 2.5 with \( f \) and \( g \) as single valued mappings is a generalization of Theorem 3.1 of [2].

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