Cellular Automata Approach in Optimum Shape of Concrete Arches under Dynamic Loads

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Abstract
Traditional methods in determination of optimum shape of structures don’t scale well. This paper discusses the application of cellular automata (CA) to study of optimum shape in concrete arches under dynamic loads by cellular automata and presents a novel approach for that. In this paper, samples of semi-circular, obtuse angel, four- centered pointed, Tudor, ogee, equilateral, catenaries, lancet and four-centered arches are modeled. Then they are analyzed and optimized under acceleration–time components of Elcentro earthquake. Using cellular automata model and provided rules, the mentioned arches are analyzed and optimized. The results of error range and time of analysis in cellular automata model and FEM software compared. According the results, in CA method, precision is less but it has less time of analysis and optimization.

Keywords: optimum shape, arch, concrete, dynamic load, tensile stress, cellular automata.

1. Introduction
A growing interest in the preservation of heritage and historic structures has created a need for efficient method for analysis of arches that are the principal components of these structures. Arch is the oldest type of construction and have been around for thousands years. They were originally built of stone or brick but now built of reinforced concrete or steel [2]. Archs are mostly known as structures as the load carrying system. In the last decades, various studies have been conducted by researchers on the response and evaluation of masonry and concrete arch under different actions [5]. Concrete arches have been used to span covering of...
considerable length in many different applications. Structural efficiency is attributed to the curvature of the arch, which transfers vertical loads laterally along the arch to the abutments at each end [6].

There has been some research on brick masonry under dynamic loads [8]. Dynamic or time history analysis is an analytical method for determining reflections during the earthquake in structures, but the most important structural properties should be known prior to analysis of structure [4]. In the field of structural optimization, one of the modern method is CA. Dynamic analysis and optimization of arches need to consume a long time; it is necessary to use a proper computational model such as cellular automata to analyze and optimize the arches in less time and also for more acceptable results. Cellular automata is a decentralized computation model and one of the applications that require data processing is cellular automata in which a very large volume of data must be processed in a short time and the results of the processing must be reapplied to the automaton [11]. It is a good method for computation and simulation of complicated behaviors by local data [18].

At the present study, modeling, analyzing and optimizing complicated behaviors of semi-circular, obtuse angle, four-centered pointed, Tudor, ogee, equilateral, catenaries, lancet and four-centered arches, under dynamic load using cellular automata are conducted. Furthermore, the ability of analyzing and optimizing of every arch after one time of modeling in a so much shorter time is highlighted.

2. ARCH MODELING USING FEM SOFTWARE

By using FEM software, arch modeling has been conducted. Furthermore, dynamic analysis has been conducted applying north-south horizontal accelerations of Elcentro earthquake in which the time, maximum acceleration, maximum velocity and maximum displacement are 31.98(s), 0.31(g), 33 (cm/sec) and 21.4 (cm), respectively (fig.1) and SOLID65 is used for analysis in this stage. In FEM software, the base and top thickness, maximum tensile stress and weight of structure have been defined as design variable, state variable and objective function, respectively. Regarding the extra time for analysis and optimization, the optimization has been conducted in design optimum processor by means of Sub problem approximation method. This is an estimating method for variable designing, state and objective function via curve fitting tool. It is a general method for solving many engineering problems [9].

![Fig.1. North-south horizontal component of Elcentro earthquake](image-url)
2-1 Geometrical Modeling

According to shape optimization design variables, such as base thickness \((t_0)\) and top thickness \((t_1)\) as parameters, all the key points are defined as follows (fig.2):

- Point 1: \((0, 0)\)
- Point 2: \((R, 0)\)
- Point 3: \((-R, 0)\)
- Point 4: \((0, R)\)
- Point 5: \((R+t_0, 0)\)
- Point 6: \((-R-t_0, 0)\)
- Point 7: \((0, R+t_1)\)

![Fig.2. Geometrical model of semicircular arch](image)

In arch modeling, the tolerance increases because the thickness decreases from base to top [6]. It should mention that in modeled arch, the thickness decreases from base \((t_0)\) to top \((t_1)\) linearly and also arch thickness of axis is 20 (cm) in the length direction. The motion of support nodes is zero and dynamic force has no effect on them. Specifications of used concrete in arch modeling are shown at table 1. The efficient factors in the inelastic nonlinear analysis have been shown in table 2. In the present paper, arch radius limit \((R)\), maximum tensile stress, base and top thickness in optimum state are considered as 4-8 (m), 4000-4100 (KN/m²), 0.85-1.36(m) and 0.22-0.39(m) respectively for all modeled arch.

<table>
<thead>
<tr>
<th>Table 1. Concrete characteristics [3]</th>
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<td>Density ((\rho)) (Kg/m³)</td>
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<td>2400</td>
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<th>Table 2. Effective coefficient in non elastic and nonlinear analysis [3]</th>
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<tr>
<td>Allowable Tension Stress ((f_t)) N/mm²</td>
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<td>2.4</td>
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3. SHORT HISTORY, FORMAL DEFINITION AND PROPERTIES OF CELLULAR AUTOMAT

Cellular automata is the computational model which can simulate the process of growth by describing a complex system by simple individuals following simple rules. This concept of simulation growth was introduce by John Von Neumann [12] and subsequently developed by other researchers in many field of science [10, 1,118]. He was interested to male relation between new computational device - automata theory - and biology. His mind was preoccupied with generating property in natural events [14]. He proved that CA can be general. According to his findings, CA is a collection of cells with reversible states and ability of computation for every thing. Although Van rules were complicated and didn’t strictly satisfy computer program, but he continues his research in two parts: for decentralizing machine which is designed for simulation of desirable function and designing of a machine which is made by simulation of complicated function by CA [15]. Wolfram has conducted some research on problem modeling by the simplest and most practicable method of CA architecture too. In 1970, “The Game of Life” introduced by Conway and became very widely known soon. At the beginning of 1980, Wolfram studied one-dimension CA rules and demonstrated that these simple CAs can be used in modeling of complicated behaviors [16, 17].
3.1 Definition
CA is characterized by (a) cellular space (b) transfer rule [12].
For CA, cell, the state of cell in time t, sum of neighbors state at time t and neighborhood radius are denoted by i, \( S_i'^t \), \( \eta_i'^t \) and r, respectively. Also, the rule is function of \( \phi(\eta_i'^t) \).

3.2 Change state rules
Each cell changes its state, spontaneously. The primary quality of cells depends on primary situation of problem. By these primary situations, CA is a system which has certain behavior by local rules. The cells which are not neighbors, have no effect on each other. CA has no memory, so present state defines the next state [18].

Quadruple CA is as CA= (Q, d, V and \( \Phi \)), where Q, d, V and \( \Phi \) are collection of possible state, CA dimension, CA neighborhood structure and local transferring rule, respectively.

For 1-d CA, amount of i cell (\( 1 \leq i \leq n \)) at t is shown by \( a_i(t) \) and is calculated by this formula:

\[
a_{i}(t+1) = \Phi \left[ a_{i-1}(t), a_i(t), a_{i+1}(t) \right]
\]

In this formula, if \( \Phi \) is affected by the neighbors, it is general. If \( \Phi \) is a function of neighbor’s cell collection and central cell, it is totalistic.

\[
a_{i}(t+1) = \Phi \left[ a_{i-1}(t), a_i(t), a_{i+1}(t) \right]
\]

Fig. 3 shows an example of typical neighborhoods for one and dimensional cellular automata.

4. ARCH MODELING USING CA
According to definition of neighborhood radius and state reversal rule in three state 1-d CA, the data for each arch will be analyzed to find the rules of simulation of arch behavior. To achieve this aim, 50 to 200 samples of each arch radius, base and top thickness and maximum tensile stress were chosen and analyzed by two and three state algorithm (figure 4 defines two state algorithm completely). After one billion accomplishments, for 256 two-state rules and one million three-state rules, some models were provided for each arch. For example, figures 5 and 6 define semicircular rules and tensile stress efficiency, respectively.
Fig. 4. An algorithm for finding two-state 1-D cellular automata model for arch behavior modeling
5. TEST OF CELLULAR AUTOMATA MODEL

Maximum tensile stress for 50 to 200 samples (according to algorithm in figure 6) has been provided. The error percent has been compared with another analyzed model in FEM software.
Fig.6. An algorithm for analysis of arch behavior using two-state 1-D CA for maximum tensile stress

5.1 Test Of CA Model For Semicircular
Maximum tensile stress was achieved for 50 to 200 samples of semicircular arches by CA. Figure 7 define comparison between maximum tensile stress in FEM and CA model. The mean of error percent in semicircular arch is 12.485%. Figure 8 represent error percent of each sample.
Moreover, Fig. 9 illustrates the diagram of comparison between time of maximum tensile stress computation using CA and FEM software, respectively.

![Diagram showing comparison between maximum tensile stress using FEM software and CA model in semicircular arch](image1)

**Fig.7.** Comparison between maximum tensile stress using FEM software and CA model in semicircular arch

![Diagram showing error percent of maximum tensile stress computation by CA to FEM software in semicircular arch](image2)

**Fig.8.** Error percent of maximum tensile stress computation by CA to FEM software in semicircular arch
6. ARCH OPTIMIZATION USING CA

In this stage, by means of CA model for each arch top and base thickness were optimized. Considering optimized maximum tensile stress which is 4100(KN/m²), the range of radius, top thickness and maximum tensile stress in each arch are considered as input, so arch base thickness will be provided. In the next stage, size of arch radius, base thickness and maximum tensile strain are considered as input. So arch top thickness will be provided (arch base thickness optimization is defined in figure 10).
Fig. 10. Algorithm of arch thickness optimization using two-state 1-D cellular automata
6.1 TOP THICKNESS OPTIMIZATION IN SEMICIRCULAR ARCH USING CA

In this stage, 50 to 200 semicircular arch samples were chosen for top thickness optimization. Their optimum maximum tensile stress range, arch radius and base thickness were 4000 to 4100 (KN/m²), 4 to 8 meter and 0.85 to 1.36, respectively. Afterward, the top thickness was calculated and compared with top thickness in FEM software (fig.13). The mean of error percent of top thickness calculation was 12.339%. Figure 11 and 12 show error percent of each sample in CA toward FEM software and comparison of optimization time of top thickness optimization in semicircular arch.

![Figure 11. Comparison between maximum tensile stress of semicircular arch using FEM software and CA model](image)

![Figure 12. Error percent of top thickness optimization in semicircular arch using CA model towards FEM software](image)
6.2 Basic Thickness Optimization In Semicircular Arch Using CA

In this section, 50 to 200 semicircular arch samples were chosen for top thickness optimization. Their optimum maximum tensile stress range, arch radius and base thickness were 4000 to 4100(KN/ m²), 4 to 8 meter and 0.22 to 0.39, respectively. After calculation of base thickness according to algorithm in figure 10, the results were compared with base thickness in FEM software (fig.16). The mean of error percent of base thickness calculation was 13.74%. Figure 14 and 15 show error percent of each sample in CA toward FEM software and comparison of optimization time of base thickness optimization in semicircular arch.

![Figure 13](image)

Fig 13. Comparison of optimum range of arch top thickness using CA model and FEM software

![Figure 14](image)

Fig.14. Comparison between optimization time of base thickness in semicircular arch using FEM software and CA model
7. CONCLUSION
The approach presented in this paper aims to overcome the previous drawbacks. The principal novelty of this research is the use of a special class of evolutionary algorithms, called Cellular Automata.
In the present paper, nine arches- semi-circular, obtuse angle, four- centered pointed; Tudor, ogee, equilateral, catenaries, lancet and four-centered arches- were modeled using FEM software and CA model. Figures 17, 18 and 19 show analysis and optimization time, the results which are provided
by CA in arch modeling and the mean of error percent for arch analysis and its optimization, respectively. Considering the results, CA model can be used in simulation of all arches. Therefore, the time of calculation decreases. Also, it can be used in dynamic response, natural frequency and response of structure under different dynamic loads. To increase models precision, the rules which are larger than 1000000 and repeated more than 1000000000 times are needed.

Fig.17. Comparison between mean of analysis and optimization time of all discussed arches using FEM software and CA model
Fig. 18. Comparison between provided rules for discussed arches using cellular automata.

Fig. 19. Comparison between the mean of error percent of analysis of tensile stress and optimization of base and top thickness for discussed arches using CA model toward FEM software.

References


