Contact pseudo-slant submanifolds of a Kenmotsu manifold

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Abstract

We study the geometry of the contact pseudo-slant submanifolds of a Kenmotsu manifold. Necessary and sufficient conditions are given for a submanifold to be a pseudo-slant submanifold, contact pseudo-slant product, mixed geodesic and totally geodesic in Kenmotsu manifolds. ©2016 All rights reserved.

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1. Introduction

Kenmotsu \cite{8} introduced a class of almost contact Riemannian manifolds known as Kenmotsu manifolds. In 1990, B. Y. Chen \cite{5,6} introduced the notion of slant submanifold, which is generalization of both the invariant and anti-invariant submanifolds. After that many research articles have been published by different authors on the existence of these submanifolds in different ambient spaces. The slant submanifolds of an almost contact metric manifolds were defined and studied by A. Lotta \cite{10}. After, these submanifolds were studied by J. L. Cabrerizo et al. \cite{4} of Sasakian manifolds.

The notion of semi-slant submanifolds of an almost Hermitian manifold was introduced by N. Papagiuc \cite{11}. Cabrerizo et al. studied and characterized slant submanifolds of K- contact and Sasakian manifolds and gave several examples of such submanifolds. Cabrerizo et al. \cite{4} defined and studied bi-slant immersions in almost contact metric manifolds and simultaneously gave the notion of pseudo-slant submanifolds. Pseudo-slant submanifolds also have been studied by Khan at al. in

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In this paper, we study contact pseudo-slant submanifolds of a Kenmotsu manifold. In section 2, we review basic formulas and definitions for a Kenmotsu manifold and their submanifolds. In section 3, we study the geometry of the contact pseudo-slant submanifolds of a Kenmotsu manifold. Necessary and sufficient conditions are given for a submanifold to be a contact pseudo-slant submanifold, contact pseudo-slant product, mixed geodesic and totally geodesic in Kenmotsu manifolds.

2. Preliminaries

In this section, we give some terminology and notations used throughout this paper. We recall some necessary fact and formulas from the theory of Kenmotsu manifolds and their submanifolds.

Let \( \tilde{M} \) be a \((2m+1)\)-dimensional almost contact metric manifold with structure \((\varphi, \xi, \eta, g)\) where \( \varphi \) is a tensor field of type \((1,1)\), \( \xi \) a vector field, \( \eta \) is a 1-form, \( g \) is the Riemannian metric on \( \tilde{M} \), which satisfy

\[
\varphi^2 X = -X + \eta(X)\xi, \quad (2.1)
\]

\[
\varphi \xi = 0, \quad \eta \circ \varphi = 0, \quad \eta(\xi) = 1, \quad \eta(X) = g(X, \xi), \quad (2.2)
\]

and

\[
g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y), \quad g(\varphi X, Y) = -g(X, \varphi Y) \quad (2.3)
\]

for any vector fields \( X,Y \) on \( \tilde{M} \). If in addition to above relations

\[
(\tilde{\nabla}_X \varphi)Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X, \quad (2.4)
\]

then, \( \tilde{M} \) is called a Kenmotsu manifold, where \( \tilde{\nabla} \) is the Levi-Civita connection of \( g \). We have also on a Kenmotsu manifold \( \tilde{M} \)

\[
\tilde{\nabla}_X \xi = X - \eta(X)\xi \quad (2.5)
\]

for any \( X,Y \in \Gamma(T\tilde{M}) \).

Now, let \( M \) be a submanifold of a contact metric manifold \( \tilde{M} \) with the induced metric \( g \). Also, let \( \nabla \) and \( \nabla^\perp \) be the induced connections on the tangent bundle \( TM \) and the normal bundle \( T^\perp M \) of \( M \), respectively. Then the Gauss and Weingarten formulas are, respectively, given by

\[
\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \quad (2.6)
\]

and

\[
\tilde{\nabla}_X V = -A_V X + \nabla^\perp_X V, \quad (2.7)
\]

where \( \sigma \) and \( A_V \) are, respectively, the second fundamental form and the shape operator (corresponding to the normal vector field \( V \)) for the submanifold of \( M \) into \( \tilde{M} \). The second fundamental form \( \sigma \) and shape operator \( A_V \) are related by

\[
g(A_V X, Y) = g(\sigma(X,Y), V) \quad (2.8)
\]

for all \( X, Y \in \Gamma(TM) \) and \( V \in \Gamma(T^\perp M) \). If \( \sigma(X,Y) = 0 \), for each \( X,Y \in \Gamma(TM) \) then \( M \) is said to be totally geodesic submanifold.
Now, let $M$ be a submanifold of an almost contact metric manifold $\widetilde{M}$, then for any $X \in \Gamma(TM)$, we can write
\[ \varphi X = PX + FX, \tag{2.9} \]
where $PX$ and $FX$ are the tangential and normal components of $\varphi X$ respectively.

Similarly for any $V \in \Gamma(T^\perp M)$, we can write
\[ \varphi V = BV + CV, \tag{2.10} \]
where $BV$ and $CV$ are the tangential and normal components of $\varphi V$, respectively.

Thus by using (2.1), (2.9) and (2.10), we obtain
\[ P^2 = -I + \eta \otimes \xi - BF, \quad FP + CF = 0, \tag{2.11} \]
and
\[ PB + BC = 0, \quad FB + C^2 = -I. \tag{2.12} \]

Furthermore, the covariant derivatives of the tensor field $P$, $F$, $B$ and $C$ are, respectively, defined by
\[ (\nabla_X P)Y = \nabla_X PY - P\nabla_X Y, \tag{2.13} \]
\[ (\nabla_X F)Y = \nabla_X^\perp FY - F\nabla_X Y, \tag{2.14} \]
\[ (\nabla_X B)V = \nabla_X BV - B\nabla_X^\perp V, \tag{2.15} \]
and
\[ (\nabla_X C)V = \nabla_X^\perp CV - C\nabla_X^\perp V \tag{2.16} \]
for any $X,Y \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$.

Furthermore, for any $X,Y \in \Gamma(TM)$, we have $g(PX,Y) = -g(X,PY)$ and $V,U \in \Gamma(T^\perp M)$, we get $g(U,CV) = -g(CU,V)$. These show that $P$ and $C$ are also skew-symmetric tensor fields. Moreover, for any $X \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$, we have
\[ g(FX,V) = -g(X,BV), \tag{2.17} \]
which gives the relation between $F$ and $B$.

A submanifold $M$ is said to be invariant if $F$ is identically zero, that is, $\varphi X \in \Gamma(TM)$ for all $X \in \Gamma(TM)$. On the other hand, $M$ is said to be anti-invariant if $P$ is identically zero, that is, $\varphi X \in \Gamma(T^\perp M)$ for all $X \in \Gamma(TM)$. By direct calculations, we obtain the following formulas;
\[ (\nabla_X P)Y = A_{FY}X + B\sigma(X,Y) + g(PX,Y)\xi - \eta(Y)PX, \tag{2.18} \]
and
\[ (\nabla_X F)Y = C\sigma(X,Y) - \sigma(X,PY) - \eta(Y)FX. \tag{2.19} \]

Similarly, for any $V \in \Gamma(T^\perp M)$ and $X \in \Gamma(TM)$, we obtain
\[ (\nabla_X B)V = g(FX,V)\xi + A_{CV}X - PA_V X, \tag{2.20} \]
and
\[ (\nabla_X C)V = -\sigma(BV,X) - FA_V X. \tag{2.21} \]
Since $M$ is tangent to $\xi$, making use of (2.5), (2.6) and (2.8) we obtain
\[ A_V\xi = \sigma(X, \xi) = 0 \] (2.22)
for all $V \in \Gamma(T^\perp M)$ and $X \in \Gamma(TM)$.

In contact geometry, A. Lotta introduced slant submanifolds as follows:
A submanifold $M$ of an almost contact metric manifold $\tilde{M}$ is said to be slant submanifold if for any $x \in M$ and $X \in T_xM - \xi$, the angle between $T_xM$ and $\varphi X$ is constant. The constant angle $[0, \frac{\pi}{2}]$ is then called slant angle of $M$. If $\theta = 0$, the submanifold is invariant submanifold, if $\theta = \frac{\pi}{2}$ then, it is anti-invariant submanifold, if $\theta \in (0, \frac{\pi}{2})$ then it is proper slant submanifold \cite{10}.

For slant submanifolds of contact manifolds J. L. Cabrerizo et al. proved the following Lemma.

**Lemma 2.1** \cite{4}. Let $M$ be a slant submanifold of an almost contact metric manifold $\tilde{M}$ such that $\xi \in \Gamma(TM)$. Then $M$ is slant submanifold if and only if there exist a constant $\lambda \in [0, 1]$ such that
\[ P^2 = \lambda(-I + \eta \otimes \xi), \] (2.23)
moreover, if $\theta$ is the slant angle of $M$, then $\lambda = \cos^2 \theta$.

**Corollary 2.2** \cite{4}. Let $M$ be a slant submanifold of an almost contact metric manifold $\tilde{M}$ with slant angle $\theta$. Then for any $X, Y \in \Gamma(TM)$, we have
\[ g(PX, PY) = \cos^2 \theta \{g(X, Y) - \eta(X)\eta(Y)\}, \] (2.24)
and
\[ g(FX, FY) = \sin^2 \theta \{g(X, Y) - \eta(X)\eta(Y)\}. \] (2.25)

**3. Contact pseudo-slant submanifold of a Kenmotsu manifold**

In this section, we study the geometry of the contact pseudo-slant submanifolds of a Kenmotsu manifold. Necessary and sufficient conditions are given for a submanifold to be a contact pseudo-slant submanifold, contact pseudo-slant product, mixed geodesic and totally geodesic in Kenmotsu manifolds.

**Definition 3.1** \cite{9}. Let $M$ be a submanifold of an almost contact metric manifold $\tilde{M}$. $M$ is said to be contact pseudo-slant submanifold of $\tilde{M}$ if there exist two orthogonal distributions $D^\perp$ and $D_\theta$ on $M$ such that:

(i) $TM$ has the orthogonal direct decomposition $TM = D^\perp \oplus D_\theta$, $\xi \in \Gamma(D_\theta)$.

(ii) The distribution $D^\perp$ is an anti-invariant, i.e., $\varphi(D^\perp) \subset T^\perp M$.

(iii) The distribution $D_\theta$ is a slant, that is, the slant angle between of $D_\theta$ and $\varphi(D_\theta)$ is a constant.

If $\theta = 0$ then, the submanifold becomes a semi-invariant submanifold.

Let $d_1 = \text{dim}(D^\perp)$ and $d_2 = \text{dim}(D_\theta)$. We distinguish the following six cases:

(i) If $d_2 = 0$, then $M$ is an anti-invariant submanifold.

(ii) If $d_1 = 0$ and $\theta = 0$, then $M$ is invariant submanifold.

(iii) If $d_1 = 0$ and $\theta \in (0, \frac{\pi}{2})$, then $M$ is a proper slant submanifold.
(iv) If \( \theta = \frac{\pi}{2} \) then, \( M \) is an anti-invariant submanifold.

(v) If \( d_2d_1 \neq 0 \) and \( \theta = 0 \), then \( M \) is a semi-invariant submanifold.

(vi) If \( d_2d_1 \neq 0 \) and \( \theta \in (0, \frac{\pi}{2}) \), then \( M \) is a contact pseudo-slant submanifold.

If we denote the projections on \( D^\perp \) and \( D_\theta \) by \( \omega_1 \) and \( \omega_2 \), respectively, then for any vector field \( X \in \Gamma(TM) \), we can write

\[
X = \omega_1 X + \omega_2 X + \eta(X) \xi.
\]

On the other hand, applying \( \varphi \) on both sides of equation (3.1), we have

\[
\varphi X = \varphi \omega_1 X + \varphi \omega_2 X,
\]

and

\[
PX + FX = F\omega_1 X + P\omega_2 X + F\omega_2 X, \quad P\omega_1 X = 0.
\]

From which, we can easily to see

\[
PX = P\omega_2 X, \quad FX = F\omega_1 X + F\omega_2 X,
\]

and

\[
\varphi \omega_1 X = F\omega_1 X, \quad P\omega_1 X = 0, \quad \varphi \omega_2 X = P\omega_2 X + F\omega_2 X, \quad P\omega_2 X \in \Gamma(D_\theta).
\]

If we denote the orthogonal complementary of \( \varphi TM \) in \( T^\perp M \) by \( \mu \), then the normal bundle \( T^\perp M \) can be decomposed as follows

\[
T^\perp M = F(D^\perp) \oplus F(D_\theta) \oplus \mu.
\]

We can easily see that the bundle \( \mu \) is an invariant subbundle with respect to \( \varphi \). Since \( D^\perp \) and \( D_\theta \) are orthogonal distribution on \( M \), \( g(\omega_1 Z, \omega_2 X) = 0 \) for each \( Z \in \Gamma(D^\perp) \) and \( X \in \Gamma(D_\theta) \). Thus, by equation (2.3) and (2.9), we can write

\[
g(F\omega_1 Z, F\omega_2 X) = g(\varphi \omega_1 Z, \varphi \omega_2 X) = g(\omega_1 Z, \omega_2 X) = 0,
\]

that is, the distributions \( F(D^\perp) \) and \( F(D_\theta) \) are also mutually perpendicular. In fact, the decomposition (3.2) is an orthogonal direct decomposition.

Let \( M \) be a \((n+1)\)-dimensional contact pseudo-slant submanifold of \((2m+1)\)-dimensional Kenmotsu manifold \( \widetilde{M} \) with \( d_2=\text{dim}(D_\theta) = 2p + 1 \) and \( d_1 = \text{dim}(D^\perp) = q \) and

\[
\begin{cases}
e_1, e_2, \ldots, e_p, e_{p+1} = \sec \theta Pe_1, e_{p+2} = \sec \theta Pe_2, \ldots, \\
e_{2p} = \sec \theta Pe_p, e_{2p+1} = \xi, e_{2p+2}, e_{2p+3}, \ldots, e_{2p+q+1}
\end{cases}
\]

be an orthonormal basis of \( TM \) such that

\[
\{e_1, e_2, \ldots, e_p, e_{p+1} = \sec \theta Pe_1, e_{p+2} = \sec \theta Pe_2, \ldots, e_{2p} = \sec \theta Pe_p, e_{2p+1} = \xi\}
\]

are tangent to \( \Gamma(D_\theta) \) and

\[
\{e_{2p+2}, e_{2p+3}, \ldots, e_{2p+q+1}\}
\]

are tangent to \( \Gamma(D^\perp) \).

Let \( M \) be a \((n+1)\)-dimensional contact pseudo-slant submanifold of \((2m+1)\)-dimensional Kenmotsu manifold \( \widetilde{M} \) with \( \text{dim}(FD_\theta) = 2p \), \( \text{dim}(FD^\perp) = q \) and \( \text{dim}(\mu) = 2k \).
be an orthonormal basis of $T^+M$ such that. Thus the orthonormal frames of the normal subbundles $F(D_0), F(D^1)$ and $\mu$ respectively are,

$$\left\{ \begin{array}{l}
  e'_1 = \cos \theta e_1, \
  e'_2 = \cos \theta e_2, \
  e'_3 = \cos \theta e_3, \
  \ldots, \
  e'_p = \cos \theta e_p, \
  e'_{p+1} = \sec \theta \cos \theta e_{p+1}, \
  e'_{p+2} = \sec \theta \cos \theta e_{p+2}, \
  \ldots, \
  e'_{2p+2} = \sec \theta \cos \theta e_{2p+2}, \
  e'_{2p+3} = \cos \theta e_{2p+3}, \
  \ldots, \
  e'_{2p+q+1} = \cos \theta e_{2p+q+1}, \
  e'_{2p+q+2} = \cos \theta e_{2p+q+2}, \
  \ldots, \
  e'_{2p+q+k+3}, \ldots, e'_{2p+q+2k+1}
\end{array} \right\},$$

and

$$\left\{ \begin{array}{l}
  e''_{2p+2}, e''_{2p+q+3}, \ldots, e''_{2p+q+k+2}, \varphi e''_{2p+q+k+2}, \varphi e''_{2p+q+k+3}, \ldots, \varphi e''_{2p+q+2k+1}
\end{array} \right\}. $$

Hence, we can easily see

$$g(e_i, e_j) = 0, \quad \text{for} \quad 1 \leq i \leq 2p+1, \quad 2p+2 \leq j \leq 2p+q+1,$$

$$g(e_i, e'_j) = 0, \quad \text{for} \quad 1 \leq i \leq 2p+q+1, \quad 1 \leq j \leq 2p+q+k+1,$$

$$g(e'_i, e'_j) = 0, \quad \text{for} \quad 1 \leq i \leq 2p, \quad 1 \leq j \leq 2p+q+1,$$

$$g(e'_i, e''_j) = 0, \quad \text{for} \quad 1 \leq i \leq 2p, \quad 2p+2 \leq j \leq 2p+q+k+1,$$

$$g(e_i, \varphi e'_j) = 0, \quad \text{for} \quad 1 \leq i \leq 2p+1, \quad 2p+q+k+2 \leq j \leq 2p+q+2k+1$$

and

$$g(e'_i, \varphi e''_j) = 0, \quad \text{for} \quad 1 \leq i \leq 2p+q+1, \quad 2p+q+k+2 \leq j \leq 2p+q+2k+1.$$

\section*{Definition 3.2.} A contact pseudo-slant submanifold $M$ of Kenmotsu manifold $\tilde{M}$ is said to be $D_\theta$-geodesic (resp. $D^1$-geodesic) if $\sigma(X,Y) = 0$ for all $X, Y \in \Gamma(D_0)$ (resp. $\sigma(Z,W) = 0$ for all $Z, W \in \Gamma(D^1)$). If for all $X \in \Gamma(D_0)$ and $Z \in \Gamma(D^1)$, $\sigma(X, Z) = 0$, the $M$ is called mixed geodesic submanifold.

\section*{Theorem 3.3.} Let $M$ be a proper contact pseudo-slant submanifold of a Kenmotsu manifold $\tilde{M}$. If $F$ is parallel, then either $M$ is a mixed-geodesic or an anti-invariant submanifold.

\textbf{Proof.} From (2.19), we obtain

$$C\sigma(X,Y) = 0$$

for any $X \in \Gamma(D_0)$ and $Y \in \Gamma(D^1)$. Replacing $X$ by $Y$ in (2.19) and taking into account of $F$ being parallel, we have

$$C\sigma(X,Y) = \sigma(Y, PX) - \eta(X)FY = 0.$$

Thus we have

$$\sigma(Y, PX) - \eta(X)FY = 0,$$

which is equivalent to

$$\sigma(Y, P^2X) = -\cos^2 \theta \sigma(X,Y) = 0.$$

This proves our assertion.
**Theorem 3.4.** Let $M$ be a proper contact pseudo-slant submanifold of a Kenmotsu manifold $\tilde{M}$. If $B$ is parallel, then either $M$ is a $D^{\perp}$-geodesic or an anti-invariant submanifold of $\tilde{M}$.

**Proof.** If $B$ is parallel, then making use of (2.20), we obtain
\[ g(FZ, FY)\xi + AC_{FY}Z - PA_{FY}Z = 0 \]
for any $Y, Z \in \Gamma(D^{\perp})$, which implies that
\[ PA_{FY}Z = 0. \]
This tell us that $M$ is either anti-invariant or $A_{FY}Z = 0$. So we obtain
\[ g(\sigma(Z, W), FY) = 0 \]
for any $W \in \Gamma(D^{\perp})$. Also by using (2.20), we conclude that
\[ g(A_{CV}Z, Y) - g(PA_{V}Z, Y) = g(\sigma(Y, Z), CV) = 0 \]
for any $V \in \Gamma(T^{\perp}M)$. This tells us that $M$ is either $D^{\perp}$-geodesic or it is an anti-invariant submanifold.

**Definition 3.5.** Given a proper contact pseudo-slant submanifold $M$ of a Kenmotsu manifold $\tilde{M}$, if the distributions $D_{\theta}$ and $D^{\perp}$ are totally geodesic in $M$, then $M$ is said to be contact pseudo-slant product.

**Theorem 3.6.** Let $M$ be a contact pseudo-slant submanifold of a Kenmotsu manifold $\tilde{M}$. Then $M$ is a contact pseudo-slant product if and only if the shape operator of $M$ satisfies
\[ A_{F_{D^{\perp}}}PD_{\theta} = A_{F_{PD^{\theta}}}D^{\perp}. \] (3.3)

**Proof.** By using (2.18), we have
\[ \nabla_{X}PY - P\nabla_{X}Y = A_{FY}X + Bh(X, Y) + g(PX, Y)\xi - \eta(Y)PX \]
for any $X, Y \in \Gamma(D_{\theta})$. This implies that
\[ g(\nabla_{X}PY, Z) = g(A_{FY}X, Z) + g(Bh(X, Y), Z) \] (3.4)
for any $Z \in \Gamma(D^{\perp})$. Replacing $Y$ by $PY$ in (3.4) and taking into account of (2.24), we obtain
\[ \cos^{2}\theta g(\nabla_{X}Y, Z) = g(A_{FZ}PY - A_{F_{PY}}Z, X). \] (3.5)
Also, from (2.13), we have
\[ -P\nabla_{Z}U = A_{FU}Z + Bh(Z, U) \]
for any $U, Z \in \Gamma(D_{\theta})$, from which
\[ -g(P\nabla_{Z}U, PX) = g(A_{FU}Z, PX) + g(Bh(Z, U), PX), \]
that is,
\[ -\cos^{2}\theta g(\nabla_{Z}U, X) = g(A_{FU}PX - A_{FPX}U, Z) \] (3.6)
for any $X \in \Gamma(D_{\theta})$. Equations (3.5) and (3.6) imply that (3.3).\qed
Example 3.7. Let $M$ be a submanifold of $\mathbb{R}^7$ defined by equation

$$M = x(u, v, s, t, z) = (u, v, v \sin \theta, v \cos \theta, s \cos t, -s \cos t, z).$$

We can check that the tangent bundle of $M$ is spanned by the tangent vectors

$$e_1 = \frac{\partial}{\partial x_1}, \quad e_5 = \xi = \frac{\partial}{\partial z},$$

$$e_2 = \frac{\partial}{\partial y_1} + \sin \theta \frac{\partial}{\partial x_2} + \cos \theta \frac{\partial}{\partial y_2},$$

$$e_3 = \cos t \frac{\partial}{\partial x_3} - \cos t \frac{\partial}{\partial y_3},$$

$$e_4 = -s \sin t \frac{\partial}{\partial x_3} + s \sin t \frac{\partial}{\partial y_3}.$$

For the almost contact metric structure $\varphi$ of $\mathbb{R}^7$, whose coordinate systems $(x_1, y_1, x_2, y_2, x_3, y_3, z)$, choosing

$$\varphi\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial y_i}, \quad \varphi\left(\frac{\partial}{\partial y_j}\right) = -\frac{\partial}{\partial x_j}, \quad 1 \leq i, j \leq 3$$

then we have

$$\varphi e_1 = \frac{\partial}{\partial y_1}, \quad \varphi e_5 = 0,$$

$$\varphi e_2 = -\frac{\partial}{\partial x_1} + \sin \theta \frac{\partial}{\partial y_2} - \cos \theta \frac{\partial}{\partial x_2},$$

$$\varphi e_3 = \cos t \frac{\partial}{\partial y_3} + \cos t \frac{\partial}{\partial x_3},$$

$$\varphi e_4 = -s \sin t \frac{\partial}{\partial y_3} - s \sin t \frac{\partial}{\partial x_3}.$$

By direct calculations, we can infer $D_\theta = S_p\{e_1, e_2\}$ is a slant distribution with slant angle $\cos \alpha = \frac{g(e_2, \varphi e_1)}{\|e_2\|\|\varphi e_1\|} = \frac{\sqrt{2}}{2}, \quad \alpha = \frac{\pi}{4}$. Since $g(\varphi e_3, e_i) = 0, i = 1, 2, 4, 5$ and $g(\varphi e_4, e_j) = 0, j = 1, 2, 3, 5$ orthogonal to $M$, $D_\perp = S_p\{e_3, e_4\}$ is an anti-invariant distribution. That is, $M$ is a 5-dimensional proper pseudo-slant submanifold of $\mathbb{R}^7$ with its usual almost contact metric structure.

References


