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Analytical Solution for Nnonlinear Vibration of Micro-Electro-mechanical System (MEMS) by Frequency-Amplitude Formulation Method

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Abstract

In this paper, one analytical technique named Frequency-Amplitude Formulation Method (FAF) isused to obtain the behavior and frequency of theelectrostatically actuated microbeams. The main aim of the work is obtaining highly accurate analytical solution for nonlinear free vibration of a microbeam and investigates the dynamic behavior of the system. Results reveal that the nonlinear frequency of oscillatory system remarkably affected with the initial conditions. In contrast to the time marching solution results, the present analytical method is effective and convenient. It is predictable that the FAF can apply for various problems in engineering specially vibration equations.

1. Introduction

Micro-electro-mechanical systems (MEMS) are batch-fabricated devices and structures at a microscale level [1]. Since its inception, MEMS technologies are of tremendous importance in various engineering fields. Because of its small size, low power consumption and high reliability, we have seen many potential applications of MEMS actuators and sensors in aerospace, optical and biomedical engineering [2–4]. Generally, an archetypal electrostatic micro-switch is one of the significant MEMS devices, which can be modeled by an electrostatically driven microbeam and a pair of fixed electrodes. From a physical point of view, the dynamical motion of the microbeam can be governed by the electrostatic as well as intermolecular forces [5]. Due to a voltage difference between the electrode and the microbeam, the electrostatic actuation is created by the induced electrostatic charges. Besides, the intermolecular force accounts for the molecular interaction of the tiny gap size between the electrode and the microbeam.

In order to advance knowledge in micro/nanotechnology, the analysis of dynamic and stability responses of various engineering models [5–12] has thus attracted intensive research attention. Recently, Fu et al. [13] investigated the nonlinear oscillation problem arising in the MEMS microbeam model by means of the energy balance method. Because this problem [13] is strongly nonlinear when subjecting to large amplitudes of motion and physical parameters, so it is hardly amenable to analytically obtain an exact solution for such a problem.

Almost all oscillation problems in engineering are modeled by nonlinear differential equations. Obtaining exact solution for these nonlinear problems is a great propose but, in most cases it is difficult to achieve them. Therefore special techniques should be applied to solve them. Many of these techniques have been performed in recent literatures such as Homotopy Perturbation Method (HPM) [14-18], Homotopy Analysis Method (HAM) [19-21], Iteration Perturbation Method (IPM) [22-24], Variational Iteration Method (VIM) [25-28], Differential Transformation Method (DTM) [29-30], Frequency Amplitude Formulation (FAF) [31-33], Max-Min Approach (MMA) [34-38].

2. Mathematical model and solution approach

Consider a fully clamped microbeam with uniform thickness h, length l, width b ($b \geq 5h$),effective modulus $\bar{E} = \frac{E}{1-\upsilon^2}$, Young's modulus E, Poisson's ratio m and density q, as shown in Fig. 1. Employing theclassical beam theory and taking into account of the mid-plane stretching effect as well as the distributedelectrostatic force, the following dimensionless equation of motion for the microbeam can be formulated via the Galerkin method [13].

$$\ddot{u}\,(a_1 u^4 + a_2 u^2 + a_3) + a_4 u + a_5 u^3 + a_6 u^5 + a_7 u^7 = 0, \quad u(0) = A,\ \dot{u}(0) = 0 \tag{1}$$

where u is thedimensionless deflection of the microbeam, a dot denotes the derivative with respect to the dimensionless time variable $\bar{t} = t\sqrt{\frac{\bar{E}\,I}{\rho b h l^4}}$ with I and t being the second moment of area of the beam cross-section and time, respectively.The expressions of the governing parameters $a_i\ (i = 1-7)$ can be written as:

$$\begin{gathered}
a_1 = \int_0^1 \phi^6\, d\zeta,\ \ a_2 = -2\int_0^1 \phi^4\, d\zeta\ ,\ \ a_3 = \int_0^1 \phi^2\, d\zeta \\
a_4 = \int_0^1 (\phi'''' \phi - N\phi'' \phi - V^2 \phi^2)\, d\zeta \\
a_5 = -\int_0^1 (2\phi'''' \phi^3 - 2N\phi'' \phi^3 + \alpha\phi'' \phi \int_0^1 \phi'^2\, d\zeta)\, d\zeta \\
a_6 = \int_0^1 (\phi'''' \phi^5 - N\phi'' \phi^5 + 2\alpha\phi'' \phi^3 \int_0^1 \phi'^2\, d\zeta)\, d\zeta
\end{gathered} \tag{2}$$

$$a_7 = -\int_0^1 (\alpha\phi''\phi^5 \int_0^1 \phi'^2\, d\zeta)\, d\zeta$$

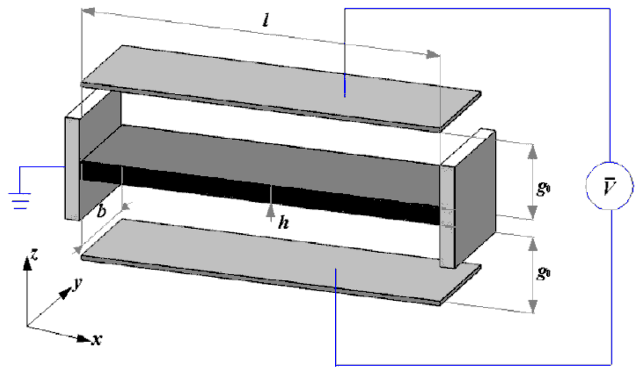

Fig. 1. Geometry for an electrostatically actuated microbeam with fixed supports

3. Application of frequency amplitude formulation

For solving Eq.(1) with frequency amplitude formulation, we use the trial functions $u_1(t) = A\cos(t)$ and $u_2(t) = A\cos(\omega t)$,which are the solutions of the following linear equations, respectively,

$$\ddot{u} + \varpi_1^2 u = 0, \quad \varpi_1^2 = 1 \tag{3}$$

$$\ddot{u} + \varpi_2^2 u = 0, \quad \varpi_2^2 = \omega^2 \tag{4}$$

The residuals are:

$$R_1(t) = (-a_3 A + a_4 A)\cos t + (-a_2 A^3 + a_5 A^3)\cos^3 t + (-a_1 A^5 + a_6 A^5)\cos^5 t + a_7 A^7 \cos^7 t \tag{5}$$

$$R_2(t) = (-a_3 A + a_4 A)\cos \omega t + (-a_2 A^3 + a_5 A^3)\cos^3 \omega t + (-a_1 A^5 + a_6 A^5)\cos^5 \omega t + a_7 A^7 \cos^7 \omega t \tag{6}$$

We introduce two new residual variables $\tilde{R}_1$ and $\tilde{R}_2$ define as[31-33]:

$$\tilde{R}_1 = \frac{4}{T_1}\int_0^{\frac{T_1}{4}} R_1(t)\cos t\, dt, \quad T_1 = 2\pi \tag{7}$$

$$\tilde{R}_2 = \frac{4}{T_2}\int_0^{\frac{T_2}{4}} R_2(t)\cos \omega t\, dt, \quad T_1 = \frac{2\pi}{\omega} \tag{8}$$

we can approximately determine ω^2 in the form,

$$\omega^2 = \frac{\tilde{R}_2 - \omega^2 \tilde{R}_1}{\tilde{R}_2 - \tilde{R}_1} \tag{9}$$

For the Eq.(1), by a simple calculation,we obtain:

$$\tilde{R}_1 = \frac{64(-a_3 A + a_4 A) + 48(-a_2 A^3 + a_5 A^3) + 40(-a_1 A^5 + a_6 A^5) + 35 A^7 a_7}{128} \tag{10}$$

and

$$\tilde{R}_2 = \frac{64(-a_3 \omega^2 A + a_4 A) + 48(-a_2 \omega^2 A^3 + a_5 A^3) + 40(-a_1 \omega^2 A^5 + a_6 A^5) + 35 A^7 a_7}{128} \tag{11}$$

Applying Eq.(9), we have:

$$\omega = \omega_{FAF} = \sqrt{\frac{-b - \sqrt{b^2 - 4ac}}{2a}} \tag{12}$$

Where a, b and c are as follows:

$$\begin{aligned} a &= -64 a_3 A - 48 a_2 A^3 - 40 a_1 A^5 \\ b &= 64 a_3 A + 64 a_4 A + 48 a_2 A^3 + 48 a_5 A^3 + 40 a_1 A^5 + 40 a_6 A^5 + 35 A^7 a_7 \\ c &= -64 a_4 A - 48 a_5 A^3 - 40 a_6 A^5 - 35 A^7 a_7 \end{aligned} \tag{13}$$

The periodic solution as follows:

$$u(t) = A\cos(\omega_{FAF} t) = A\cos(\sqrt{\frac{-b - \sqrt{b^2 - 4ac}}{2a}} t) \tag{14}$$

4. Results

In this section the result which obtained using the FAFis compared with the time marching results for the following parameters.

$A = 0.3,\ N = 10,\ \alpha = 24$

Figures 2 and 3 show the displacement of the microbeam(u(t))with FAF, EBM [13] and ODE-solver in MATLAB. Also, comparison between frequencies obtained by FAF and EBM are illustrated in figure 4.

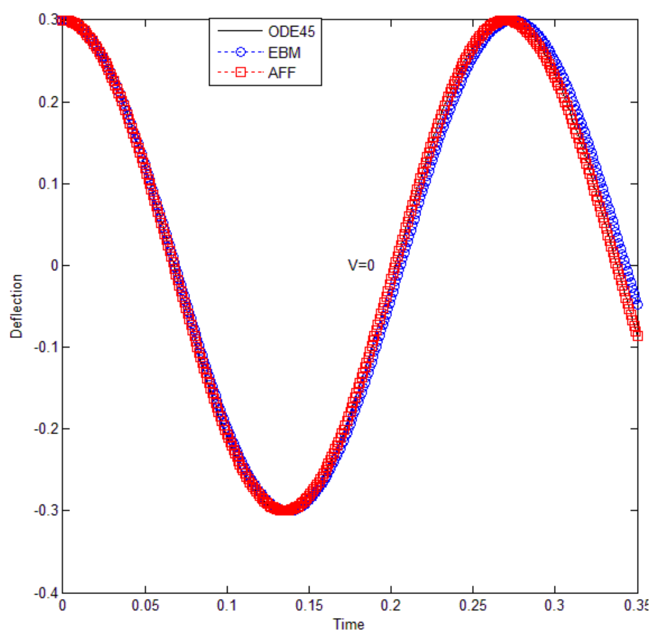

Fig2.Comparison between the FAF, EBM and ODE45 results of the tapered beam ($A = 0.3,\ V = 0$).

Figure 5 illustrates the error percentages of the analytical solution used in this work and presented in [13]. As time passes the FAF's error decrease with respect to the other analytical result. The error percentage of the analytical solutions results calculated from the following equation.

$$\%Error = \frac{\left|u_{Exact} - u_{Analytical}\right|}{u_{Exact}} \times 100 \tag{15}$$

Figure 6 shows the phase-plane of the microbeam for V=0and $A = 0.3$. In this figure the FAF and EBM compared with ODE45 solution. Moreover, Figure 7 shows the vibration behavior of the MEMS for different initial amplitudes.

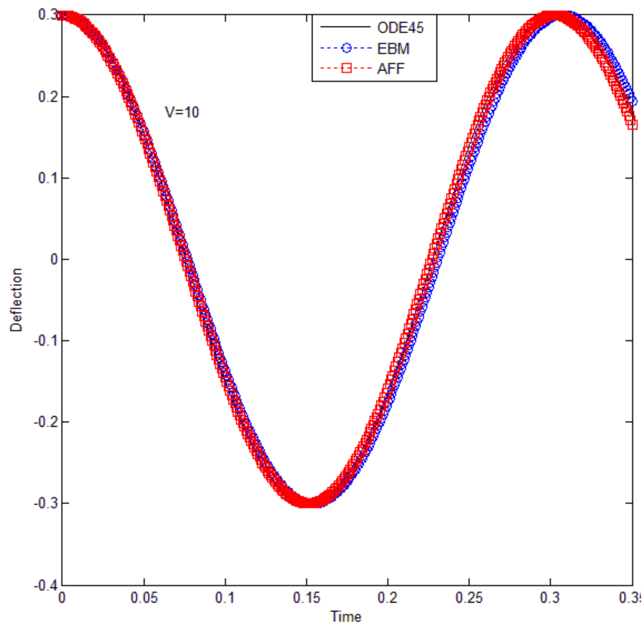

Fig3.Comparison between the FAF, EBM and ODE45 results of the tapered beam($A = 0.3$, $V = 10$).

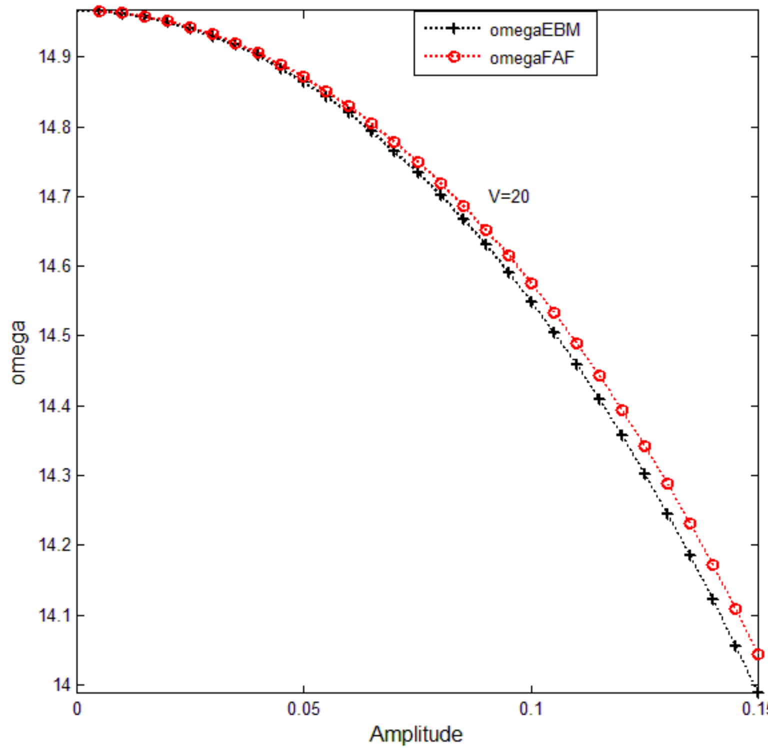

Fig 4. Nonlinear frequency versus amplitude for various V=20

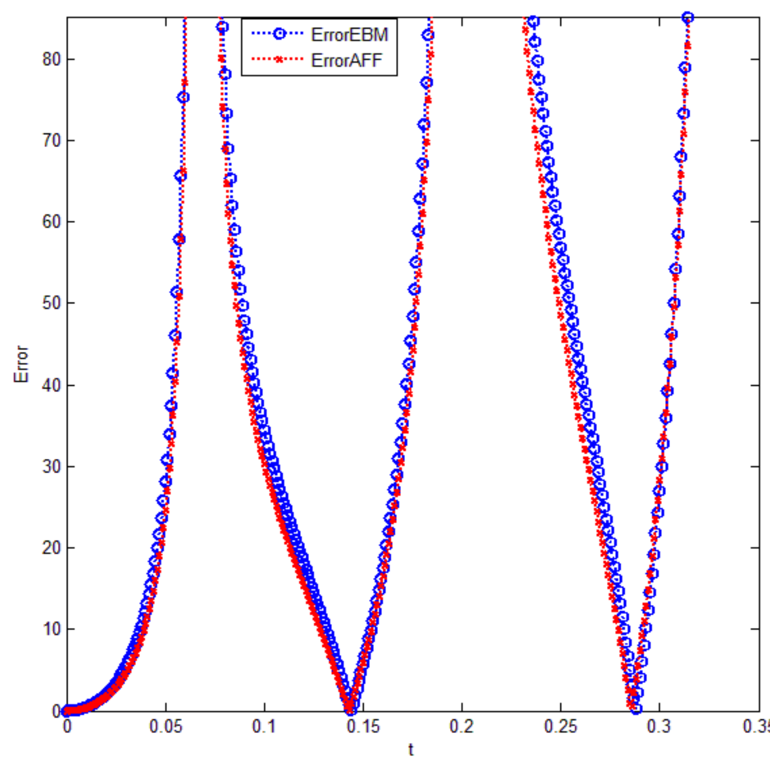

Fig 5. Error percentage for FAF and other analytical solution($A = 0.3$, $V = 0$).

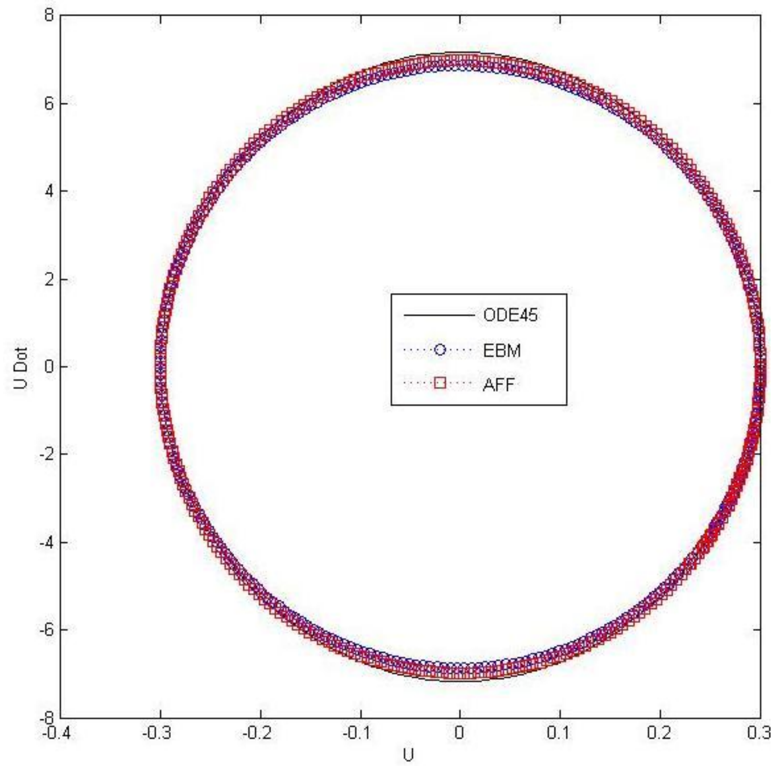

Fig 6. Phase plane of the system using exact solution and analytical solutions ($A = 0.3, V = 0$)

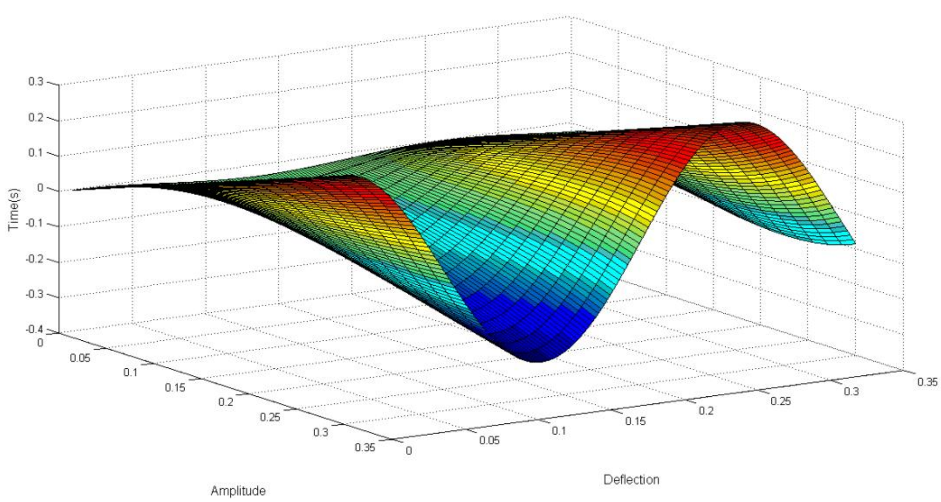

Fig 7. The vibration behavior of the tapered beam $A = 0.3, V = 10$

Conclusions

In this paper, the FAF was employed to solve the governing equation of nonlinear oscillations of microbeams. The results of FAF have excellent agreement with the results obtained by EBM and ODE45solution. The error percentage achieved by FAF decreased during the time against of the other analytical solution. This method is simple and doesn't need to programming but it is important to choose the correct frequency for solving some complicated problems. It can be approved that FAF is powerful and efficient technique in finding analytical solutions for a wide classes of nonlinear oscillator.

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