A hybrid Fuzzy MCDM Approach to Thesis Subject Selection

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Abstract

One of the most significant decisions that a doctoral student may make in the beginning of his/her research career is the selection of a thesis subject. There are also a number of conflicting criteria, including supervisor reputation, convergence of interest, student’s interest and motivation, etc., which should be considered as a part of the appropriate thesis subject selection process. For this purpose, multiple criteria decision making (MCDM) methods have been found to be useful approaches to solve this kind of problem. This paper devises a fuzzy hybrid analytical hierarchy process (AHP) and technique for order preference by similarity to ideal solution (TOPSIS) approach to the problem of thesis subject selection. Fuzzy AHP is used to calculate the weight of each criterion, and fuzzy TOPSIS is proposed to prioritize thesis subjects from the best to the worst ones. The application of fuzzy set theory allows incorporating the vague and imprecise linguistic terms into the decision process.

Keywords: Thesis subject, Fuzzy TOPSIS, Fuzzy AHP

1. Introduction

Selecting an appropriate thesis subject is one of the most complicated and time consuming problems for doctoral students, due to many feasible alternatives and conflicting objectives. The determination and evaluation of positive and negative characteristics of one alternative relative to others is a difficult task. Consequently choosing proper thesis subject in a doctorate course can improve the writing process, provide effective utilization of resources, increase productivity and enhance motivation, effort and supervisor aid.

Earning the doctorate, the highest academic degree, is a major educational, professional, and personal accomplishment, requiring years of diligent work [29]. The hard won letters “PhD” behind a person’s name signify that he or she is now capable of independent scholarly work [29].
Nowadays we have been encountered by the increasing number in international conferences and the increasing number of Doctor of Philosophy (PhD) students enrolling or graduating each year [23], with many subjects that everyone can think about them for research. For example, in the UK, the numbers of doctorates awarded in 2002 was 10,660, compared with a figure of almost half that amount (7559) in 1995 [30]. Science cannot advance unless research findings are effectively and widely communicated [10] and in this advance, PhD thesis has enormous roles. Writing a thesis/dissertation is a most formidable task for many graduate students [11]. This is not only because of the daunting size of the document but also because of the high standard to which the thesis/dissertation is held [11]. The dissertation is a key tool for teaching examining and publishing ideas. The doctoral dissertation is the capstone event of doctoral study [27]. While developing their dissertations, graduate students discover the cutting edge of the discipline, learn the intricacies of the research process, and engage in scientific dialogue with experts in the field [27]. The dissertation supervisor has a tremendous impact on the topic selection, the format decision, and the writing of the literature review but selecting one subject among subjects that fit criteria for supervisor and student is so important. Several problems, such as poor completion rates of research degrees [4] and delayed completion of thesis [13], have been found in work related to thesis in postgraduate and higher levels of education. For example, Rudd found that 40% to 50% of post graduate students in UK failed to complete their dissertation in social sciences [28]. There are many books on how to get a PhD [2], [5], [12], [24]. Most of these books emphasis the institutional and organizational forms necessary for the accomplishment of a PhD. Some of these books also discuss how to choose a research topic [2], [5] and research methodology [5], [12], presumably after the problem statement, the purpose and significance of the study have been discussed [23]. The suitable selection of thesis subject is a crucial component in the eventual success or failure of the doctoral dissertation. Many papers have been written about choosing research question and thesis subject but an important element is choosing appropriate subject among subjects that a student can choose. Many potential criteria, such as interest and motivation, job prospect, supervisor reputation, etc., should be considered in the selection procedure of a thesis subject; hence, thesis subject selection can be viewed as a multiple criteria decision making (MCDM) problem in the presence of many quantitative and qualitative criteria.

The MCDM methods deal with the process of making decisions in the presence of multiple criteria or objectives. A decision maker (DM) is required to choose among quantifiable or non-quantifiable and multiple criteria. The DM's evaluations on qualitative criteria are often subjective and imprecise. The objectives are usually conflicting and therefore the solution is highly dependent on the preferences of the DM. Besides, it is very difficult to develop a selection criterion that can precisely describe the preference of one alternative over another. The evaluation data of thesis subject alternatives suitability for various subjective criteria, and the weights of the criteria are usually expressed in linguistic terms. This makes fuzzy logic a more natural approach to this kind of problems. The technique for order preference by similarity to ideal solution (TOPSIS) is one of the well known classical MCDM methods. TOPSIS is a widely accepted multi criteria decision making technique due to its sound logic, simultaneous consideration of the ideal and the anti-ideal solutions, and easily programmable computation procedure. This technique is based on the concept that the ideal alternative has the best level for all criteria, whereas the negative ideal is the one with all the worst criteria values. In fuzzy TOPSIS, criteria values are represented by fuzzy numbers [8]. Also linguistic preferences can be easily converted to fuzzy numbers and TOPSIS allows using these fuzzy numbers in the calculation. There are many weight calculation procedures, but the AHP has some advantages. One of the most important advantages of the AHP is based on pairwise comparison [21]. Besides, the AHP calculates the inconsistency index which is the ratio of the DM's inconsistency [21]. However, sometimes large number of pairwise comparisons performed by DMs can cause impractical usage of the AHP process, especially in fuzzy AHP. To cope with this problem, the fuzzy TOPSIS technique can be used to reduce the number of pairwise comparisons and to rank
the alternatives. There are many papers about dissertation in the literature. Some of them focus on the problems for writing in another language \cite{11}. Others talk about accessibility of theses in and out of university \cite{22}. Some papers offer change in writing thesis in some areas toward 10 years ago \cite{14}, \cite{20}. As mentioned above, various researchers have studied to determine some problems for the writing dissertations. These papers always make recommendations and do not use any mathematical models. Ideally a student should know the key attributes he/she wishes to see in his/her thesis subject and their relative importance, and then be able to choose the subject that best fits his/her priorities. There are no studies that have looked into the method of thesis subject selection, and this is where this study hopes to fill the gap. In this study, evaluation of thesis subject alternatives is modeled. Considering strategic implications of the thesis selection decisions by using the AHP method, this study proposes a combined fuzzy AHP and fuzzy TOPSIS methodology for evaluating and selecting most suitable thesis subject alternatives for a student as a real world application. Fuzzy TOPSIS is used to select a thesis subject alternative and the fuzzy AHP is applied to calculate criteria weights.

The rest of the paper is organized as follows. Section 2 presents the proposed methodology accompanied by a theoretic description of fuzzy AHP procedure and fuzzy TOPSIS procedure. In section 3, a real-world case study is illustrated to prove the applicability of the proposed method. The results of ranking are also discussed in this section. Finally, concluding remarks are presented in section 4.

2. Proposed model

In this paper, we proposed a combined fuzzy AHP and fuzzy TOPSIS approach for thesis subject selection problem (see Fig. 1). In step 1, the interested areas for thesis subject are determined. In steps 2 and 3, the supervisor for each area is determined, and negotiation with supervisors is started by using face-to-face interview, respectively. In a further attempt, all the criteria for evaluation are determined by using three constant. In the step 4, the criteria for thesis selection are defined, and then, in step 5, the decision matrix is constructed. According to these criteria, the required data utilized in the comparisons are collected from the related DM again. After constructing the evaluation criteria hierarchy, in step 6, the criteria weights are calculated by applying the fuzzy AHP method. The performances of the alternatives corresponding to the criteria are performed under the setting of fuzzy set theory. In step 7, fuzzy TOPSIS is employed to achieve the final ranking results. The detailed descriptions of the major steps are elaborated in the following subsections.

![Fig. 1 A two phased methodology for thesis subject selection process](image-url)
2.1. Fuzzy AHP procedure
In the proposed methodology, the AHP with its fuzzy extension, namely fuzzy AHP, is applied to obtain more decisive judgments by prioritizing the thesis subject selection criteria and weighting them in the presence of vagueness. This study concentrates on a fuzzy AHP approach introduced by Chang [6], in which triangular fuzzy numbers are preferred for pairwise comparison scale. Extent analysis method is selected for the synthetic extent values of the pairwise comparisons. Some papers were published by Kahraman et al. [15], [16] used the fuzzy AHP procedure based on extent analysis method and showed how it can be applied to selection problems. A fuzzy number is a special fuzzy set \( F = \{ (x, \mu_F(x), x \in \mathbb{R} \} \), where \( x \) takes its values on the real line, \( R: -\infty \leq x \leq \infty \) and \( \mu_F(x) \) is a continuous mapping from \( R \) to the closed interval \([0, 1]\). A triangular fuzzy number (TFN) expresses the relative strength of each pair of elements in the same hierarchy and can be denoted as \( M = (l, m, u) \), where \( l \leq m \leq u \). The parameters \( l; m; u; \) indicate the smallest possible value, the most promising value, and the largest possible value respectively in a fuzzy event. Triangular type membership function of \( M \) fuzzy number can be described as in Eq. 1.

\[
\mu_M(x) = \begin{cases} 
0 & x < l \\
\frac{(x - l)}{(m - l)} & 1 \leq x \leq m \\
\frac{(u - x)}{(u - m)} & m \leq x \leq u \\
0 & x > u
\end{cases}
\]

When \( l = m = u \), it is a non fuzzy number by convention. A linguistic variable is a variable whose values are expressed in linguistic terms. The concept of a linguistic variable is very useful in dealing with situations, which are too complex or not well defined to be reasonably described in conventional quantitative expressions [17], [32], [33]. In this study, the linguistic variables that are utilized in the model can be expressed in positive TFNs for each criterion as in Fig. 2.

The linguistic variables matching TFNs and the corresponding membership functions are provided in Table 1. Proposed methodology employs a Likert Scale of fuzzy numbers starting from 1 to 9 symbolized with tilde (~) for the fuzzy AHP approach. Table 1 depicts AHP and fuzzy AHP comparison scale considering the linguistic variables that describes the importance of criteria and alternatives to improve the scaling scheme for the judgment matrices.
By using TFNs via pairwise comparison, the fuzzy judgment matrix $\tilde{A}(a_{ij})$ can be expressed mathematically as in Eq. 2:

$$
\tilde{A} = \begin{bmatrix}
1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\
\tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & 1
\end{bmatrix}
$$

The judgment matrix $\tilde{A}$ is an $n \times n$ fuzzy matrix containing fuzzy numbers $\tilde{a}_{ij}$.

$$
\tilde{a}_{ij} = \begin{cases} 
1, & i = j \\
(1, 3, 5, 7, 9) & \text{for } i < j \\
(1, 1, 3, 5, 7) & \text{for } i > j
\end{cases} 
$$

Let $X = \{x_1, x_2, \ldots, x_n\}$ be an object set, whereas $U = \{u_1, u_2, \ldots, u_n\}$ is a goal set. According to fuzzy extent analysis, the method can be performed with respect to each object for each corresponding goal, $g_i$, resulting in $m$ extent analysis values for each object, given as $M_{g_1}^1, M_{g_2}^1, \ldots, M_{g_m}^n, i = 1, 2, \ldots, n$, where all the $M_{g_j}^i (j = 1, 2, \ldots, m)$ are TFNs representing the performance of the object $x_i$ with regard to each goal $u_j$. The steps of Chang's extent analysis [6] can be detailed as follows [1, 15, 16]:

**Step 1:** The fuzzy synthetic extent value with respect to the $i$th object is defined as:

$$
S_i = \sum_{j=1}^{m} M_{g_j}^i \otimes \sum_{j=1}^{m} \left(\sum_{j=1}^{m} M_{g_j}^i\right)^{-1} 
$$

To obtain $\sum_{j=1}^{m} M_{g_j}^i$, perform the fuzzy addition operation $m$ extent analysis values for a particular matrix such that operation $m$ extent analysis values for a particular matrix such that

$$
\sum_{j=1}^{m} M_{g_j}^i = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right)
$$

and obtain $\left(\sum_{j=1}^{m} M_{g_j}^i\right)^{-1}$, perform the fuzzy addition operation of $M_{g_j}^i$ ($j = 1, 2, \ldots, m$) values and such that

$$
\sum_{j=1}^{m} M_{g_j}^i = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right)
$$

and then compute the inverse of the vector in Eq. 6 such that

$$
\left(\sum_{j=1}^{m} M_{g_j}^i\right)^{-1} = \left(\frac{1}{\sum_{j=1}^{m} l_j}, \frac{1}{\sum_{j=1}^{m} m_j}, \frac{1}{\sum_{j=1}^{m} u_j}\right)
$$

**Step 2:** The degree of possibility of $M_2 \geq M_1$ is defined as:

$$
V(M_2 \geq M_1) = \sup \left[\min (\mu_{M_1}(x), \mu_{M_2}(y)) \right] \quad (8)
$$

$\gamma \geq \chi$

and can be equivalently expressed as follows:
\[ V(M_2 \geq M_1) = \text{hght}(M_1 \cap M_2) = \mu_{M2}(d) = \begin{cases} 1, & \text{if } m_2 \geq m_1, \\ 0, & \text{if } l_1 - u_2, \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise} \end{cases} \tag{9} \]

where \( d \) is the ordinate of the highest intersection point \( D \) between \( \mu_{M_1} \) and \( \mu_{M_2} \). To compare \( M_1 \) and \( M_2 \), both the values of \( V(M_2 \geq M_1) \) and \( V(M_2 \geq M_2) \) are required.

**Step 3:** The degree possibility of a convex fuzzy number to be greater than \( k \) convex fuzzy numbers \( M_i \) \( (i = 1, 2, \ldots, k) \) can be defined by Eq. 10.

\[ V(M \geq M_1, M_2, \ldots, M_k) = V(M \geq M_1) \text{and } V(M \geq M_2) \text{ and } \ldots \text{ and } V(M \geq M_k) = \min V(M \geq M_i) \]

\( i = 1, 2, 3, \ldots, k. \tag{10} \)

Assume that:

\[ d'(Ai) = \min V(S_i \geq S_k) \tag{11} \]

For \( k = 1, 2, \ldots, n; k \neq i \). Then, the weight vector is given by as in Eq. 12:

\[ W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T \tag{12} \]

![Fig. 3 Intersection point “d” between two fuzzy numbers \( M_1 \) and \( M_2 \)](image)

where \( A_i \) \( (i = 1, 2, \ldots, n) \) has \( n \) elements.

**Step 4:** The normalized weight vectors are defined as:

\[ W = (d(A_1), d(A_2), \ldots, d(A_n))^T \tag{13} \]

where \( W \) is a non fuzzy number.

### 2.2. Fuzzy TOPSIS procedure

In the following subsection, some basic important definitions of fuzzy sets from Zimmermann [33], Buckley [3], Zadeh [32], Kaufmann and Gupta [17], Yang and Hung [31] and Chen et al. [9] are reviewed and summarized. It is often difficult for a DM to assign a precise performance rating to an alternative for the criteria under consideration. The advantage of using a fuzzy approach is to assign the relative importance of criteria using fuzzy numbers instead of precise numbers. This subsection extends TOPSIS to the fuzzy environment.

Let \( \tilde{a} = (l_1, m_1, u_1) \) and \( \tilde{b} = (l_2, m_2, u_2) \) be two TFNs, then the vertex method is defined to calculate the distance between them, as Eq. 14:

\[ d(\tilde{a}, \tilde{b}) = \frac{1}{\sqrt{2}} \left[ (l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2 \right] \tag{14} \]

The problem can be described by following sets:

(i) A set of \( J \) possible candidates called \( A = \{A_1, A_2, \ldots, A_J\} \)

(ii) A set of \( n \) criteria, \( C = \{C_1, C_2, \ldots, C_n\} \)

(iii) A set of performance ratings of \( A_j \) \( (j = 1, 2, 3, \ldots, J) \) with respect to criteria \( C_i \) \( (i = 1, 2, 3, \ldots, n) \) called \( X = \{x_{ij} \mid i = 1, 2, 3, \ldots, n, \ j = 1, 2, 3, \ldots, J\} \)

(iv) A set of importance weights of each criterion \( w_i \) \( (i = 1, 2, 3, \ldots, n) \)

Considering the different importance values of each criterion, the weighted normalized fuzzy-decision matrix is constructed as:

\[ \tilde{V} = [\tilde{v}_{ij}]_{n \times J} \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, J \text{ where } \tilde{v}_{ij} = x_{ij} \cdot w_i \tag{15} \]
According to the briefly summarized fuzzy theory above, fuzzy TOPSIS steps can be outlined as follows:

**Step 1:** Choose the linguistic ratings \( \tilde{x}_{ij} \ (i = 1, 2, \ldots, n; j = 1, 2, \ldots, f) \) for alternatives with respect to criteria. The fuzzy linguistic rating \( \tilde{x}_{ij} \) preserves the property that the ranges of normalized TFNs belong to \([0,1] \); thus, there is no need for normalization. Let \( \tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}) \), \( \tilde{x}_{ij}^+ = (a_{ij}^+, b_{ij}^+, c_{ij}^+) \) and \( \tilde{x}_{ij}^- = (a_{ij}^-, b_{ij}^-, c_{ij}^-) \). We have

\[
\tilde{r}_{ij} = \begin{cases} 
\tilde{x}_{ij} (\pm) \tilde{x}_{ij}^+ = \frac{a_{ij}}{c_{ij}^+}, b_{ij}, c_{ij} \quad \text{if } C_i \text{ is benefit criterion } & \quad c_{ij}^+ = \max_i c_{ij} \\
\tilde{x}_{ij} (\pm) \tilde{x}_{ij}^- = \frac{a_{ij}^-}{c_{ij}^-}, b_{ij}^-, c_{ij} \quad \text{if } C_i \text{ is cost criterion } & \quad a_{ij}^- = \min_i a_{ij} 
\end{cases} 
\tag{16}
\]

**Step 2:** Calculate the weighted normalized fuzzy decision matrix. The weighted normalized \( \tilde{v}_{ij} \) value calculated by Eq. 15.

**Step 3:** Identify positive ideal \( (A^+) \) and negative ideal \( (A^-) \) solutions. The fuzzy positive ideal solution \( (FPIS, A^+) \) and the fuzzy negative ideal solution \( (FNIS, A^-) \) are shown in Eqs. 17 and 18.

\[
A^+ = \{\tilde{v}_{i1}, \tilde{v}_{i2}, \ldots, \tilde{v}_{im} \} = \left\{ \left( \frac{\max v_{ij}}{j}, \frac{\min v_{ij}}{j} \right) \mid i = 1, 2, \ldots, n; j = 1, 2, \ldots, f \right\} 
\tag{17}
\]

\[
A^- = \{\tilde{v}_{i1}, \tilde{v}_{i2}, \ldots, \tilde{v}_{im} \} = \left\{ \left( \frac{\min v_{ij}}{j}, \frac{\max v_{ij}}{j} \right) \mid i = 1, 2, \ldots, n; j = 1, 2, \ldots, f \right\} 
\tag{18}
\]

where \( I' \) is associated with benefit criteria and \( I'' \) is associated with cost criteria.

**Step 4:** Calculate the distance of each alternative from \( A^+ \) and \( A^- \) using Eqs. 19 and 20.

\[
D_j^+ = \sum_{i=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_{i1}^+) \quad j = 1, 2, \ldots, f 
\tag{19}
\]

\[
D_j^- = \sum_{i=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_{i1}^-) \quad j = 1, 2, \ldots, f 
\tag{20}
\]

**Step 5:** Calculate similarities to ideal solution

\[
CC_j = \frac{D_j^-}{D_j^+ + D_j^-} \quad j = 1, 2, \ldots, f 
\tag{21}
\]

**Step 6:** Rank preference order. Choose an alternative with maximum \( CC_j^+ \) or rank alternatives according to \( CC_j^+ \) in descending order.

### 3. Case study

A real world case problem is selected to illustrate the application of the proposed approach. The selected case is a student in a university of Tehran, in Iran. Therefore a two phased methodology was designed to tackle with the problems that could be seen in the selection process (see Fig. 1). First the student defines the specific areas of interest for thesis research, including management, marketing and industrial engineering. Then define supervisors for these areas. In step 3, the student visits all supervisors, negotiates with them to define all theses subject and talks about possibility acceptance of them for research in PhD degree. The student could get acceptance for working with three supervisors. After that she negotiates with them and defines all theses subject. Student uses face-to-face interview with each supervisor. Finally the student defines four subjects (alternatives) with the three different supervisors which are denoted as \( A_1, A_2, A_3 \) and \( A_4 \) respectively. In step 4, first, a detailed questionnaire related with the data regarding the qualitative and quantitative criteria for the thesis subject selection model was prepared. Twenty two criteria including Freedom to work , Time conscious, Job prospect, Convergence of interest, Reputation/Subject knowledge/Publications, Personal relationship with the professor, Social networks, Can take a stand, Number of thesis guided, Commitment and involvement, Student’s interest and motivation, Student’s Reputation/Subject knowledge/Publications, Access to resources (such as other theses, books, papers, etc) and Collecting data, Publish in high quality journal, Chance to finish on time, Subject’s importance, Subject’s creativity, Subject’s flexibility, Subject’s up to date, Acceptance chance for higher
degree, income, cost are determined. The first ten criteria are about supervisor and were applied by Ray [26]. After a set of interviews, by using three constant, nine criteria were determined to perform the analysis. Because of importance of supervisor in thesis, six criteria are about supervisor and three of them are about student. Nine criteria are: Freedom to work, Convergence of interest, Reputation/Subject knowledge/Publications, Personal relationship with the professor, Social networks, Commitment and involvement, Student's interest and motivation, Publish in high quality journal, Subject's importance and creativity, which are denoted as $C_1$, $C_2$, $C_3$, ..., $C_9$, respectively. Freedom to work means that the professor is open to ideas; he/she is flexible about adopting alternative approaches and have free time for discussion about thesis. It’s so important for asking questions. Convergence of interest means the matching of interest of the student and the professor. Reputation/Subject knowledge/Publications are basically related to the reputation of the professor in his or her field.

### Table 2. Pairwise comparisons of selection criteria via TFN

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>Priority Weight (W)</th>
</tr>
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<tr>
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<td>1,3,5</td>
<td>1/7,1/5,1/3</td>
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<td>1/7,1/5,1/3</td>
<td>0.087</td>
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<td>1,3,5</td>
<td>1/5,1/3,1</td>
<td>1,1,1</td>
<td>0.173</td>
</tr>
</tbody>
</table>

Number of thesis guided by the professor is good criteria for this survey or number of his/her papers. Personal relationship with the professor is defined as intimate and understanding relationship with the professor. Social networks mean the professors’ social network and relationship with other professors in the institute and outside of the university. Commitment and involvement is defined as professors’ enthusiasm in guiding the thesis, emotional investment and supervisor's effort. Student's interest and motivation is defined student tension, need and scale of effort for terminate the thesis. Publish in high quality journal means that the subject has how much chance for publishing in top related journals. It is related to the quality of thesis and importance of subject. It also is a key factor for getting PhD and reputation. Subject’s importance and creativity is basically a factor that student must consider for define the subject. It’s related to the student researches in interested fields before defining subject and Subject’s creativity is defined as how much creativity is in thesis subject. It relates to the student viewpoint. We consider them as a one criterion. After determining all selection criteria and alternative theses subject, the paired comparisons were made by using the TFNs to tackle the
ambiguities involved in the process of the linguistic assessment of the data. The student filled this table 2 for reaching importance of the alternatives and criteria.

3.1. Results

The aim of using fuzzy AHP is to determine importance weight of the criteria that will be employed in fuzzy TOPSIS method. Table 2 depicts the pairwise comparison matrix set by TFNs that matches linguistic statements of data. The fuzzy values of paired comparison were converted to crisp values via the Chang’s extent analysis as mentioned before. First, the fuzzy synthetic extent values were calculated by using Eq. 4 with the help of Eqs.5–7. Equations 8–9 were applied to express the degree of synthetic extent values. To have a weight vector given by as in Eq.12, Eqs.10–11 were applied by comparing the fuzzy numbers. After normalizing weight vector defined as in Eq. 13, the obtained priority weight vector of criteria is figured out in the last column of Table 2. Phase I is called as the fuzzy AHP phase essentially. Phase II, which is called as the fuzzy TOPSIS phase, starts establishing fuzzy evaluations of the alternative theses subject (A1, A2, A3, A4) with respect to the individual criteria by using TFNs again.

| Table 3: The comparison of alternatives according to the criteria |
|-------------------|---|---|---|
|                  | A1      | A2      | A3      | A4      |
| C1                | (6,7,8) | (4,5,6) | (1,1,1) | (8,9,10) |
| C2                | (4,5,6) | (6,7,8) | (6,7,8) | (2,3,4)  |
| C3                | (8,9,10)| (4,5,6) | (2,3,4) | (6,7,8)  |
| C4                | (2,3,4) | (2,3,4) | (8,9,10)| (6,7,8)  |
| C5                | (4,5,6) | (8,9,10)| (6,7,8) | (4,5,6)  |
| C6                | (6,7,8) | (4,5,6) | (2,3,4) | (4,5,6)  |
| C7                | (2,3,4) | (8,9,10)| (6,7,8) | (2,3,4)  |
| C8                | (2,3,4) | (6,7,8) | (8,9,10)| (2,3,4)  |
| C9                | (4,5,6) | (6,7,8) | (8,9,10)| (2,3,4)  |

This is a decision matrix for ranking alternatives and indicates the performance ratings of the alternatives according to the criteria. We use the linguistic scales and their corresponding fuzzy numbers: (1,1,1)-very poor , (2,3,4)-poor, (4,5,6)-fair, (6,7,8)-good, (8,9,10)-very good. Table 3 shows the comparison of alternatives according to the criteria. After constructing decision matrix, normalized decision matrix is calculated. The normalized decision matrix is obtained by using Eq.16. In our calculation, all the criteria are defined as benefit criteria. As an example, the maximum value of the criterion ‘C1’ is fuzzy numbers (8,9,10) on alternative A4. The normalization calculation for alternative A1 is,

\[
(6, 7, 8)/(10, 10, 10) = (6/10, 7/10, 8/10) = (0.6, 0.7, 0.8).
\]

The weighted normalized fuzzy decision matrix can be obtained multiplying the normalized decision matrix by the weights of the criteria matrix (Table 2) which is found by using fuzzy AHP. Table 4 shows weighted normalized decision matrix. The positive ideal solution (A+) and negative ideal solution (A−) are determined by using the weighted normalized values. Equations 17–18 are used to determine the positive ideal solution and negative ideal solution. The positive TFNs are in the range[0,1]. Hence the fuzzy positive ideal reference point (FPIS,A+) is (1, 1, 1) and fuzzy negative ideal reference point (FNIS,A−) is (0, 0, 0). In the last step, the relative closeness to the ideal solution is calculated. The relative closeness to the ideal solution is defined on Eqs.19–20. Equation 14 is used to calculate distances to ideal solutions. Table 5 summarizes the results. The higher the closeness means the better the rank, so the relative closeness to the ideal solution of the alternatives can be substituted as follows:

\[
A_4 > A_3 > A_2 > A_1\]

A4 is defined as the best alternative for this selection that a student can choose for thesis subject in PhD.
4. Concluding remarks

One of the most important problems for a PhD student is choosing appropriate thesis subject. In this paper, two phased methodology based on fuzzy AHP and fuzzy TOPSIS for selecting the most suitable thesis subject was proposed, whereas TOPSIS used fuzzy AHP result weights as input weights. For dealing uncertainty and improving lack of precision in evaluating criteria and thesis subject alternatives, triangular fuzzy numbers were used. In TOPSIS method, the score option can provide better insight to the DM by taking into account both the differences and similarities of the alternatives according to the best and the worse alternatives. The fuzzy numbers enabled DM to get better results in the overall importance of criteria and real alternatives. As a result of the study, we find that the proposed method is practical for ranking thesis subject alternatives with respect to multiple conflicting criteria.

References


Table 4. Weighted normalized decision matrix

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<tr>
<th>Alternative</th>
<th>A₁</th>
<th>A₂</th>
<th>A₃</th>
<th>A₄</th>
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</thead>
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<tr>
<td>C₁</td>
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<td>0.010</td>
<td>0.012</td>
<td>0.006</td>
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<td>C₂</td>
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<tr>
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<td>0.050</td>
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<tr>
<td>C₄</td>
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<td>0.029</td>
<td>0.039</td>
<td>0.019</td>
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<tr>
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<td>0.004</td>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>C₆</td>
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<td>0.071</td>
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<tr>
<td>C₇</td>
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<td>0.119</td>
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<tr>
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<td>C₉</td>
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Table 5. The results

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<tr>
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<th>D⁻</th>
<th>CC⁺</th>
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<tr>
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</table>


